Techno-economic evaluation of flexibility services provision via residential li-ion batteries co-located with rooftop solar generation

By

Carmine Calabrese

Submitted to the Department of Electrical Engineering, Electronics, Computers and Systems
In partial fulfillment of the requirements for the degree of
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Abstract

In this dissertation an analysis was done on the potentiality flexibility services for domestic loads on a low voltage power system. Flexibility of an electric power system is the range of possible changes in the generation or the load in order to adapt the system to the forecasted changes in load and generation patterns. Flexibility allows the electric power system to gain stability and reliability since the system will be able to adapt and be in balance in real time even with a big share of renewable energy sources in the energy production mix. The purpose of this dissertation is to test and prove the impact of the optimization of domestic batteries for flexibility services, using mixed integer linear programming to optimize dwellings net demand according to different optimization scenarios. This study models and solves a system of 55 houses with PV and batteries under different control scheme (optimization scenarios) both individually optimized and as an aggregated load to get the best possible economic outcome. The results of the optimization were then the input for the power flow analysis over the whole network, to check its stability. The first step done was to build a simple model for a dwelling, without even considering the power transfer efficiency or the optimization in an economic point of view. The model then grew in complexity and its objective became the best economic outcome. Different scenarios were then built for the household model: a fully optimized system with a time of use tariff for the imported energy, the same system but with a flat tariff, a system where the battery could only charge from the surplus of solar production, a system where all the dwellings were optimized together to cope with grid stability issues and the base case system where no battery or PV is considered. The optimization scenario that offers the best economic outcome is the one where the import power is bought through a time of use tariff, power imported from the grid can be used to charge the batteries and power export to the grid at a fixed price is allowed. The system that has the lowest cost for the users is the individually optimized system. In conclusion once the power flows were performed though it was possible to understand that the system that provides the best economic outcome is not the best system from a power grid stability point of view, it actually is the system with the worst power flow outcome in terms of voltage drop, reaching a voltage that was lower than 0.94pu in some of the buses at some of the timesteps. This means that according to UK's standards the grid would be not considered stable, nevertheless the aggregated system where all the dwellings are optimized at the same time to get the best economic outcome, and as well to get a stable system power flow wise is the second best option on the economic point of view and it has no voltage drop below 0.94pu, thus it is considered a stable system in the UK, and the economic difference is not too big. When comparing the results also with the base case it is easy to notice that the impact of DSR on a system is proved, and furthermore it is possible to prove the importance of the aggregation of loads on a low voltage, residential area, network. But in order to have people participating in DSR and willing to cooperate for the power network stability some economic incentives are needed.

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1 Chapter One – Introduction

This chapter will give a brief introduction to clarify what flexibility and demand side response is, going through a literature review to give an idea of the state of the art of DSR and flexibility, focusing on the general status in Europe, mainly in Italy, Germany and United Kingdom.

1.1 Flexibility definition

Flexibility of an electric power system is the range of possible changes in the generation or the load in order to adapt the system to the forecasted changes in load and generation patterns.[1] Flexibility allows the electric power system to gain stability and reliability since the system will be able to adapt and be in balance in real time even with a big share of renewable energy sources in the energy production mix. Flexibility can be sold as a set of different services – i.e. DSR (demand side response) – on the electricity market.

1.2 Why flexibility is needed

Nowadays in the EU due to environmental issues, such as the greenhouse gas emissions, there are some policies to increase the green energy generation, reducing in the meantime the usual approach to the generation, of having large capacity power plants fired by fossil fuels. Some goals are as well set, to be reached step by step, until 2050 from the EU, and internally for some of the countries. Due to the increase of distributed generation, mainly renewable generation (solar and wind) which are not predictable and not dispatchable, a system that can adapt its load to the changes in the generation at all time is needed.[2] Non-dispatchable generation means that we cannot have a full control on the generation pattern, we cannot turn on and off the generators in order to meet the fluctuation in the electricity needs.[11] Adding flexibility on the grid, that can be provided in different ways: with demand side response (DSR), with energy storage (i.e. batteries, or hydro pumping for storage) and with interconnections (mainly with other countries) gives the system the possibility to adapt the load to the generation instead of adapting the generation to the load. Another way to add flexibility to the grid could be the addition of small and fast responding generators powered by fossil fuels, with low inertia, thus a fast response. One of the problems added to the grid with the increase of renewable generation is the lack of inertia (the ability of the system to remain unchanged unless actions from the outside force it to change) due to the decrease in big capacity power plants with heavy rotating generators. Inertia is one of the things that keeps the system stable as the rotating machines that generate electricity have the possibility to ramp up or down their output with a response which is not too slow if the variation is small enough; in the case of solar there is no real control on the output power because it depends on whether the panel is hit by the sun or not.[10][11] Nevertheless, flexibility is important even in systems with low penetration of non-dispatchable generation and with big inertia installed: the ability of a load to be shiftable, and the chance to have forecasts on the use of the loads in the system provide the possibility of deferring the installation of new capacity on the network and to use the grid in a more efficient way, leading to economical savings. Out of the possible ways to provide flexibility in a system, this dissertation will focus mainly on the DSR, analyzing the penetration of DSR in the EU and some of the European countries systems and markets, then focusing on the UK; an analysis of what flexibility in general could bring to the UK system (and an analysis of the system itself) will follow.

1.3 Demand side response definition

Changing in the demand to respond, thus adapt, to some inputs that could be the electricity price or the power generation level. There are different approaches to this: with some incentives on the bills of the consumer to incentivize them to actively participate in the DSR adapting their load to the electricity tariffs avoiding the highest prices peaks; or it can be done remotely, controlling the loads of the users without the need for them to be active and to do anything to adapt, with the least possible discomfort for the customers, the load to the economic needs of who is in charge of controlling those loads. The person or company (mainly a company, an aggregator) that oversees to remotely control households' loads will give money to pay back the loss in comfort to the costumer, but still will provide a service to the DSO (distribution system operator), TSO (transmission system operator), and the network in general that will grant them an income.

1.4 Main players in electricity markets

- Generators: the main energy producers, the holders of the power plants, nowadays mostly fossil fuels fired ones;
- Retailers: who is in charge to bid in the market to buy the electricity and then sell it to the consumers;
- BRPs: balance responsible parties, can be a producer, a major consumer, a supplier or a trader, they are responsible to keep in balance the load and the generation, they pay imbalance fees when imbalances occur; [13]
- TSO: transmission system operators, the company(\ies) in charge of the HV transmission level grid, that interfaces the production with the distribution;
- DSO: distribution system operators, the company(\ies) in charge of usually MV and LV distribution network, they connect the consumers with the transmission, moreover some generators at distribution level can be part of the network;
- Aggregators: companies that gather costumers together in order to be able to manage a minimum market required power of load to offer flexibility services (DSR) and sell these services in the appropriate markets; [4]
- Costumers: final users that pay the retailers to use the electricity; in case of aggregators in the markets are also the ones who benefit from an aggregator to both reduce their bills and get money for the flexibility service offered.

All these players take part every day to the electricity market, and the penetration of DSR and aggregators is dependent on the rules of the market that vary from country to country. The major problems to be solved in order to integrate aggregation in the system are:

- An aggregator can reduce the demand of energy, thus the need for generation, and this way the generators would have losses on their income;[2]
- An aggregator by shifting or shedding some loads can cause imbalances in the BRPs forecasting that will make them pay for those wrong forecasts.[2]

These two main issues cause conflict of interests between the parties and it is going to be the country, with their regulations for a fair electricity market, to come over these problems.

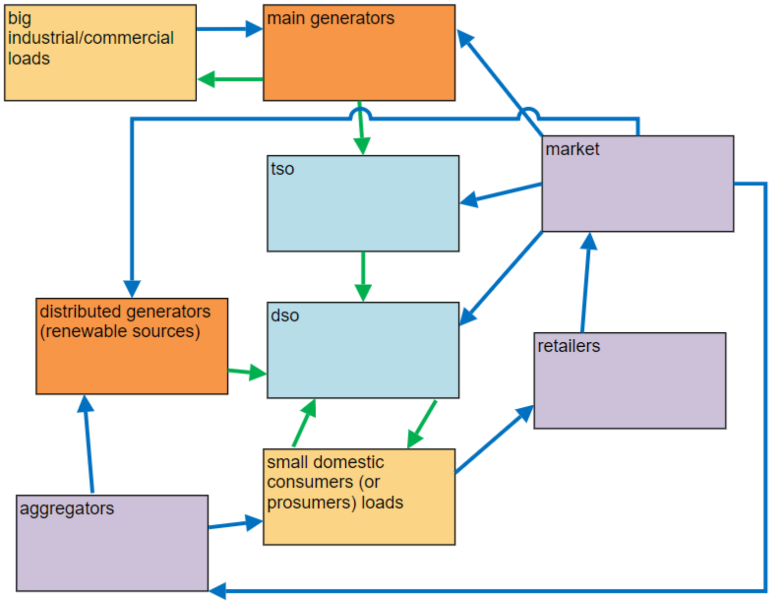


Figure 1 Electricity system and markets players [8]

In the schematics the green arrows are the energy flow, while the blue ones show the financial flow. Typically, the flow used to be from the top of the ladder to the bottom, but now there are possibilities to have bidirectional distribution network system, due to the distributed generators and the prosumers that can inject energy in the grid, while the consumers can anyway provide flexibility services but not actual energy. The prosumer/consumer figure is raising its importance, and it is expected for this to continue in the future, when the electric load of the dwellings will grow even more because of electric heating and electric vehicles replacing the gas heating system and petrol fired cars. Money is flowing through the electricity market that interconnects the other players, it manages all the money flow: incoming money from the retailers that get paid by the costumers, money from the retailers to the generators and money to TSOs and DSOs for the network usage fees which are their income and as well money paid to the aggregators for the flexibility services offered to the grid. The aggregators will buy this flexibility from the prosumers and eventually the distributed generators, making a pool of capacity that can be sold in the appropriate electricity markets. Other than this there must be a lot of interaction between the players, forecasting, metering and coordination/negotiation are part of the everyday energy markets. In the schematics the BRPs are missing because as mentioned before they can be independent players but usually they work in cooperation with some of the players that already take part to the market, being just a service that one of these players offer, selling it to the markets, with the risk of paying fees in case of imbalances though. The industrial loads in the end can operate separately from the market buying their electricity directly from the producer and being physically connected either to them or most probably direct to the HV transmission system. The next paragraph will explain what happens in the market, that here is simplified and represented by one only block, while in reality it is much more complex and divided in several categories, according both to the type of offered service and to the time window when this service is traded.

1.5 Electricity market

The electricity market has a complex structure and complex rules as well, a full description of the laws and of the market is not in the scope for this work, so a brief description of the main features and laws to the markets will be enough to demonstrate what DSR and flexibility in general can bring to the electricity market. First thing to consider is that the electricity market is a competitive deregulated market where prices are "decided" at all time by the market itself and there is competition among the players. The electrical market has some categories, depending on the offered service and on the time window when the electricity is traded. According to the service sold there are three categories of markets:

- Capacity market: TSO must ensure that there is enough capacity for a reliable system operation at competitive prices;[19]
- Energy market: the actual wholesale electricity market where bids and auctions take place for buying and selling energy;[19]
- Ancillary services market: market for any service in support of power system operation (the ancillary services will be discussed later).[19]

According to the time windows for the operations in the market there are four categories of markets:

- Futures (or forward) market: a market that is open long time before the delivery of the service and allows participants to hedge to lower the risks that a real time-market can cause, bids are placed at the forecasted price for the moment of the expected delivery of the service;
- Day-ahead market: a market that is open up to 11am of the day before the actual delivery day, in order to try to balance in advance with better forecasting than the ones available when trading in the forward market;
- Intraday market: a continuous market almost real-time with the delivery of the service, the gate closes one hour before each half an hour settlement period where the service is physically delivered, this market provides the players with the last chance to be in balance with load and production;
- Balancing market: the market in which all the imbalances must be balanced by the TSO.

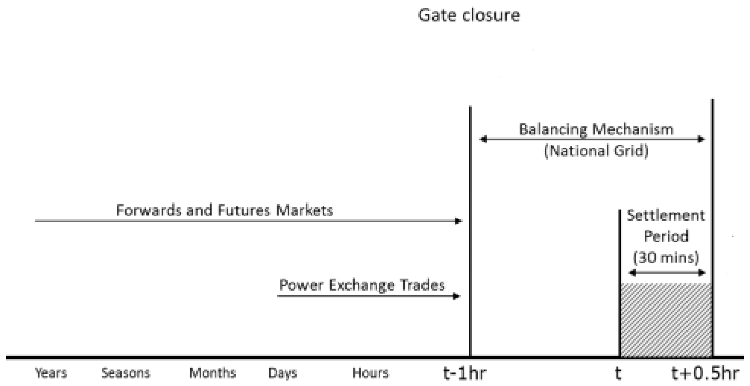


Figure 2 UK electricity market timeline [20]

In the UK's electricity market over 90% of the trades takes place as bilateral contracting in the futures and forwards market, 5% of the demand is traded as power exchanges within 48 hours before the service delivery, while the balancing services operated by the TSO account for the 2% of the total energy demand. In the UK, markets are oriented in a way to incentivize the parts to play on the competitive wholesale market and balance the load and generation, trying to minimize the TSO balancing intervention. Generators and suppliers provide notifications of their generation and demand before 11.00 am for the next operational day (05:00-05:00) to the TSO in order to inform the TSO of their plans for the dispatch of the electricity and allow it to eventually take part to the market and resolve possible transmission constraints. At gate closure, generators and suppliers must provide final notifications of their expected generation and demand, which may or may not be the same of their final contracted positions that have to be notified as well to the TSO at the same gate closure time. They may also bid on the variation of the generation or supply that they might do in the half-hour settlement period. Besides the specific UK and electricity market there are some common rules that regulate each competitive commodity market and drive the price for the commodity according to the demand and the supply. The market is obviously affected by the behavior of the buyers and the sellers, the main driver for them to increase the demand, or the production, of a commodity is the price of the commodity itself. The demand against the price is represented by an inverse function graph, it is possible to notice from the graph the dependency of the demand on the price of the commodity itself, and that people will increase their demand as the commodity decreases its price. The supply against price on the other hand will be higher when the price will be higher and this is a direct relationship between supply and price that is represented on the next graph showing supply against price behavior; this happens because the market price has to be high enough as compared to the production price, in order to make it worth it for the producer to increase the production, thus the supply, of the commodity. A competitive market (a market in which buyers and sellers’ actions cannot influence the price of the commodity) is in equilibrium when the quantity that the consumers are willing to buy and the quantity that the suppliers are willing to produce match, this is represented in the next graph, that shows how to get the equilibrium point and find the right price for a certain demand and supply, this is obtained overlapping the two previous graphs; the point of the interception of the curves is the market equilibrium point or market clearing price. In this scenario if any external action happens that prevents the price to be set at the equilibrium price there might be an excess of supply or an unsatisfied demand; considering that a competitive market is free to operate and the price is actually set by the market itself from time to time, this problem will be automatically solved: if the price settles at a higher price than the equilibrium point there will be a decrease in the demand which will cause a surplus of production (excess of supply scenario) , the suppliers in this case will lower the price of the commodity in order to reach the equilibrium and be able to sell what they produced; on the other hand if the price settles at a lower price than the equilibrium point one, then there will be a decrease in production (unsatisfied demand scenario) because it will not be worth to produce for a low price, in this case price will increase to the equilibrium point in order to satisfy demand and production. In conclusion in a competitive market, a price equal to the equilibrium point will give the biggest advantage both to suppliers and consumers at the same time. Even though these laws are for any competitive market, because the electricity markets are deregulated competitive ones, these general laws apply to them, in order to settle the price, at all times, for the electricity.

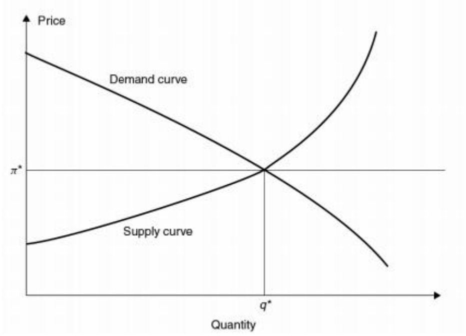


Figure 3 Market equilibrium point [21]

The reason why the electricity market has high price volatility, thus playing on this market may lead to high investment risks is that the electricity demand, and nowadays production as well, are affected by many agents, such as the weather or a fault that could occur in the generation, transmission or distribution network that make the accurate forecasting of demand and production (that, in the electricity markets must always match to be in a balance situation) hard to be performed; this can lead to sudden imbalance that will affect then the price of the electricity. Because of this reason a risk assessment strategy when taking part to the electricity market needs to be done; there are different approaches and strategies in order to take part to the market with the lowest possible risk in the investment, to make profits and prevent losses. One of the most important strategies is the hedging, which is the chance to buy or sell a product in advance, using the forecasted price expected for the contracted delivery time of the service. This may lead to have an economic income to the seller or the buyer depending on the actual market price for the service while it is being delivered, nevertheless even if the costumer might have advantages in buying in advance at a possible lower price, there are many risks: the amount of energy bought in advance could not match the real time demand. So hedging, to be successful needs accurate forecasting, but anyway until the moment the service will be delivered (actually until the gate closure time) there is always the chance to either buy more energy or sell the expected energy surplus at a different market price than when the first bid was placed, causing again the possibility of an income for the customer or the seller, depending on the changes in the market price. Whoever is a player on the market must consider the risks when trading, thus they should perform a risk assessment, for instance as presented in [22]; the risk assessment is out of the focus of this work so it will not be explained in the details, just a definition of two possible strategies, value at risk and conditional value at risk (VaR and CVaR) taken from [22] will be given, the full explanation with a practical example is in the referenced paper. “VaR is a methodology developed by the financial industry to provide quantification for a company’s portfolio’s exposure to risk. VaR summarizes the expected maximum loss over a target horizon within a given confidence interval. For example, a company with current day stock valuation of $100M could state that with 95% certainty under normal market conditions, the valuation would lose at most U.S.$ 11M by the end of the next market day” [22]. “Conditional Value-at-Risk is also known as the mean excess loss. CVaR overcomes at least one of VaR’s limitations: measuring the potential losses exceeding VaR. By calculating the mean of the loss exceeding the VaR value, CVaR provides a better indication of the potential losses exceeding the assumed confidence level. CVaR is defined as the conditional expectation of losses given that the loss exceeds a threshold value (VaR value). For example, a 95% confidence CVaR value provides the mean of the expected losses for the potential loss values that exceed the 95%-VaR value” [22]. So, in the end, the entrance of DSR in the electricity market could affect the price volatility and the strategies of the market’s players and change the whole approach to the electricity market. This happens because it is possible to have a better load forecasting and the chance to adapt the load to an eventually bad long-term forecasting instead of buying more energy or selling the surplus energy. Price volatility will still be affecting anyway the electricity market because the generation mix for electricity is going to be more and more weather dependent with the increase in the integration of the renewable energy sources, and anyways even if DSR will see a high deployment in the future, not all the loads, and not at all times are available for DSR. Thus, DSR can mitigate price volatility, help bidding in the market in advance with possible non very accurate load forecast that can be smoothed out, or eventually be an important resource in the ancillary services and balancing markets.

1.6 Status of DSR in Europe

Europe is undertaking a process to support DSR since few years ago, with some directives that are now transposed in each of the member states to aware the governments of DSR and to help DSR entrance in the EU countries’ markets. This process though is quite slow, especially in some of the states and is quite advanced in others, the Energy Efficiency Directive (2012/27/EU) requires member states to:

(Art. 15.4)

- “Ensure the removal of incentives in transmission and distribution tariffs that are detrimental to the overall efficiency of the generation, transmission, distribution and supply of electricity or those that might hamper participation.”
- “Ensure that network operators are incentivized to improve efficiency in infrastructure design and operation, and that tariffs allow retailers to improve consumer participation in system efficiency, including DSR, depending on national circumstances”

The directive establishes consumer access as well to energy markets, either individual or through aggregation:

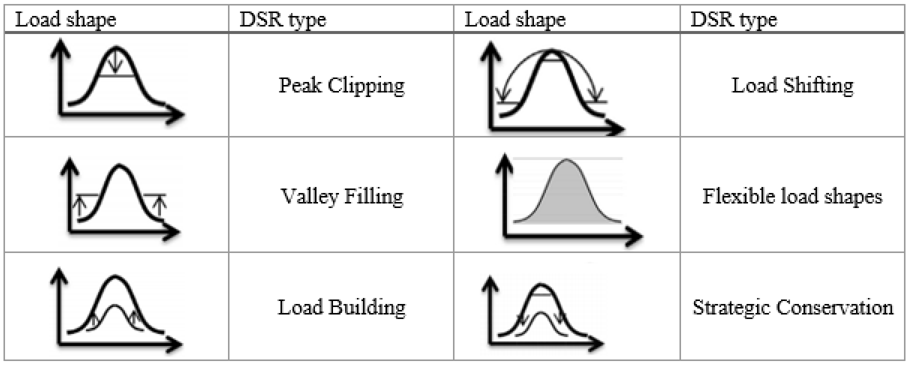
(Art. 15.8)

- “Member states shall ensure that national regulatory authorities encourage demand side resources, such as DSR, to participate alongside supply in the wholesale and retail markets”
- “Subject to technical constraints inherent in managing networks, member states shall ensure that TSOs and DSOs, in meeting requirements for balancing and ancillary services, treat DSR providers, including aggregators, in a non-discriminatory manner, on the basis of their technical capabilities”
- “Member states shall promote access to and participation of DSR in balancing, reserves and other system services markets, inter alia by requiring national regulatory authorities […] in close cooperation with demand service providers and consumers, to define technical modalities for participation in these markets on the basis of the technical requirements of these markets and the capabilities of DSR. Such specifications shall include the participation of aggregators”

This directive’s transposition period ended on the 5th of June 2014, nevertheless there is still a lot of misunderstanding about the DSR, and the system is still far away from the one described in the directive, in some of the countries the transposition happened by name but not by facts. Integration of DSR in Europe is different from country to country, and the Europe itself can be split in at least three main groups according to the status of DSR deployment:

- States where DSR reforms have still to take place seriously. In these countries DSR is legal but the structure of the system has not changed making it impossible to DSR to take part to the market. The role of the aggregators is not well defined, Italy is one of the countries belonging to this group, even though Italians officially declared that they are aware of the problem and that changes in the system are in the plans for the near future;[3]
- States that are in process of enabling DSR but only through the retailers. The aggregators can only be service providers to retailers and not be independent parties providing offering to the costumers. In this way market offerings will be the one positive for the retailers, and this means that they might not be concerning about the consumers' benefit. Germany is part of this group of countries, though it is going through a regulatory review and things may change in 2017-18;[3]
- States that enable both DSR and independent aggregation. These markets have made progresses into adjusting the market entry requirements to facilitate consumers participation and indeed, even if further work needs to be done, in these countries the MW of DSR more than tripled between 2013-15. UK is part of this group.[3]

An important figure in this process and in the markets is the one of the aggregators, since the minimum capacity to enter in some countries' markets is high, for the small consumers such as domestic ones the aggregator can help the entrance in the markets. "The aggregator is a third-party company specialized in electricity demand side participation, in practice aggregators contract with the individual demand sites, whether they're industrial, commercial or residential consumers, and aggregate them to operate as a single DSR provider to the markets."[4] The aggregator, differently from the consumers (at least the residential ones) takes all the risks (and possible advantages) of being exposed directly to the market prices and placing bids on the market, thus being exposed to the fluctuation of the prices of electricity and the peaks in the prices. The aggregator anyways is not object of charges for the use of the network as it is not producing or consuming any energy; it is a figure in between the costumers and the market, that buys flexibility services from the costumers and sells them in the appropriate markets in different ways depending on what it is allowed to do. Before analyzing the status in three European countries (Italy, Germany and the UK) here follow some details about what services to the grid DSR can provide and how. DSR, as aforementioned, can be directly activated changing the habits of the costumers that proactively react to some tariffs or can be remotely activated using some communication infrastructure, metering and forecasting by a third party which is the aggregator.[6] About what it can provide, it can change the shape of the load in several ways, adapting it to real time grid's needs, and this flexibility in order to be placed on the market needs forecasting and prediction. Having the possibility of changing the load profile not only gives flexibility and reliability to the grid, but it can make it possible to avoid congestion on the network during peak times causing less stress to the equipment and can provide frequency response as well in order to keep the system stable. There are other options to solve the issue with the frequency response (keep the load and the generation balanced in order to keep the frequency stable, otherwise causing failures on the grid) but considering the economic and environmental costs, and as well the speed of the response, DSR is probably the best way to adapt the load to the generation. Enabling DSR in the system can help the introduction of non-dispatchable renewable sources of energy, reducing the use of the fossil fuel source of energy for the peak demand and spinning reserve as well, since the load can be smoothed out and the frequency response handled via DSR. Furthermore, the use of the system could become more efficient and avoiding the peak congestions could allow to defer or even avoid some investments for improving the network, making DSR becoming an important resource even for dispatchable generation when it comes to plan the future investments for the grid. Moreover, adapting the demand to the generation could reduce the general cost of electricity supply mitigating price volatility. The following graphs will show how DSR can act on the load profiles.



Load shape	DSR type	Load shape	DSR type
	Peak Clipping		Load Shifting
	Valley Filling		Flexible load shapes
	Load Building		Strategic Conservation

Figure 4 DSR possible services [7]

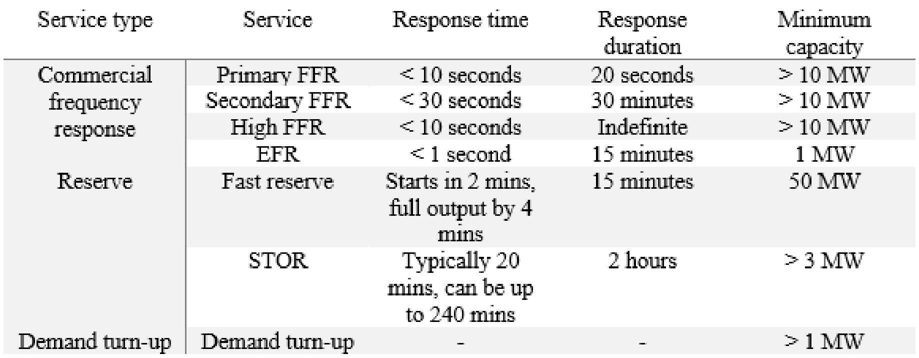
When a consumer becomes a prosumer (for instance with the use of a PV panel plus a backup battery and eventually owning an EV as well) and sells its flexibility to an aggregator, that is when DSR is the most effective because not only load can be increased or decreased according to the needs of the system, but also there can be injection of energy in the grid whether this is required, and anyways the addition of the load can still become stored energy into the battery of the EV or the backup battery of the PV, while energy can be stored in heating (of the house for instance, even better with an electric hot water tank) when there is no EV or battery storage on the consumer side, and the heating system of the dwelling is electric.

1.7 Ancillary services

Before the analysis of the DSR status in Italy, Germany and the UK a brief description of what ancillary services are and how they work in the UK is provided in order to clarify what services can be provided by DSR in general. “Ancillary Services refers to a range of functions which the system operator (SO) uses to ensure system balancing, stability and security” [12]. In the UK there are several kinds of ancillary services, but the most relevant ones concerning DSR are the frequency response, reserve and demand turn-up. In the UK, balancing services are provided alongside with ancillary services and some of the ancillary services are for balancing mechanism units only or are divided into services offered by balancing mechanism units and services offered by non-balancing mechanism units.

- Frequency response: the TSO must control the frequency to a level of +/- 1% of nominal (50Hz) value; it can be dynamic or non-dynamic, dynamic frequency response is a continuously provided service used to manage the normal second-by-second changes on the system, Non-dynamic frequency response is typically a discrete service triggered at a defined frequency deviation. Frequency response is provided either via commercial firm frequency response (FFR), or enhanced frequency response (EFR), or via non-commercial mandatory frequency response; our focus will be on firm frequency response mainly and enhanced frequency response as well.

  - FFR is open to both balancing and non-balancing mechanism (BM) providers, including DSR and aggregators that are pre-qualified and can provide a minimum of 1MW capacity; providers can offer other balancing services out of the FFR windows; FFR is divided in three sub-categories based on response time:[14]
    - Primary response: response provided within 10 seconds for maximum 20 seconds;
    - Secondary response: response provided within 30 seconds for maximum 30 minutes;
    - High frequency response: response provided within 10 seconds for undefined maximum duration.
  - EFR is open to both BM and non-BM providers including aggregators for a minimum of 1MW capacity and maximum of 50MW, they must be ready to respond within one second to frequency variation and operate in frequency sensitive mode at the start of their tendered window; EFR is dispatched automatically.[15]
- Reserve: the TSO needs the possibility to source extra power either increasing the generation or decreasing the demand in order to cope with unforeseen demand increase or unavailable generation; there are a number of reserve services, but the most interesting ones are the fast reserve and the STOR (short term operating reserve).
  - Fast reserve provides rapid and reliable delivery of active power, this service can be provided by both BM and non-BM providers, including aggregated DSR and storage providers, they can offer other balancing services outside the fast reserve windows; the technical requirements are that the active power delivery must start within 2 minutes of the instruction, the reserve should be sustainable for a minimum of 15 minutes and the minimum capacity is 50 MW;[16]
  - STOR occurs when it is economic to access to sources of extra power to help manage actual demand At certain times of the day (typically morning and evening peak windows), it can be provided by both BM and non-BM participants, including aggregators, it is possible to provide other balancing services outside the STOR windows, it needs a minimum capacity of 3 MW and response must be sustained for a minimum of two hours.[17]
- Demand turn-up: it is a service that encourages large energy users and generators to either increase demand or reduce generation at times of high renewable output and low national demand, it is open to any technology that has this flexibility including generator, storage, and aggregate loads as well; it is not possible to provide any other balancing service alongside with demand turn-up; the minimum capacity is 1MW and the minimum capacity of aggregated loads is 0.1MW. [18]



Service type	Service	Response time	Response duration	Minimum capacity
Commercial frequency response	Primary FFR	< 10 seconds	20 seconds	> 10 MW
	Secondary FFR	< 30 seconds	30 minutes	> 10 MW
	High FFR	< 10 seconds	Indefinite	> 10 MW
	EFR	< 1 second	15 minutes	1 MW
Reserve	Fast reserve	Starts in 2 mins, full output by 4 mins	15 minutes	50 MW
	STOR	Typically 20 mins, can be up to 240 mins	2 hours	> 3 MW
Demand turn-up	Demand turn-up	-	-	> 1 MW

Figure 5 Ancillary services market in the UK [12]

A short summary of the status of DSR in Italy, Germany and the UK, and how to possibly improve the system will follow. When analyzing the situation in three European countries the focus will be on the integration of DSR in the markets, including balancing market (to cope with the imbalance in the system), ancillary services market and wholesale energy market.

1.8 DSR in Italy

A decrease of consumption in parallel with a rapid growth in use of renewable generation is taking place into Italian electricity system, gas and hydro are Italy's main resources for flexibility while frameworks for DSR participation in any market are still not in place. Only the interruptible contracts program is the exception but is a dedicated DSR program out of the balancing market. The minimum size to take part is 1MW but aggregation is not allowed, payments are attractive and related to availability rather than utilization as the program has been called few times in the last decade. Italian NRA (national regulatory authority) in the 2015-18 period included in the strategic guidelines the evaluation of DSR mechanism for market development.

A summary of the situation in Italian electricity markets concerning DSR follows:

- Primary frequency control is an uncompensated service and is mandatory for non-intermittent generators bigger than 10MW;
- Secondary frequency control and tertiary reserve are paid but not accessible to load curtailment;
- For the interruptible contracts, aggregation is not allowed;
- In the spot market consumers should belong to the same market zone and bid a minimum of 1TWh; participation fee is as well very high: 7,500 euros for registration to the platform plus 10,000 euros as yearly fee.

TSO in Italy is concerned by increasing issues with frequency and system security and is seeking for DSR, regulator is aware as well that the rules must change.

The main market barriers for DSR in Italy are:[3]

- DSR and aggregation are not enabled within the ancillary services, balancing, or wholesale markets;
- The interruptible contracts for large consumers cannot be considered a DSR program and is barely ever activated.

The main market enablers for DSR in Italy are:[3]

- The largest consumers can act directly and receive a payment for contracting with TSO for interruptible contracts;
- Very large consumers can participate directly into the spot market if they wish to pay 17,000 euros in the beginning and 10,000 euros annually fees from then on;
- TSO, DSOs and regulators are aware of the issues and a review is underway.

1.9 DSR in Germany

Germany’s situation now is unique in Europe for having almost all the markets open to DSR and at the same moment having barriers that make participation barely impossible, both provided by retailers and individual aggregators, anyway the government is aware of this and is making new rules to change this situation. Germany has a huge need in flexibility, thus in DSR, according to their plans of having 35% of renewable energy supply by 2020 and phasing out nuclear by 2022.

A summary of the situation in German electricity markets concerning DSR follows:

- DSR is legal in Germany but aggregation is for retailers only and faces anyway entry barriers;
- Wholesale market and re-dispatch market are closed to DSR;
- Intra-day market is open for consumers through their retailer, if that service is offered;
- There is no capacity market;
- DRS and aggregation are legal in German balancing market but due to entry barriers it is hard to estimate the participation, which is anyways expected to be low;
- Balancing and ancillary services markets are open to DSR but its entrance in the markets faces big barriers:
    - For entering the interruptible loads program in Germany, the minimum bid is of 50MW per consumer;
    - In primary control reserve the technical modalities are still designed around generation;
    - In secondary control reserve and in minute reserve there are potential risks in increases of grid tariffs for deviations from their normal (as flat as possible and over 10GWh annual consumption for being considered large users) energy consumption profile.

The main market barriers in Germany in general are due to:[3]

- The non-enabled independent aggregation (aggregators must have bilateral contracts with the retailers before engaging with a consumer, and retailers are potential competitors on the market);
- The network fee system that penalizes large consumers for changing their load profile;
- The required ability to be activated for a duration of 4 hours for minute reserve and 12 hours (up to 60 over the weekend) for secondary reserve;
- The 50MW minimum bid for interruptible load service;
- Tests for prequalification are required at an individual asset level, instead of being required at the aggregated level.

The main market enablers in Germany are:[3]

- The fact that in theory the market is open (ancillary services market is open to DSR, and it is possible to bid in aggregated load);
- Minimum bids do not exceed 5MW (except for the interruptible load service).

1.10 DSR in the UK

UK was the first country to open several of its markets to consumer participation in EU; nowadays all balancing services markets are open to DSR and aggregated load is accepted. Even if markets are open 2015 results are worse than 2013-14 and if this trend keeps going on the UK will no longer be a viable market for DSR, that is because several procedural and operational requirements (such as bidding, measurement, baseline) are still inappropriate for DSR. Considering the state of the market and of the large transmission-connected renewables, the opportunity for DSR is now higher than ever. In the UK there is a highly competitive retail market and a lot of small aggregators, some of them are even taking out supply licenses.

A summary of the situation in UK's electricity markets concerning DSR follows:

- BRP and aggregator relationship issue (how to cope with imbalances in BRPs forecasting due to aggregation of DSR loads on the network? Who should pay for that problem?) is not solved yet, but the problem now still doesn't have any impact (at the moment the BRPs in the UK have to pay for the imbalance, regardless of the reason that caused it);
- The capacity market introduced in 2014 is not placing DSR on an even plane with generation;
- In the balancing and ancillary services markets DSR have access to all the programs;
- In the wholesale market DSR only directly participates in the day-ahead and intraday markets in the form of flexibility of retailers and large industrial customers that already are trading members;
- DSR can participate in capacity market in theory, but practically this market is still too much in favor of generation;
- STOR (short term operating reserve), has a new regulation that basically cuts off DSR participation;
- Ofgem's (NRA – national regulatory agency) approach is to incentivize network innovation supporting DSR if it is cost efficient;
- In UK independent aggregation is enabled;
- It is possible to aggregate loads from all over the country, the customer though must inform the retailer about the participation.

The main market barriers for DSR in UK are:[3]

- Capacity market has too many big barriers at all levels to DSR;
- NRA and TSO failed in maintaining any fully viable entry point for DSR;
- STOR bidding process must change lowering requirements in order to be accessible to DSR.

The main market enablers for DSR in UK are:[3]

- Markets are open to DSR;
- Aggregation and independent aggregation are enabled;
- The financing scheme is encouraging new companies to enter the market;

- DSOs’ surrounding regulation encourages innovation and energy efficiency;
- Frequency response services markets allow DSR to participate.

1.11 Flexibility analysis in the UK

Flexibility technologies can change a lot the paths to make new investments in electricity system; nowadays one investment to consider when planning renovation of the electrical system should be the flexibility technologies. These technologies can provide less risk on the investments and it is possible to make a ‘least worst regret’ analysis that will show how the impact of the flexibility technologies will affect the plan on renewing the electric system. First, when we consider the electric system, we must know that the investments that are done nowadays will affect the future much more than the time being for few reasons:

- The length of the life of the electrical components;
- The actual time past between the plan and the actuation of the plan itself;
- The change in the conditions considered while planning and the actual ones due to the time taken to get the renewed system online and to the change in technology and price of the technology itself.

Indeed to make the best decision for not having to regret it, when it comes to install more power generation due to grid stability issues or more load increase, investing in flexibility technology, instead of investing in new plants, can really give us a hand as it will shift the needs for new installation of additional capacity once we achieve a higher efficiency in the system that we’re already using. Nevertheless, the delay in renewing the system will affect the flexibility technology deployment and impact on the effective cost reduction. One of the main reasons why investing in flexibility is highly recommended is that the inertia of the system is dropping as more non-dispatchable, renewable generation is coming to the grid and due to nationwide environmental goals, the big capacity plants fired by fossil fuels are shutting down, thus a flexible system is important to overcome this lack of inertia.

In the UK “an analysis of electricity system flexibility for Great Britain” by Carbon trust and Imperial College London was carried out using the ‘least worst regret’ analysis and considering four main solutions to increase flexibility in the network:[2]

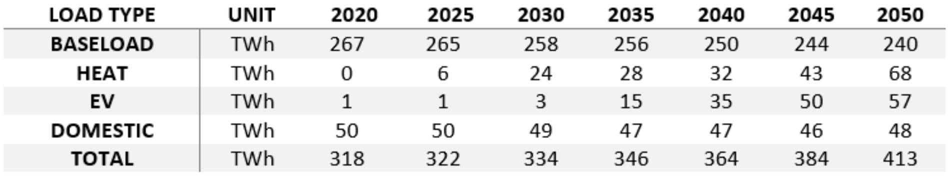
- Demand side response of controllable loads (whether they are industrial, commercial or residential loads);
- Energy storage technologies (any kind of energy storage, from batteries to compressed air or pumped hydro);
- Interconnections (mainly to other countries) that can move electricity to reduce demand or supply;
- More flexible CCGTs in order to operate more dynamically and to be able to ramp faster (low inertia) to match the peak demands.

Analyzing each of the flexibility technologies’ possible future evolution:

- Flexibility enables lower investment in low carbon generation to meet the 2050 carbon target;
- Flexibility from DSR, storage and interconnectors, reduces reliance on conventional peaking assets, thus reduces the cost of the system;

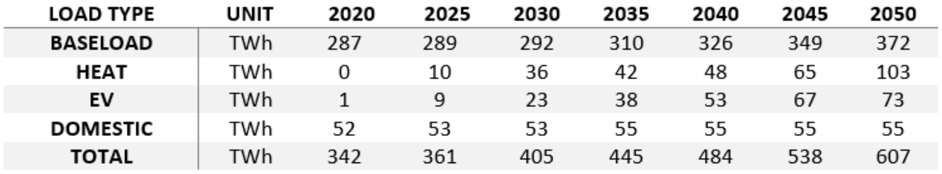
- Flexibility technologies defer necessary investments in transmission and distribution network reinforcement;
- DSR is the most cost uncertain technology thus an increase of 1GW per year as maximum constraint was added, nevertheless the optimal level in the analysis almost always reaches the maximum level, thus DSR optimal deployment might be much higher than the one considered in this paper;
- The range of deployment of storage is driven more by the uncertainty in the cost of other technologies (DSR) than in the cost of storage itself;
- Interconnections with other countries is a key source of flexibility and although the current capacity is optimal (in this analysis a net zero energy flow on the interconnectors is considered, which is nowhere close to be true, so probably interconnectors capacity must be anyway higher than the predicted) any delay in the deployment would increase the costs a lot;
- There is no need of more flexible CCGTs, though there is the need for traditional CCGTs (increasing in capacity for some years and then decreasing) for covering the demand when intermittent generation is in troubles other than to be used as flexible technology;
- OCGTs and reciprocating engines have an important role in providing flexibility (peak demand) and ensuring adequate capacity particularly when DSR and storage are expensive.

Expectation on the demand of energy across different loads in the UK according to the less demanding and the more demanding scenarios are represented in the following charts.



LOAD TYPE	UNIT	2020	2025	2030	2035	2040	2045	2050
BASELOAD	TWh	267	265	258	256	250	244	240
HEAT	TWh	0	6	24	28	32	43	68
EV	TWh	1	1	3	15	35	50	57
DOMESTIC	TWh	50	50	49	47	47	46	48
TOTAL	TWh	318	322	334	346	364	384	413

Figure 6 Least demanding scenario [2]



LOAD TYPE	UNIT	2020	2025	2030	2035	2040	2045	2050
BASELOAD	TWh	287	289	292	310	326	349	372
HEAT	TWh	0	10	36	42	48	65	103
EV	TWh	1	9	23	38	53	67	73
DOMESTIC	TWh	52	53	53	55	55	55	55
TOTAL	TWh	342	361	405	445	484	538	607

Figure 7 Most demanding scenario [2]

It is possible to notice that there is a big growth (and well expected too) in EV load and even bigger in Heat driven electricity load, while domestic loads are quite stable (a bit less or a bit more depending on the scenarios) and the baseload is seriously affected by the scenario in a range of roughly -10% for the least demanding scenario and +30% for the most demanding scenario. The changes in the baseload is anyway not even close to be comparable with the growth of the EV and Heat electricity loads, for an overall increase in the total load of +30% in the least demanding scenario up to +77% in the most demanding one.

1.12 Industrial-commercial load and residential load difference

The main difference between industrial-commercial loads and residential loads is clearly the capacity of the loads, that makes the industrial-commercial (usually) big loads take part to the market in a different way with their own tariffs and their own way of being flexible in the most efficient way for providing themselves the biggest income balancing the consumption needed for the activity and the possibility of participating into DSR. Even if these loads are by far the biggest portion of a country's load, the domestic loads still have a lot to offer and are rarely considered in DSR nowadays. The domestic loads to be able taking part to a whichever market need an aggregator otherwise they would never reach the size for the minimum bids, and are harder to control because of the need to have under control a lot of dwellings with all of their electrical loads, that is why nowadays it is becoming popular the idea of controlling "only" the domestic heating (space and/or water heating) when it is electric, EVs and as well controlling eventually batteries and PV panels if installed in the houses. The complexity of the domestic load and the need for aggregation makes the DSR participation challenging, and this means that there needs to be a fast, possibly real-time, communication between each single dwelling and the aggregator in order to be able providing DSR, especially for what it concerns some kinds of frequency response. Domestic loads are as well harder to predict since they rely on the users and what they do with all their electrical appliances, they vary a lot seasonally, but even the daily variation is big, plus each dwelling provides a different load profile and each household needs to have the comfort they want even when offering DSR. On the other hand the industrial-commercial load profile is much more predictable, and the seasonal difference tends to be lower (heating and cooling are usually a small part of the load for industrial loads but can reach roughly the half of the domestic demand) and there might not be need for comfort even though there is the need for being productive. A barrier for the domestic loads to join DSR is the need of remote controlling and the trustfulness of the aggregators that is crucial as the aggregators have all the data of the electric loads of the dwellings, which must be kept private, and not everyone is happy to share data or to give up a bit of comfort in change of an economic income. Therefore, there is need for clear regulation and incentives in order to give appeal to DSR and help domestic participation through an aggregator to DSR. The next table shows what policies could help domestic loads to take part to DSR through an aggregator and which ones would raise a barrier to this as well.

Non DSR friendly policies	DSR services provision	DSR friendly policies
All hours	*Resource availability*	Peak hours only
Arbitrary	*Event trigger*	Needs-based/transparent
Instantaneous	*Advanced notice*	Multiple hours
Unlimited	*Event duration*	Fixed/targeted
None	*Event limits*	Daily/annual limits
Overly complex	*Technology requirements*	Adequate/reasonable
Complex/biased	*Baseline*	Simple/accurate/fair
None	*Aggregation*	By total portfolio
Energy only	*Payments*	Availability and energy
Severe	*Non-compliance penalties*	Reasonable

Figure 8 Friendly and non-friendly policies for DSR deployment [6]

It is clear that transparency and simplicity together with incentives and possibility to aggregate are the main enablers, as well as not overdemanding availability and duration of the service together with an advanced notice of use. It is as well clear that the opposite are the main barriers for domestic user to participate to DSR. A key feature for the UK's domestic load participation to DSR is the roll out of smart meters within 2020, that is one of the most important technical enablers for domestic DSR to happen.

1.13 Domestic load analysis in the UK

The domestic load is a big part of UK's load and now the increasing integration of non-dispatchable (renewables, usually distribution level) energy sources on the grid while it is consequently decreasing the use of big, dispatchable and old fossil fuel fired, power plants make the situation in UK a scenario in which the DSR for domestic load could become very important and this is a scenario in need of DSR. This is why an analysis of the domestic load in UK today and the expectations for the future are possibly crucial for the future of UK's electricity system. The first thing to notice is how domestic load can be split in different categories, and then that not all of them are suitable for DSR, depending as well on costumers' wills and needs. These categories, and their percentual weight over the whole domestic load are:

- Water heating (7%);
- Space heating (21%);
- Lighting and appliances (with sub-categories) (72%, of which):
    - Computing (8%);
    - Cooking (16%);
    - Lighting (16%);
    - Cold (16%);
    - Wet (18%);
    - Consumer electronics (26%).

Considering that the overall domestic load is roughly the 35% of UK's load and is expected to remain constant over the time (with changes in categories though that will increase the DSR available load), out of this load the suitable one for DSR would approximately be the 52.5% (water heating, space heating, cold and wet). The reason why only these categories were chosen as DSR accessible load is because these kinds of loads allow some lag between the demand and the actual delivery of the service and thus are more suitable to provide flexibility rather than lighting and consumer electronics for instance that are loads over which the costumer is hardly willing to cede the control to someone else. When analyzing the domestic loads not only the yearly load is important, but as well the daily load profile is, and even more important it is to consider the seasonality of these loads, so all the DSR potential from domestic load is heavily affected by this. That is why some expected load profiles for the DSR suitable categories are provided in this analysis. The electric space and water heating demand (ESWH) is expected to grow in 2030 as well as the wet appliances load, while the load due to cold appliances is expected to slightly decrease. Besides the lighting and the computing loads all the other domestic non-DSR loads are as well expected to be higher than now.

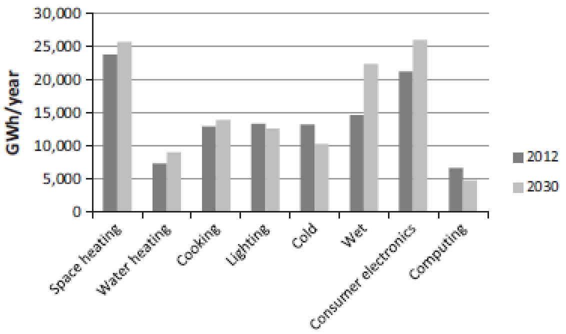


Figure 9 2012-2030 domestic demand comparison [5]

This change in domestic load is due to the fact that while the number of dwellings increases, the efficiency of these appliances increases as well. In some cases, the efficiency growth overcomes the dwellings' growth, in others it happens the opposite. Moreover, changes in costumers' habits affect the load changes as well.

A category by category analysis is following.

- ESWH demand is expected to increase even though heating systems' efficiency increases, because it is expected that the heating system (whether they are for water or space) are going to switch from gas to electricity;
- Cold demand is expected to decrease due to its efficiency improvement being greater than its growth in appliances;
- Wet demand is expected to increase due to the big increase in the use of tumble dryer, overcoming the increase in efficiency;
- Lighting demand is expected to be lower because of the wider use of more energy efficient bulbs rather than the standard ones;
- Consumer electronics demand increases because of the coming of new devices in the market (Set Top Box and gaming consoles) and an increase on the use of the TV as well, all of this is greater than the steps forward on the efficiency side (and the loss of interest in some devices such as DVD players);
- Home computing is going to decrease its demand because of the reductions in energy consumption of desktops and monitors that outweighs the increase in laptop and multi-function devices use;
- Cooking load is expected to grow due to the increase in the household numbers and the changes in the usage of the electric cooking devices (hobs and oven are decreasing while the use of microwave and kettle is increasing).

Focusing on the DSR available load, a growth in 2030 is expected, even if the cold appliances load decreases, the overall DSR load still slightly grows, the next graph will show the variation on DSR category loads change in the future.

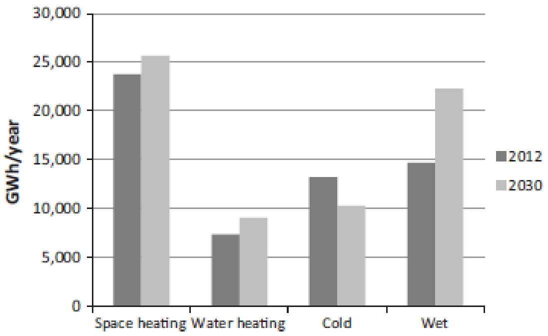


Figure 10 2012-2030 domestic DSR available demand comparison [5]

As aforementioned not only the average demand is important but the seasonal and daily demand are as well critical especially when it comes to heating, cold and wet loads. It is clear that heating loads will be much higher during the winter rather than during the summer, while the opposite happens for the cooling appliances, the wet appliances indeed follow the same pattern as the heating ones due to the fact that dryers are used in the winter but not in the summer. Three graphs showing the daily load profile of the ESWH, cold and wet appliances for the typical winter and summer profiles are following.

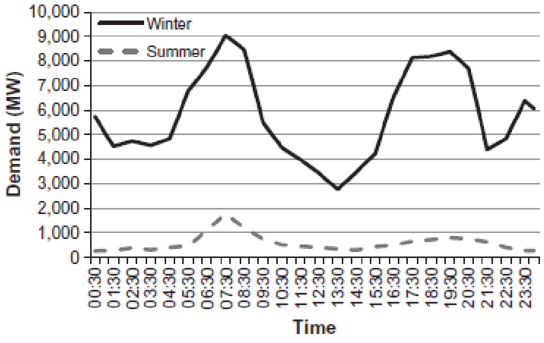


Figure 11 Daily ESWH load profile expected in 2030 [5]

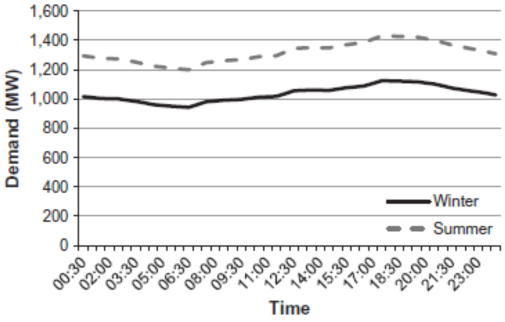


Figure 12 Daily cold load profile expected in 2030 [5]

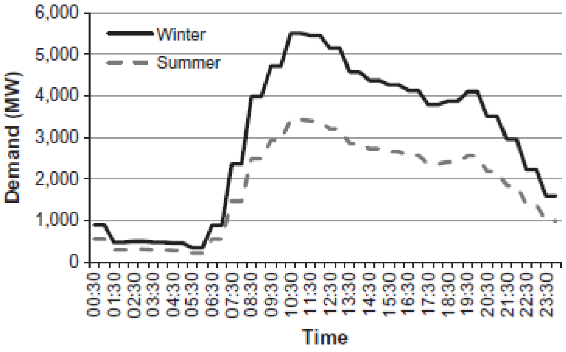


Figure 13 Daily wet load profile expected in 2030 [5]

The three profiles show how each of the DSR possible domestic loads categories changes daily both during summer and during winter. The morning and afternoon peaks for ESWH are well known to be happening due to the customers' use of their heating system when they wake and then whey comeback home after the working time, while during the day the load is lower due also to a higher ambient temperature. The cold appliances load in the UK is roughly flat due to the fact that there is no need for ambient cooling and as well the seasonal variation is not as high as in the ESWH appliances. The wet appliances load profile though is very peculiar, it is clear that during the night the load reaches the minimum, probably due to the fact that people find noisy those appliances so they don't like them to be working in the night or perhaps because of the fact that they are not present to initiate wet appliance operations, while there are peaks in the morning and basically at lunch and dinner time as well when the dishwasher is probably in use, while the dryers are used typically after the washing machines. Summing up all of the three load profiles it is possible to find the daily overall DSR accessible domestic load profile which is shown in the following diagram.

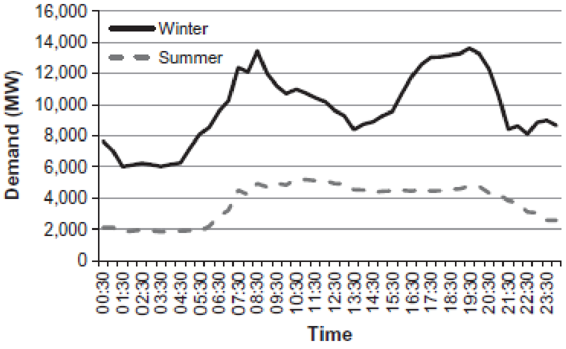


Figure 14 Overall domestic DSR accessible load profile expected in 2030 [5]

The changes in the load from today to 2030 will keep basically flat the overall domestic electricity demand at the 35% of the whole system electricity demand but the DSR accessible domestic demand will increase from the 52.5% to the 55% out of the whole domestic demand, these are yearly consumption based numbers, while it is clear that the amount of DSR accessible domestic load varies a lot during a single day, a week, a month, a season and finally a year.

2 Chapter Two – Methodology

This chapter will focus on the mathematical methodology used to model and solve the problems for the single dwellings and for the aggregated loads scenarios before running the power flow; this method is called Linear Programming, and the actually used method is Mixed Integer Linear Programming.

2.1 Linear programming

The method used to model and solve the optimization problem for the control of the loads is the linear programming known also known as linear optimization because linear programming is a way to model and solve linear problems aimed to maximize or minimize an objective function. A definition for linear programming is the following: "Linear programming is a mathematical method that is used to determine the best possible outcome or solution from a given set of parameters or list of requirements, which are represented in the form of linear relationships." [23] Linear programming is used to solve problems in several environments, from economic problems to engineering problems, from power problems to logistic problems, all of these problems are solved with a different approach to linear programming, but yet with the same tool. To solve the problems in linear programming, depending on their complexity each different solver uses a different algorithm, in this dissertation there will be no discussion on any of these algorithms and the way they work, but it is good to know that there are different approaches, and each and any of them has its strengths and weaknesses. The most important part when formulating a linear problem is to keep in mind that it is a simplified model of the truth, that has to try to be as close to reality as possible, but needs to keep its simplicity of being linear; being linear means that the unknown variables cannot be multiplied one with another, and not even with themselves (the power of the unknown variables terms both in the constraints and in the linear function must be one). A general linear programming problem is an optimization problem in which a linear function needs to be optimized subject to some linear inequalities; an example of formulation of a linear problem is the following [24]:

$$Maximize\ x_1 + x_2$$

$$Subject\ to$$

$$4x_1 - x_2 \leq 8$$

$$2x_1 + x_2 \leq 10$$

$$5x_1 - 2x_2 \geq\ -2$$

$$x_1, x_2 \geq 0$$

This is a very simple problem, indeed it is a two variables problem, which can be solved easily graphically, whilst real problems are usually much more complex and have more than two variables, thus more than two dimensions, so it becomes almost impossible to solve them graphically; nevertheless all the discussions about this problem are still valid for a more complex one. First of all this is a problem written in the standard form as it is a linear maximization problem with a set of linear inequalities as constraints, some of the constraints are "special" constraints, these are the last two constraints, the nonnegative ones, which are not always used in linear programming but they must be used if a problem is written in the standard form. Any couple of values of the variables x1 and x2 that satisfies all the constraints is called a feasible solution, whereas a set of values that do not withstand the constraints is called an infeasible solution, a problem can be as well feasible of infeasible if it has at least a feasible solution or it has none feasible solutions. The set of feasible solution will shape graphically, in whichever dimensional space, a convex region called feasible region, or simplex as well (that is why the first ever used algorithm to solve linear programming problem was called the simplex algorithm). The function to be maximized (or minimized in some cases) is called the objective function, and is the function that is relating all the unknowns one another, a whatever value of this function, within the simplex, is an objective value, while the optimal solution or optimal objective value is the maximum (or minimum) value that the objective function can reach within the simplex region. Setting the constraints (boundaries) to the problem is a very important step because a problem that does not have enough constraints might be an unbounded problem which will not return an optimal finite solution, and a problem with too many constraints could cause on the other hand an infeasible problem since the constraints might negate one another. The next figures will show the graphic representation of this problem, and his graphic solution, with the 3 feasible solution out of which one is the optimal.

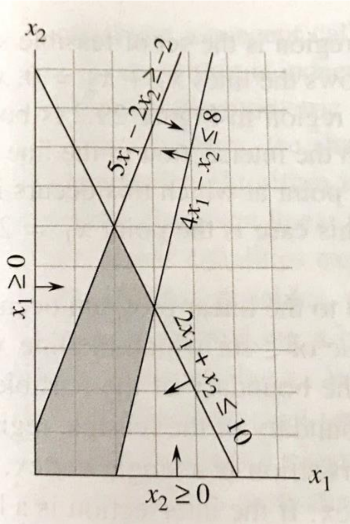


Figure 15 linear constraints building up the simplex [24]

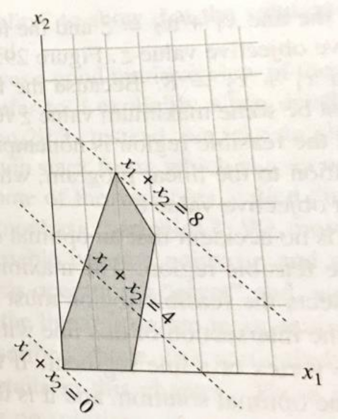


Figure 16 objective values against the simplex [24]

The first picture shows how the simplex is built for this problem, all the constraints are linear, indeed on the graphic the constraints are straight lines, and all the lines which represent the constraints also have an arrow pointing towards the direction that satisfies the constraint itself, the intersection of these constraints, keeping in mind the direction of the arrows gives us the simplex, the shaded area, which is the area that contains all the feasible solutions; the edges and vertices of the area are part of the simplex as well. The second picture shows the simplex and some possible values for the objective function as well, the three parallel dotted lines are the lines that represent a specific value for the objective function, thus all the points in each of the line have the same objective value; also in this case since these three lines are straight lines it is possible to notice that objective function is a linear function, and so the problem actually is a linear programming problem. In a well modelled problem like this, only one value can be the optimal value, and in this case is the optimal objective value is 8 which is the maximum value that the objective function can reach within the simplex, and according to the constraints the values of x1 and x2 can only be 2 and 6 respectively. The fact that the solution is a vertex of the simplex is no surprise and it is not casual; in a convex region the optimal value for the objective function must be on one edge, if that happens on a single point then it is on a vertex and there is only one optimal solution; otherwise if that happens to be one of the segments bounding the simplex, since the objective value is the same on that straight line, it must be the same even on the vertices, thus, again, the solution is still on the vertex of the simplex, obviously in this case there is not a single optimal solution but a set of optimal solutions for the unknown variables aiming to the same objective value. Graphically it is not easy to represent a problem which is larger than a bi dimensional problem, though what it has been said so far will still work on more complex problems, so even in multidimensional problems the solution will be on the boundary, and if it is a single solution on one only vertex. Even though, as aforementioned, this dissertation will not focus on the algorithms used for solving linear programming problems, this is just a brief paragraph naming the main ones and the approach used to solve the problems, the chapter will then analyze the methods for linearizing some typical nonlinear functions that have been used to solve some of the tasks of the following chapter. The eldest and most known algorithm to solve the linear programming problems is the simplex algorithm which starts at some vertex and after a series of iterations it checks the value of the objective function with the neighboring vertices until it finds the optimal solution which can be the maximum or the minimum value for the objective function. It is nowadays still one of the most used algorithms, though on very complex problems it might become very slow to find a solution, as the time taken to solve it might raise exponentially with the number of unknown variables, another well-known algorithm is the ellipsoid algorithm, that was supposed to be better than the simplex algorithm but in the end it is slow in practice. Another class of polynomial-time algorithm (rather than the ellipsoid algorithm) is known as interior-point method. As the name suggests this algorithm opposite to the simplex one, starts from within the simplex and it does some iteration to find the solution, so some of the intermediate solutions find by this algorithm might not be on a vertex, but the optimal solution will still be on a vertex of the simplex. There are other classes of linear programming problems such as the integer linear programming problems, in which all the variables are integer; the class of problems solved in the optimization tasks regarding this dissertation are mixed integer problems, which are the most common, in which only some variables (the least amount possible as they increase the time to solve the problem) are integer variables, and usually they are binary variables to set up different scenarios within the evolution of the problem.

2.2 How to linearize special sets of equations

In order to linearize some nonlinear constraints or functions, there is the need for some integer linear programming tricks that will turn the whole problem into a mixed integer linear programming problem. This is possible because of the availability of computer codes to solve this class of problems, integer variables can be introduced in a linear problem for solving either/or constraints, to solve if/then (aka conditional) constraints and to linearize a nonlinear function using a technique called piecewise linear approximation; there are other tricks useful for other scenarios, but these three main ones are the ones that were used in the solving of the following tasks, so the focus will be on explaining how these trick work and how they can be applied. With regards to either/or constraints it is possible to consider following model [25]:

$$Minimize \sum_{j \in J} c_j * x_j$$

$$Subject\ to$$

$$\sum_{j \in J} a_{1j} * x_j \leq b_1 \ (1)$$

$$\sum_{j \in J} a_{2j} * x_j \leq b_1 \ (2)$$

$$x_j \geq 0\ \forall j \in J$$

$Where\ at\ least\ one\ of\ the\ conditions\ (1)\ or\ (2)\ must\ hold$

In a linear programming problem, all the constraints must hold so the condition that at least one of the constraints must hold cannot be formulated; to solve this issue the introduction of a binary (integer value that can be only 0 or 1) variable is needed. Consider a binary variable “y”, the constraints can be rewritten in this way:

$$\sum_{j \in J} a_{1j} * x_j \leq b_1 * y\ (1)$$

$$\sum_{j \in J} a_{2j} * x_j \leq b_1 * (1 - y) \ (2)$$

This way when y is 1 the constraint number one must hold, while when y is 0 the constraint number 2 must hold. The conditional constraint model can be treated almost in the same way as the either/or constraint because in the end in case of a set of two conditional constraints such as [25]:

$$if \sum_{j \in J} a_{1j} * x_j \leq b_1 \ (1) is \ statisfied,$$

$$then \sum_{j \in J} a_{2j} * x_j \leq b_1 \ (2) \ must \ also \ be \ satisfied.$$

Is equivalent to:

$$either \sum_{j \in J} a_{1j} * x_j > b_1 \ (1)$$

$$or \sum_{j \in J} a_{2j} * x_j \leq b_1 \ (2) \ must \ hold$$

Where the lesser equal of the first constraint is changed into a greater than, so once the if/then constraint is transformed into an either/or constraint, the same technique used before can be used to solve this constraint, adding a binary value to multiply the first constraint, and the negation of that binary variable (1 – y) to multiply the second constraint. In some cases the bounds “b” in the previous formulas might be different in the two constraints, thus there might be the need of an upper bound “M1” and “M2” to multiply the binary variable in the two constraints in order to have one only of the constraints to still be valid without affecting the model. The piecewise linear approximation of a function is the hardest trick to implement, an example of need for piecewise linear approximation is the one that follows [3]:

$$Minimize \sum_{j \in J} f_j(x_j)$$

$$Subject \ to$$

$$\sum_{j \in J} a_{ij} * x_j \lesseqgtr b_i \ \forall \ i \ \in I$$

$$x_j \geq 0 \ \forall j \in J$$

In this model the objective function can be a whatever function, even a nonlinear one; in order to implement the piecewise linear approximation though, if it is a nonlinear function it still must be a separable function, which can be defined as a sum of function of scalar variables; in this kind of functions, the nonlinear functions can be approximated by piecewise linear ones, and this way it is possible to have a mixed integer linear programming problem. An example of separable function and non-separable function is the following:

$$x_1^2 + \frac{1}{x_2} - 2 * x_3 = f_1(x_1) + f_2(x_2) + f_3(x_3)\ separable\ function$$

$$x_1 * x_2 + 3 * x_2 + x_2^2 = f_1(x_1, x_2) + f_2(x_2)\ non - separable\ function$$

To show how piecewise linear approximation is implemented is better to use an easier function to be approximated such as $f(x) = \frac{x^2}{2}$; the figure below shows the actual dotted curve as well as the piecewise linear approximation represented by the straight lines, the points where the slope of the linear function changes are called breakpoints. The approximation can be done in several ways, the method use in the modelling for the next chapter is the weighted sums using a lambda formulation that will be explained below the picture.

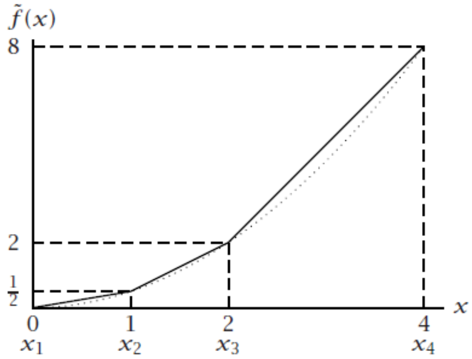


Figure 17 piecewise linear approximation of f(x) = (x^2)/2 [25]

Considering the four breakpoints (x1 = 0, x2 = 1, x3 = 2, x4 = 4) and the corresponding function values (f(x1) = 0, f(x2) = 1/2, f(x3) = 2, f(x4) = 8), any point in between two breakpoints is a weighted sum of these two breakpoints, for instance $x = 3 = \frac{1}{2} * 2 + \frac{1}{2} * 4$ and the approximated function value $\bar{f}(3) = 5 = \frac{1}{2} * 2 + \frac{1}{2} * 8$. Let λ1, λ2, λ3, λ4 be four nonnegative weights such that their sum is 1, the piecewise linear approximation of a function can then be written as:

$$\lambda_1 * f(x_1) + \lambda_2 * f(x_2) + \lambda_3 * f(x_3) + \lambda_4 * f(x_4) = \bar{f}(x)$$

$$\lambda_1 * x_1 + \lambda_2 * x_2 + \lambda_3 * x_3 + \lambda_4 * x_4 = x$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$

with the added constraint that at most two adjacent λs can be greater than zero.

3 Chapter Three – Optimization of domestic loads and power flow analysis

This chapter is about the work done in the company, to model and optimize a household connected to a LV network, which has a PV and a battery as well. The system of 55 households is then tested in the standard IEEE European LV network test to check the results. It will go through all the tasks tackled during the months of working in the company and for each task results, plots, equations and code snippets will be showed and commented.

3.1 A simple model of a household

In the first part of the work the focus was on building a reasonably accurate model for the household system, for a single household since the model will be the same no matter how many households will be taken into account. The approach to start building the model was to start from a very easy model, both easy to build and easy in a way that it was not an accurate model, to then be able topping up the model and growing in complexity until a good model was built. The idea was to model the behavior of a house having its load, a PV and a battery installed, optimizing the control of the PV and the battery to make self-consumption, thus to import as less power as possible from the grid. In this model the efficiency of the power transfer and the losses were not taken into account at all so the formulas for this model are the following ones:

$$x_{import}(t) - x_{export}(t) = c_{demand}(t) - c_{solar}(t) + x_{battery}(t)\ (1)$$

$$x_{soc}(t+1) - x_{soc}(t) = x_{battery}(t) * \Delta_t\ (2)$$

$$x_{battery}(t) \leq c_{battery\_max}$$

$$x_{battery}(t) \geq -c_{battery\_max}$$

$$x_{soc}(t) \leq c_{soc\_max}$$

$$x_{soc}(t) \geq 0$$

$$if\ x_{import}(t) > 0\ then\ x_{export}(t) = 0\ and\ viceversa$$

Where the X's are the unknown variables, while the C's are the constant (known, input) variable to solve the linear programming optimization problem; and where x_{import} is the imported power from the grid at each time step, x_{export} is the exported power to the grid at each time step, c_{demand} is the load (kW) at each time, c_{solar} is the solar power output of the PV at each time, $x_{battery}$ is the power exchanged with the battery system that can be positive (a load) when it charges or negative (generation) when it discharges powering up the system, at last x_{soc} is the state of charge (energy stored at each time, kWh) of the battery at each time step. The last if/then constraint is necessary to state that if power is being imported from the grid, it cannot be exported at the same time, and it is, as stated in the chapter about linear programming, a nonlinear constraint that needed a trick to be linearized. At this moment the resolution for load and generation was of one sample per hour which is quite a poor resolution, but in the end that will not affect the model, the only difference will be in the number of timesteps, not in the equations, nor in the coding for the model (except for the length of the variables). This task was tackled coding in python and using the library PuLp to build the model and then to solve it, all the task, except for the power flow ones were tackled this way, the short pseudo code for this simple model is the following one:

```
Solving a model with battery and PV to import less energy from the grid with 100%
efficiency

1.  The minimization problem is declared
2.  Variables are declared, each variable has its boundaries, its name, its
    dimension (in this case a vector), and its type (all of them are real
    numbers except one which is an integer number)
3.  The objective function is added to the program to minimize the import
    energy from the grid
4.  For loop over the timesteps

        A constraint for the overall system instantaneous power transfer is
        added

        A constraint for the instantaneous battery energy transfer and state of
        charge variation is added

        Two constraints are added to solve the either/or condition that the
        power at each instant of time can be either imported or exported

5.  A constraint for the initial value of the state of charge to be 0 is added
6.  Problem is solved
7.  Store significant values on a file
8.  Return flag value to check if the problem is properly solved
9.  Return of the significant values
10. Return plots of the significant values
```

With these few lines of code it was possible to build the whole model and to solve it as well. Once the code was built it was the time for running the code and checking the result, first of all it was important to check the flag returned by the solver in order to know whether the problem was actually solved or not, and in case it was solved it was needed the flag to know also if the solution was the optimal one, whilst if the problem was not solved the flag was useful to know whether there were feasibility issues or unbounded variables. Once the model was complete and run and the returned flag was optimal the results were then stored in different variables and in a file, and results were also displayed, with the aid of graphs. The results of the modelling of this task are the ones in the following graph.

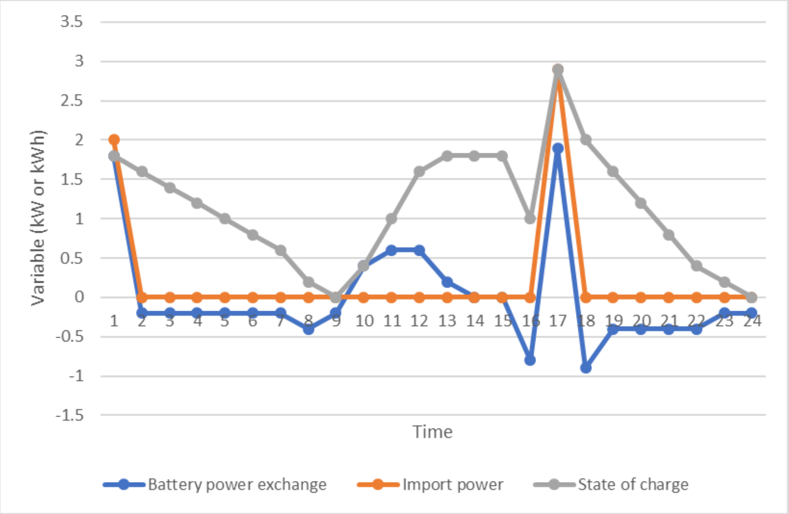


Figure 18 Results for the model

In this graph the export power is not showing because anyway the objective function was not including any term for export, thus the export itself was obviously null since nothing was requiring the model to export power to the grid. It is possible to notice how the variables are evolving in time and that the power is imported mostly to charge up the battery, the difference in the peaks between the import power and the battery power exchange is the actual load at the moment. When the battery power exchange is negative it means that the battery is exporting power to the load, and this happens most of the time keeping the import power to zero so affecting the grid the least possible, the battery at some point gets charged directly from the PV, and this happens when the solar production is higher than the load itself. The grey line is the evolution in time of the state of charge of the battery (energy, kWh) that starts from zero and at the end of the day all the energy from the battery is also drained and it ends again to zero, when power is exported from the battery the state of charge lowers its value, while when power is imported in the battery the state of charge raises its value, this behavior is the expected one since the battery charges and discharges when it imports and exports power respectively. The total energy imported over a day in this simple model was 4.9 kWh and considering a tariff for the import of the energy the price over the day paid by this household for the power imported from the grid would've been 1.36£. The import power tariff, the export power tariff, the load and the solar production used for this model and the following ones until an actual accurate model for the household was built were hardcoded as it follows (one value represents one timestep with 1 hour resolution).

The data for the total power imported over a day and the price paid for powering the daily load are useful for comparing the model while it evolves in complexity, which is why they were computed and thus reported here.

3.2 Efficiency and losses are included in the model

In the second task a more realistic model started to take shape since a fixed value for the efficiency was taken into account at this point and as well the battery standby losses and the losses in the power transfers with the grid as well, the fixed value for the efficiency, without taking into account the power transferred chosen was 0.9 and the standby losses were set to be 0.05 kW while the losses when importing from the grid were set to 0.025 kW. The model was now designed to make sure that if the battery is charging, it cannot be discharging in the same timestep, furthermore the standby losses for the battery were designed to happen only if the state of charge of the battery was greater than zero, because obviously if there is no energy stored in the battery there must be no losses due to this, another problem faced was the one with the import power losses, because these losses occurred only when the power was imported from the grid, otherwise there wouldn’t be any import power losses directly to the load, and the ones for the battery were already accounted, indeed the import power losses were designed to be added to the load at each time step, only if in that time step the load would be powered from the grid. The pseudo code for modelling and solving the problem is the following:

```
Solving a model with battery and PV to import less energy from the grid with 90%
efficiency and considering power conversion and power transfer losses

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its
   dimension (in this case a vector), and its type (all of them are real
   numbers except three which are integer numbers)
3. The objective function is added to the program to minimize the import
   energy from the grid
4. For loop over the timesteps

       A constraint for the overall system instantaneous power transfer is
       added

       A constraint for the instantaneous battery energy transfer and state of
       charge variation is added, including efficiency and losses

       Two constraints are added to solve the either/or condition that the
       power at each instant of time can be either imported or exported

       A constraint to state that the overall power transferred from the
       battery is the difference of charging and discharging at each timestep

       Two constraints are added to solve the either/or condition that the
       battery at each instant of time can be either charged or discharged

       A constraint to state that there are battery losses only when the state
       of charge of the battery is greater than 0 is added

       Two constraints to state that if the power is imported from the grid,
       and not from the battery the losses are only the power transfer losses
       and not the power conversion losses as well

5. A constraint for the initial value of the state of charge to be 0 is added
6. Problem is solved
7. Store significant values on a file
8. Return flag value to check if the problem is properly solved
9. Return of the significant values
10. Return plots of the significant values
```

The code had some changes from the one before that are easy to spot; first of all there are more variables, and more decisional binary variables used to linearize the nonlinear constraints, variables for the only charging and discharging of the battery, for the losses and for the overall load including losses for instance were added to the problem, furthermore the problem itself is more complex, there are more constraints, equality and inequality ones, and some of the constraints are changed, what did not change is the objective function since the goal of this model is again to optimize self-consumption importing the minimum possible energy from the grid, regardless of what it could be the price optimized energy import. In the constraint considering the energy balance (state of charge constraint, the efficiency of the battery was considered, both in charging and in discharging, using the same fixed 0.9 value, and as well the standby losses are considered. The delta time for this problem, and up to the point the model switches from one hour resolution to fifteen minutes resolution is 1, because the conversion from power to energy is done switching from kW to kWh.

The results of the problem for this model are the following:

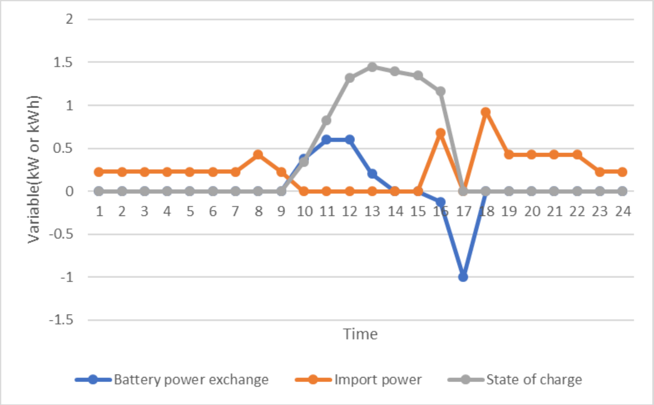


Figure 19 Results for the model

It is very easy to notice the difference in the results between these first two models, but to make it even more clear the next graph will compare the state of charge of the battery in the two different models.

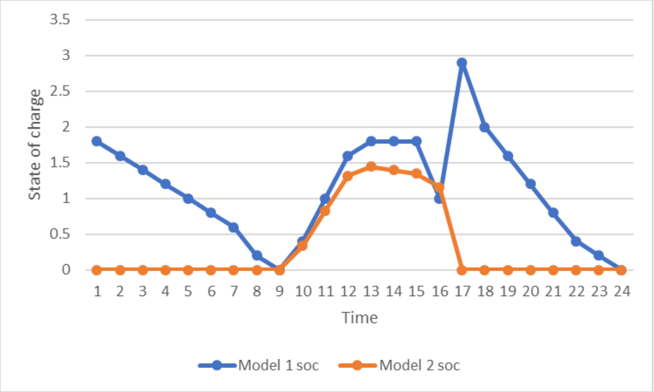


Figure 20 State of charge comparison between the first and the second model

From this graph the difference is very easy to spot, and the behavior is the expected one, there energy is imported in the battery to be used in the shortest amount of time, since the import of the energy and the export as well is affected by the efficiency, and the stored energy is affected by the losses, so importing a huge amount of energy, and keeping it stored for a long time would cause big standby losses. Another significant value that can be compared is indeed the total energy imported which is now greater and it is 5.98 kW while the cost for this model is 1.16£ that is indeed lower than the price for the first model.

3.3 State of charge constraint is added to test model response

In the task no major changes to the model and to the code were made, it was only added a constraint on the state of charge, to force it to be 2kWh at time 16, in order to check that the model was responding to the changing inputs and conditions of the constraints in the expected way, indeed the code for the task 3 is the same as it is for task two with one added constraints as it possible to see.

```
Solving a model with battery and PV to import less energy from the grid with 90%
efficiency and considering power conversion and power transfer losses and a set
value for the state of charge

    1.  The minimization problem is declared
    2.  Variables are declared, each variable has its boundaries, its name, its
        dimension (in this case a vector), and its type (all of them are real
        numbers except three which are integer numbers)
    3.  The objective function is added to the program to minimize the import
        energy from the grid
    4.  For loop over the timesteps

            A constraint for the overall system instantaneous power transfer is
            added

            A constraint for the instantaneous battery energy transfer and state of
            charge variation is added, including efficiency and losses

            Two constraints are added to solve the either/or condition that the
            power at each instant of time can be either imported or exported

            A constraint to state that the overall power transferred from the
            battery is the difference of charging and discharging at each timestep

            Two constraints are added to solve the either/or condition that the
            battery at each instant of time can be either charged or discharged

            A constraint to state that there are battery losses only when the state
            of charge of the battery is greater than 0 is added

            Two constraints to state that if the power is imported from the grid,
            and not from the battery the losses are only the power transfer losses
            and not the power conversion losses as well

    5.  A constraint for the initial value of the state of charge to be 0 is added
    6.  A constraint to set the value of the state of charge to a fixed one at a
        defined timestep is added
    7.  Problem is solved
    8.  Store significant values on a file
    9.  Return flag value to check if the problem is properly solved
    10. Return of the significant values
    11. Return plots of the significant values
```

The only difference from the code for the previous model is the constraint, stating that the state of charge at a certain timestep (16th) must be equal to a certain value (2 kWh). This model is not more complex, nor more accurate than the previous one when it comes to represent the real behaviour of the household but it gives the chance to check the difference in the results, and to see that it is consistent with the changes in the code, meaning that actually the model is responding to the new inputs. A constraint like that is anyway important because there might be users that at a certain point in the day want to have a certain amount of energy stored in the battery for whichever reason, so it is reasonable to test the model against this kind of constraints, not only to check its consistency but also because it is something that might be required in the control system of the household. The next graph shows the results for this model, that are noticeably different from all the previous models.

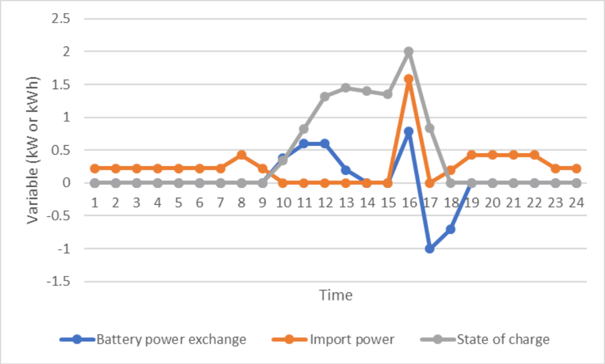


Figure 21 Results for the model

As expected not only the graph shows a much different evolution in time for the variables taken into account, but as well a huge there is a spike in the import power (positively as well in the battery power exchange) and in the state of charge at time 16 in order to cope with the 2 kWh constraint for the state of charge at time 16. Not only the shape of the variables over time and their values have changed, but also the total power energy imported is now 6.15 kWh that is higher than the amount in the previous model and the daily price for the imported energy is now 1.23£ which again is higher than the price for the second task model. The increase in the energy demand and price was well expected because now the model is requiring more energy at time 16 to be stored in the battery than it was asking in the second task, thus more energy must be imported from the grid and as well the price increases because more energy needs to be bought.

3.4 A change in the optimization process to take price into account

In this new step, again the model will not change at all, because no additional constraint will be added, this actually means that the complexity of the model will still be the same, and the constraint on the specific value of the state of charge will still hold. The difference in this step is made by the change in the objective function, which will totally change the result of the simulation because the goal will no longer be to reduce the amount of import power from the grid, but it will actually be the optimization of the daily cost to be as low as possible. The following one is the new objective function, where p_{import} is the import price for the energy and x_{import} is the imported energy.

$$minimise \sum (p_{import}(t) \cdot x_{import}(t)) \forall t \in T$$

Now since there is a variable tariff for the energy price, the one showed in the task 1 paragraph the algorithm will try to increase the energy import when the price is low to prevent importing when the price is high, giving a totally different shape to the solution. Not only this will significantly change, but also the daily price of energy and the amount of energy imported because even though the constraints are the same, the problem is not even close to be the same (in terms of linear programming, the simplex is still the same, but the function to optimize is a different function).

```
Solving a model with battery and PV to minimize the energy price allowing only energy import from the grid with 90% efficiency and considering power conversion and power transfer losses and a set value for the state of charge

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its dimension (in this case a vector), and its type (all of them are real numbers except three which are integer numbers)
3. The objective function is added to the program to minimize the cost of energy considering a flat tariff for import energy
4. For loop over the timesteps

      A constraint for the overall system instantaneous power transfer is added

      A constraint for the instantaneous battery energy transfer and state of charge variation is added, including efficiency and losses

      Two constraints are added to solve the either/or condition that the power at each instant of time can be either imported or exported

      A constraint to state that the overall power transferred from the battery is the difference of charging and discharging at each timestep

      Two constraints are added to solve the either/or condition that the battery at each instant of time can be either charged or discharged

      A constraint to state that there are battery losses only when the state of charge of the battery is greater than 0 is added

      Two constraints to state that if the power is imported from the grid, and not from the battery the losses are only the power transfer losses and not the power conversion losses as well

5. A constraint for the initial value of the state of charge to be 0 is added
6. A constraint to set the value of the state of charge to a fixed one at a defined timestep is added
7. Problem is solved
8. Store significant values on a file
9. Return flag value to check if the problem is properly solved
10. Return of the significant values
11. Return plots of the significant values
```

As it is possible to see from the code for this task, there are no changes in the constraints as previously stated, not even in the amount of variables, the only difference is in the objective function; the minimization problem will now have as objective function not only the sum of the import power over the day (or import energy, it still is the same since the resolution still is 1 hour) but the sum of the import energy times the time of use tariff price to be minimized. Because of this change the model is now aware of the energy market, and this is a huge difference in the approach to the problem solution, because the control scheme is now responding directly to the energy market prices in order to find the optimal solution. Obviously to be able doing this in a proper way a simple model will not be enough, because to be able to play an important role in the market there is a need for forecasting in the price evolution over the day (even though the shape of the price evolution is roughly similar one day to another, the values might differ a lot, reaching even negative values, furthermore there would probably be changes even in the shape during the weekends, holidays or with the changing of the seasons during a year), in the solar generation (this will depend a lot on the weather, thus it is not something very reliable, and a forecasting is absolutely needed) and last but not least in the loading profile of the household. The forecast of these three variables would allow to build a model that can adapt and respond in advance to the changes in the tariffs, in the load, and in the solar generation.

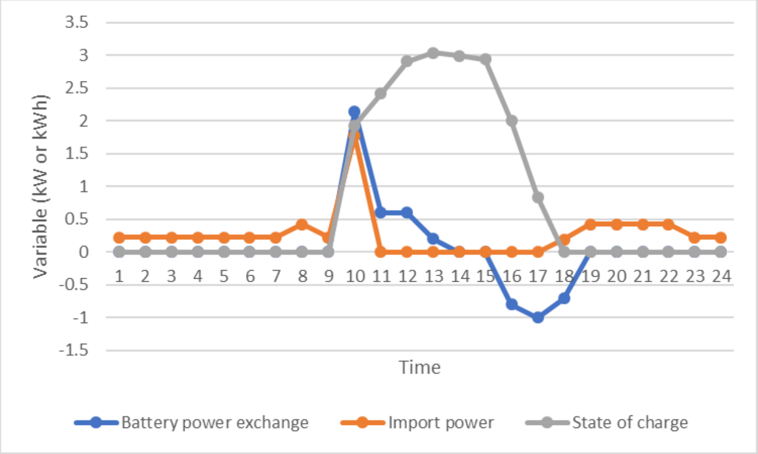


Figure 22 Results for the model

As expected the result is absolutely different from the previous one even if the constraints are the same: in this model there is a spike in demand from the grid at time 10, when the import price for energy from the grid is at its lowest time before increasing, this means that the energy is bought as much as possible, as late as possible, in order to have the cheapest price and the minimum standby losses, and this is exactly the expected behavior for the problem solution. It is noticeable as well that the state of charge reaches much higher values in order to cope with the constraint forcing it to be 2 kWh at time 16 without the need for extra energy import when the price is higher, and as well the spike in the import power is higher than before. In this scenario the total imported energy is 6.33 kWh and the daily price for the energy is 0.78£, both these results were expected as well, since the objective function is no longer minimizing the energy import, but the daily price for the energy consumption, the energy import is now higher than before, not much, but still is higher, while the price is much lower (0.78£ compared to 1.23£).

3.5 Export of energy back to the grid is now allowed

Even in task 5 there is no change in the model, but a change in the objective function, that now reached its final evolution, in order to take into account not only the price for the imported energy but also the price at which the exported energy is sold when it’s sent back to the grid, this implies that now the model is completely responding to the market prices, both for importing the energy and for exporting the energy, while the model is kept constant, this way it is possible to notice the effects on the system response to the changes applied on the objective function, before increasing the complexity in the system to reach the least ideal (or the most real) system possible, accounting both the model and the objective function. The following one is the new objective function, where p_{import} is the import price for the energy and x_{import} is the imported energy, obviously p_{export} is the price for the exported energy and x$_{export}$, the exported energy; the value for the exported energy times the price in the sum is negative because the goal is to minimise the cost, thus maximise the income, and selling the energy is not a cost, is an income instead.

$$minimise \sum (p_{import}(t) \cdot x_{import}(t) - p_{export}(t) \cdot x_{export}(t)) \forall t \in T$$

The new code on the other hand is the following:

```
Solving a model with battery and PV to minimize the energy price allowing both
energy import and energy export with 90% efficiency and considering power
conversion and power transfer losses and a set value for the state of charge

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its
   dimension (in this case a vector), and its type (all of them are real
   numbers except three which are integer numbers)
3. The objective function is added to the program to minimize the cost of
   energy considering a flat tariff for energy import and a flat tariff for
   energy export
4. For loop over the timesteps
       A constraint for the overall system instantaneous power transfer is
       added
       A constraint for the instantaneous battery energy transfer and state of
       charge variation is added, including efficiency and losses
       Two constraints are added to solve the either/or condition that the
       power at each instant of time can be either imported or exported
       A constraint to state that the overall power transferred from the
       battery is the difference of charging and discharging at each timestep
       Two constraints are added to solve the either/or condition that the
       battery at each instant of time can be either charged or discharged
       A constraint to state that there are battery losses only when the state
       of charge of the battery is greater than 0 is added
       Two constraints to state that if the power is imported from the grid,
       and not from the battery the losses are only the power transfer losses
       and not the power conversion losses as well
5. A constraint for the initial value of the state of charge to be 0 is added
6. A constraint to set the value of the state of charge to a fixed one at a
   defined timestep is added
7. Problem is solved
8. Store significant values on a file
9. Return flag value to check if the problem is properly solved
10. Return of the significant values
11. Return plots of the significant values
```

In this case again there is no change in the variables or in the constraints and the only change is in the objective function that now includes the export energy and the export price in its formulation. Even though the system is now different, the results in the end is not affected, that is because the chosen price for the export energy (a flat tariff of 0.0538 £/kWh, the one showed when explaining task one, which is the real price for the energy export in the UK) is too low so it is not convenient to import more power when it is cheaper in order to sell it back to the market to have a gain out of this. Nevertheless, a graph with the result will now follow, the total cost is still 0.78£ as before and the total energy imported is still 6.33 kWh as before, as nothing changed.

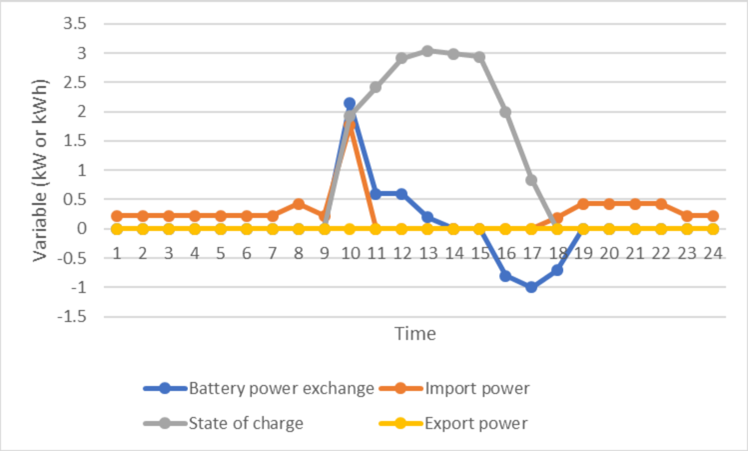


Figure 23 Results for the model

It is clear from the results that not only it is the same as before, but as well that even if now the objective function is taking into account the possibility to export the energy back into the grid, this does not happen as the export power is stuck at zero at all times. To check that this new objective function is responding in the proper way to the inputs and to see some difference in the behavior of the system, the same system was run with a different flat tariff price for the export price (a higher one, which value is 0.126968£). This way it is possible to see a difference in the results and to check that the behavior of the system is the expected one; obviously this price is not even close to be real (roughly 2.5 times higher than the real price) but it was used only to test the model and check the results, and to validate the model itself before taking any further step; another important value at this point is the total energy discharged during the day from the battery, as it will be useful to check the ninth task model against it. At this point, before editing the export price, the total energy discharged from the battery during the day was 2.51 kWh.

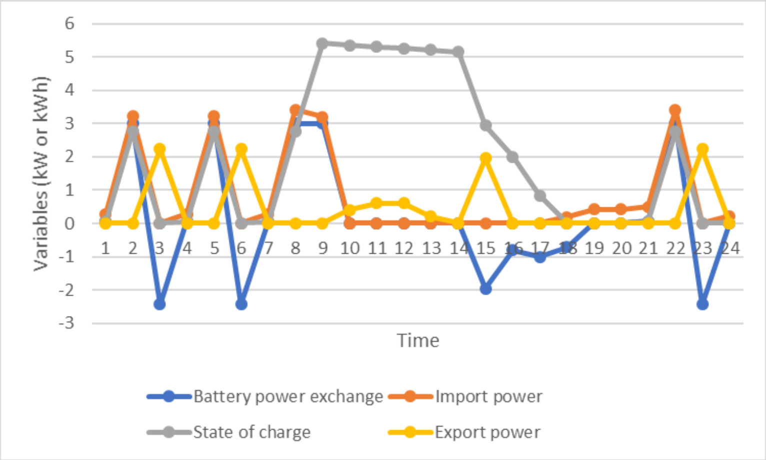


Figure 24 Results for the model with modified export price

It is very easy to notice that the shape now is absolutely different because the export price is convenient the behavior of the system is now to buy as much energy as possible when it’s cheap and to sell it back when it’s economically worth it, the most important thing to notice is that the export is actually happening and the shapes of the variables are reasonable, this means that the changes made in the objective function were consistent and that the model is working. The only reason why this change in the price was made actually to countercheck that the model was behaving properly, because with an inconvenient price no difference was noticed, thus it was impossible to tell whether the changes in the code affected the model or not. The total cost for the energy in this scenario lowered to 0.72£, and this is another signal that the system is working properly, because if the price was not lower than before it would mean that the optimal solution would’ve still been the previous solution, but this change in the results is actually giving proof that the responses from the model to the new inputs and objective function are right. Obviously, the amount of energy imported increased a lot reaching 19.13 kWh and as well the amount of energy discharge from the battery increased to 11.781 kWh. From this moment on, no changes were made to the objective function, this last one was the most complex objective function needed to be able modelling properly the final system; indeed this objective function is market sensitive, and allows import and export when it’s most convenient for the system, and since the final network is going to be optimized to have the lowest price possible, this is the best objective function that could be built. On the other hand the model still needs to grow in complexity, and the next three tasks are useful to build the final model, which is the closest one to reality, in order to have the best model the behavior of a real battery, different models will be built and checked against each other to find out not only the most accurate one, but the optimal one according also to the computing power (actual time taken) needed to solve the problem, considering that the fastest solution, when accurate enough, is the best solution for this kind of analysis. Since the final model will be anyway very complex, having a 15 minutes resolution and the overall amount of 55 single systems (and as well 55 interconnected systems) so it will take time to solve it, thus an approximated model, which is fast, is better than a slow one closer to reality.

3.6 A better approximation for the efficiency

To improve the model built so far there was the need for testing on a real battery, a Solax triple power T45 battery rated 3.6 kW and 6.3 kWh, with the data from the test performed on the battery it was possible to study the behavior of the battery in different situations, depending both on the state of charge of the battery and on the power exchanged from the battery, both when charging and when discharging. In this way it was possible to improve the model for the efficiency of the power exchanged with the battery in the model built so far, and also to limit the power transfer allowed depending on the state of charge. Once the data were available the first thing to was to check the results of the test with regards to power transfer in order to build the closest to reality approximation for the power transfer efficiency. After reading the data from the file it was possible to plot the graphic showing the DC power against the AC power, which, if the efficiency was constant would be a straight line, and if the efficiency was 100% it would have a 45 degrees angle with the x, and the y axis.

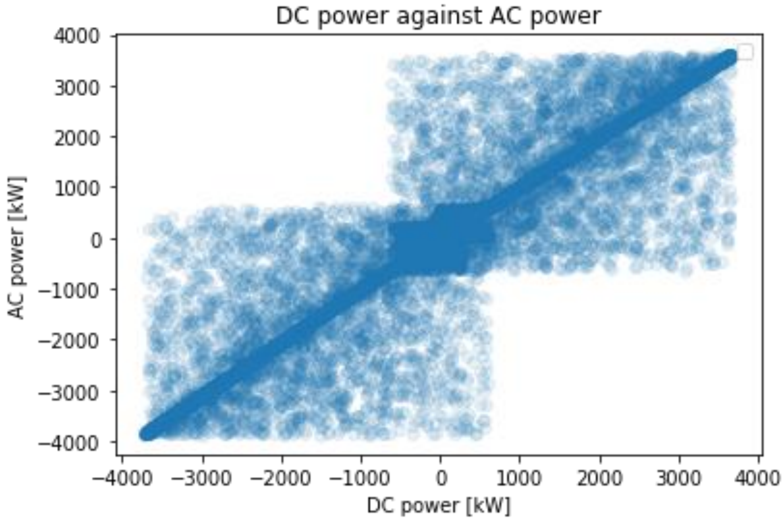


Figure 25 DC power against AC power before filtering

All the spots on the figure are the points coming out directly from the test results without any filtering operation, it is clear that there is a lot of noise in the data and that a filtering operation is needed, anyways it is possible to notice as well that where the dots are more dense it means that there are more points in the dataset matching that result. Thus it is clear that for high power transfer the efficiency is linear, or roughly linear, while for small power transfer it is not, indeed (both negative and positive side) there is almost a straight line after a certain amount of power transfer, while it is confused what happens with low power transfers. Reading the file that has the results of the test it is possible to see that the tests are performed starting from a certain time and that a setpoint for the power transfer is given and hold for a time interval, the first 4 pieces of data per each setpoint given are affected by a transient due to the change in the power level setpoint to the battery, thus they produce a large amount of noise, in order to overcome this problem a code filtering out the results of the test according to this issue was written. Anyway, this filtering was not enough because some of the results were anyways meaningless, especially when the power setpoint was close to 0, it was possible to have values of the efficiency much higher than 1 or even smaller than 0 (negative). Thus, after noticing that the lowest efficiency reached from the battery was 0.6 another filter was applied in the code, cutting out all the results that had efficiency lower than 0.6 and higher than 1.

```
Program built to check the real efficiency against power transfer behavior of a
real battery

1. Data reading from a file
2. Storing the data from the file in different variables
3. Creating variables with unknown length (vectors) to be able manipulating
   and storing the data from the file
4. For loop from the beginning of the test on the battery until the end

      If condition to check if the power is imported or exported and to check
      that the efficiency is within reasonable values and to wait for the
      transient to be over every time the setpoint power transfer value from
      the battery is changed
```

```
                Using 3 variables to store the import power transfer values
                that withstand the if conditions stated above in a vector

5. For loop from the beginning of the test on the battery until the end

        If condition to check if the power is imported or exported and to check
        that the efficiency is within reasonable values and to wait for the
        transient to be over every time the setpoint power transfer value from
        the battery is changed

                Using 3 variables to store the export power transfer values
                that withstand the if conditions stated above in a vector

6. For loop over the length of the import power vectors built at step 4

        Build a vector with the power transfer efficiency for importing

7. For loop over the length of the export power vectors built at step 5

        Build a vector with the power transfer efficiency for importing

8. Print the plot of the import and of the export power conversion efficiency
```

At this point a plot for the import power efficiency and the export power efficiency was done, in order to check if the data were clean from the noise, and also to see how the efficiency evolves against the power transfer, in order to build a function to approximate the evolution of the efficiency and check it against the real behavior of the battery.

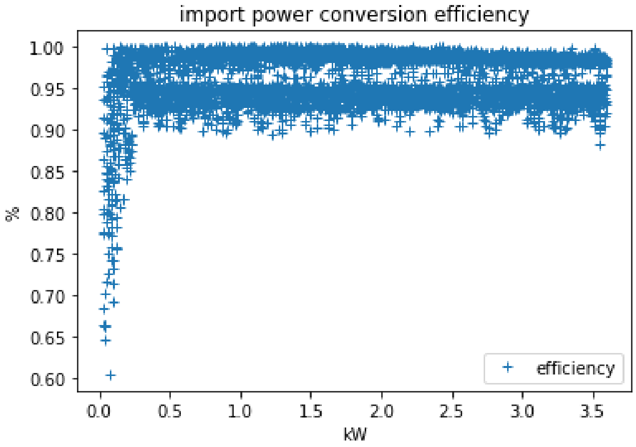


Figure 26 Import power efficiency against power, after filtering

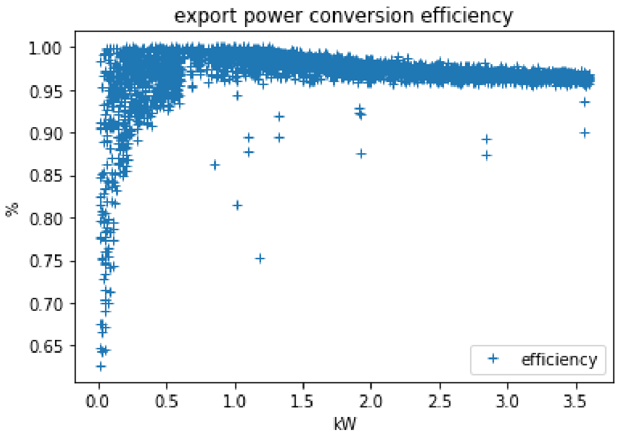


Figure 27 Import power efficiency against power, after filtering

As it is possible to see from the plots, the data are now much cleaner, considering the huge amount of data, there is still some noise and some values that actually are not as meaningful but it is now possible to understand the behavior of the battery when it comes to power conversion efficiency against the power transfer. As mentioned before it is possible to consider a linear behavior (constant value for the efficiency) when the power transferred is high enough (approximately when the power transfer is greater than 100 W) while when power transfer are lower the efficiency grows in a linear way, but the behavior of the product of the power transfer times the efficiency is an exponential behavior. With this in mind using the linear programming, it was possible to have an approximation for the efficiency, both for the export and for the import power; it is also clear from the plots that even though the values are similar, they do not reach the same final value, and as well the speed at which the efficiency grows (angle of the straight line approximating the efficiency growth) is slightly different.

```
Program built to model the export power conversion efficiency against the export
power transfer

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its
   dimension (in this case a vector), and its type (a real variable and an
   integer one)
3. The null objective function is added to the problem (an objective function
   must be added even if there is no need for it)
4. For loop over the length of the export power conversion efficiency vector

       The equation of the power conversion efficiency which is split in two
       parts with and either/or condition (efficiency = 1.85*power transferred
       + 0.6 if power transfer is lower than 100W, efficiency = 0.97 if power
       transfer is greater than or equal to 100W)

       The constraint to solve the either/or condition is added

5. The problem is solved
6. A plot of the data is returned
```

These few lines of code are enough to build an approximation to the export power conversion efficiency, there is no objective function, this is fine because there is no need for finding an optimal value in this problem, as long as the constraints are fine, the result for approximating the efficiency will be the right one.

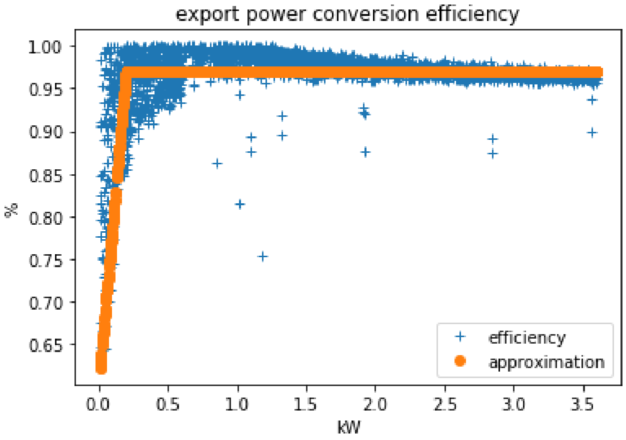


Figure 28 Export power efficiency approximation vs export power efficiency

A similar approach was used to approximate the import power conversion efficiency as well.

```
Program built to model the export power conversion efficiency against the export
power transfer

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its
   dimension (in this case a vector), and its type (a real variable and an
   integer one)
3. The null objective function is added to the problem (an objective function
   must be added even if there is no need for it)
4. For loop over the length of the import power conversion efficiency vector

       The equation of the power conversion efficiency which is split in two
       parts with and either/or condition (efficiency = 1.8*power transferred
       + 0.6 if power transfer is lower than 100W, efficiency = 0.96 if power
       transfer is greater than or equal to 100W)

       The constraint to solve the either/or condition is added

5. The problem is solved
6. A plot of the data is returned
```

The main difference between the two models is in the final value of the efficiency which is 0.96

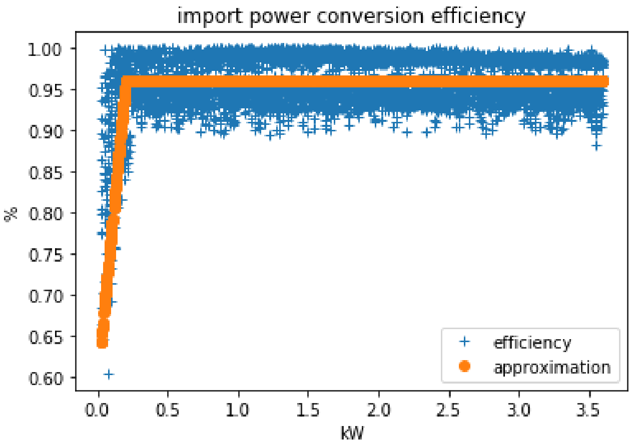


Figure 29 Import power efficiency approximation vs import power efficiency

It is now possible to see that the model built for the approximation up to this point is giving good results, as the orange function is a good approximation of the blue, real, one in both cases. This task and the approximation of the power conversion efficiency was only a step to move closer to the task eight in which an overall piecewise linear approximation of the product of the power conversion times the power conversion efficiency, according to the amount of power exchanged will be done. But an approximation and a study of the behavior of the battery and its efficiency was needed to be able to move further.

3.7 A limit on the power exchange is established

In this task the analysis of the real battery moved from the power conversion efficiency to the maximum power transfer allowed, with regards to the rated, against the state of charge of the battery. The rated power of the battery is not always the maximum allowed power transfer, because when the battery is almost fully charged the maximum charging power allowed decreases from the rated power and the same happens when the battery is almost empty, it is impossible to use the full rated power to export from the battery to the load. Reading the data from the test on the real battery, without the need to filter them out in this case (even though there is a bit of noise as it will be possible to see in the results) it was possible to store the data for the maximum power transfer possible, both when importing and when exporting, at each state of charge value from 0% to 100% and also to store the maximum overall value for importing and for exporting, which will be needed later on for bounding the linear programming problem solved to make the approximation of the power transfer against the state of charge.

```
Program built to get the power transfer limit according to the state of charge of
the battery

    1. Data reading from a file
    2. Storing variables from the file into vectors
    3. Build temporary variables to compare against when finding the results
    4. Creating variables with unknown length (vectors) to store the final results
       in these variables
    5. For loop with a variable going from 0 to 100

          For loop over the length of the variables obtained from the file
```

```
        If condition to check the state of charge of the battery

            If condition to obtain the maximum power transfer possible
            according to the state of charge when charging

                Storing the data of the maximum power transfer
                allowed

            If condition to obtain the maximum power transfer possible
            according to the state of charge when discharging

                Storing the data of the maximum power transfer
                allowed

        Storing the data into the vectors created before

        Refreshing the temporary variables
```

6. `Plotting the significant data`

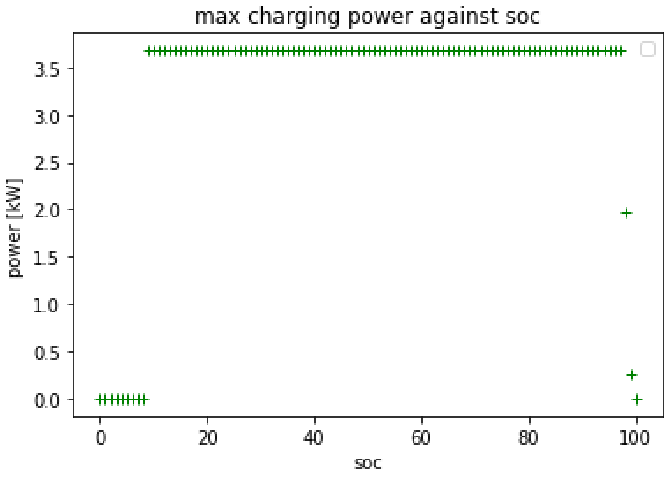


Figure 30 Maximum charging power against state of charge

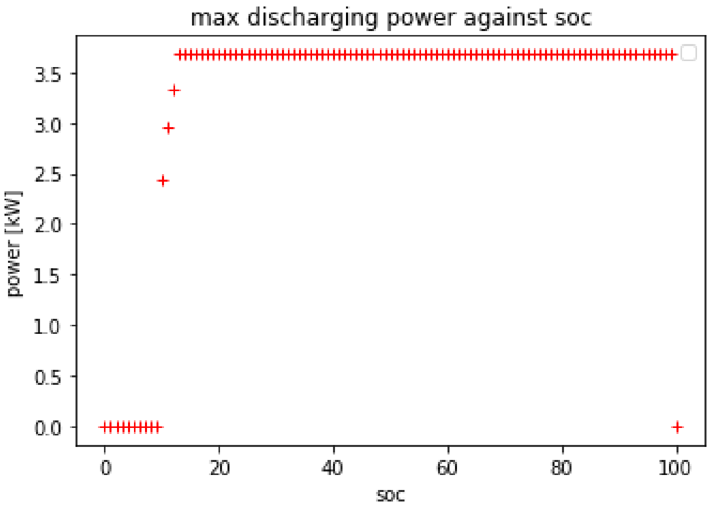


Figure 31 Maximum discharging power against state of charge

These two plots show the results for the maximum power transfer allowed when charging or discharging, against the state of charge, in both of the plots there is some noise due to the large amount of data, and some of them was probably not very accurate, indeed the in plot of the charging power it is obvious that for low level of state of charge it is not zero the maximum power allowed, but it is indeed the rated power, while the behavior after 90% of state of charge is accurate and expected, a reduction on the maximum power transfer allowed; on the other hand the plot for the maximum discharging power allowed is much more accurate, there is a mistake only at the value of state of charge 100% where the maximum power transfer allowed, while discharging, is the rated value. It was now possible to realize how to model the power transfer depending on the state of charge; using the following formulas (with values of 0.1 for lambda 1 and 0.9 for lambda 2) the model was then linearized, built and solved using PuLp.

$$x_{discharge}(t) \leq \left(0.5 \cdot k_{SOC}^{low}(t) + k_{SOC}^{mid}(t) + k_{SOC}^{high}(t)\right) \cdot c_{discharge}^{max}$$

$$x_{charge}(t) \leq \left(k_{SOC}^{low}(t) + k_{SOC}^{mid}(t) + 0.5 \cdot k_{SOC}^{high}(t)\right) \cdot c_{charge}^{max}$$

$$\lambda_1 \cdot k_{SOC}^{mid}(t) + \lambda_2 \cdot k_{SOC}^{high}(t) \leq \frac{x_{SOC}(t)}{c_{SOC}^{max}}$$

$$\lambda_1 \cdot k_{SOC}^{low}(t) + \lambda_2 \cdot k_{SOC}^{mid}(t) \geq \frac{x_{SOC}(t)}{c_{SOC}^{max}}$$

$$k_{SOC}^{low}(t) + k_{SOC}^{mid}(t) + k_{SOC}^{high}(t) = 1$$

$$k_{SOC}^{low}(t) + k_{SOC}^{mid}(t) + k_{SOC}^{high}(t)\ are\ binary\ values$$

The first two equations are the one giving the maximum power transfer allowed when discharging (the first) and when charging (the second). In those equations the maximum allowed value must be lower than or equal to the rated discharge/charge power multiplied either by 1 or by 0.5 depending on which of the “k’s” is active. The k’s are binary variables, and only one of them can be 1 at a time, they represent the state of charge of the battery, if the battery is in low charge the k^{low} will be active, if the battery is in the midrange state of charge the k^{medium} will be active, and if the battery is in high charge then the k^{high} will be active. The other two equations are the constraint needed to trigger one of the k’s to be active (value = 1) and the other ones to be zero. The value chosen for the low charge status is below the 10% of the state of charge, and for the high charge range is above the 90% of the state of charge, thus the midrange value for the state of charge is between 11% and 89% of the state of charge of the battery (this in translate in the values of $\lambda_1 = 0.1$ and $\lambda_2 = 0.9$). These equations were then transposed into code to solve the model to make the approximation of the power transfer allowed by the battery according to its state of charge.

```
Program built to model the charging and discharging maximum power transfer
allowed from the battery according to its state of charge

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its
   dimension (in this case a vector), and its type (four are integer numbers,
   and two are real numbers)
3. The null objective function is added to the problem (an objective function
   must be added even if there is no need for it)
4. For loop over the length of the vectors

      A constraint is added to limit the value of the sum of the three “$k_{SOC}$”
      as in the equation above

      A constraint is added to limit the maximum discharge power as in the
      equation above

      A constraint is added to limit the maximum charge power as in the
      equation above

      Two constraints are added to force the values of the three “$k_{SOC}$”
      according to the state of charge as in the equations above

      A variable to store the state of charge of the battery is added

5. Problem is solved
6. Return flag value to check if the problem is properly solved
7. Return plots of the significant values
```

Once the problem was solved the plotted solution was overlapped with the previous plots in order to check if the model was correct, the following plots are the final plots, showing that (considering the problem already mentioned in the original plots) the model is behaving in an appropriate way, and this new set of constraints is ready to be implemented in the model of the battery.

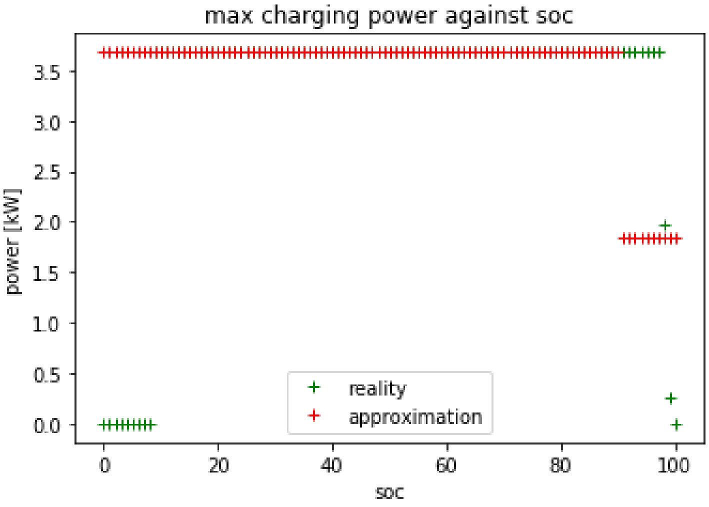


Figure 32 Maximum charging power against state of charge, reality and approximation comparison

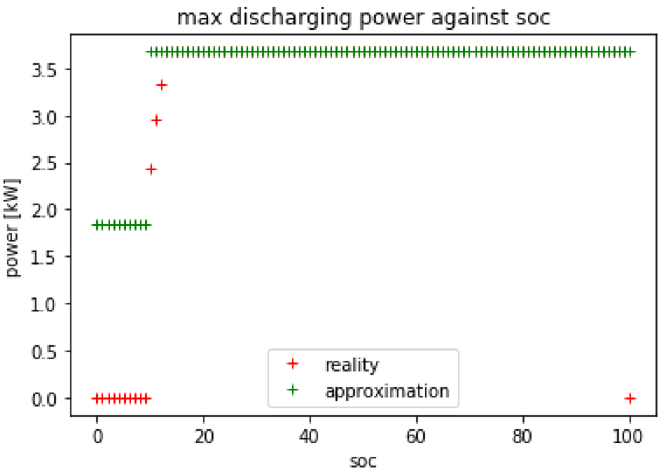


Figure 33 Maximum discharging power against state of charge, reality and approximation comparison

The approximation is obviously a simplified version of the reality, but this way it is possible to limit the power transfers in a more realistic way than it was before (actually before there was no limit at all, except for the rating of the battery itself) so the results are the expected ones, as it is possible to see from the plots, this means that the new lines of code are ready to be added to the existing model for the new tasks, in order to make as similar as possible to the reality.

3.8 A piecewise linear approximation to the power transfer including the efficiency

Once the problem of the power transfer against the state of charge was solved, it was time to get back to the efficiency in order to complete the model of the power conversion including the formulation for the efficiency. To make the model the most accurate possible, preventing it to be too heavy from a computational point of view, and preventing as well to have too many nonlinear constraint changed in order to make them linear, the best way was to have a model for the power conversion, when importing, and when exporting, which included already the formulation for the efficiency, and make then a piecewise linear approximation to this function. The other solution would be to implement exactly the formulations for the efficiency in the model, causing products of variables, which means non linearities, which means more computational power required and more time needed to get a solution to the problem. The first thing done to solve this problem was to see find the shape of the function representing the AC power vs the DC power both when exporting, and when importing.

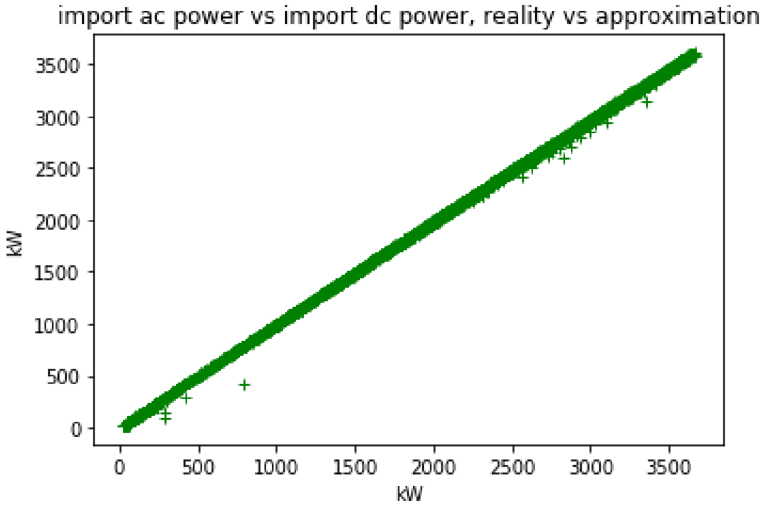


Figure 34 Import AC vs DC power, reality

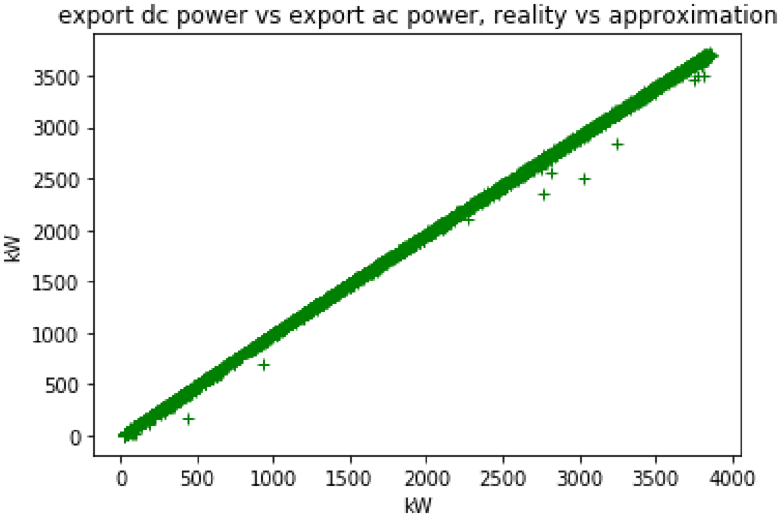


Figure 35 Export AC vs DC power, reality

The two images though as expected, seem to be a straight line, as if the efficiency was linear, and that is because the data are already filtered in this case, and as well the nonlinear part is a very small region of the whole curve, to see the non-linearity a zoom on a low power transfer region was then done.

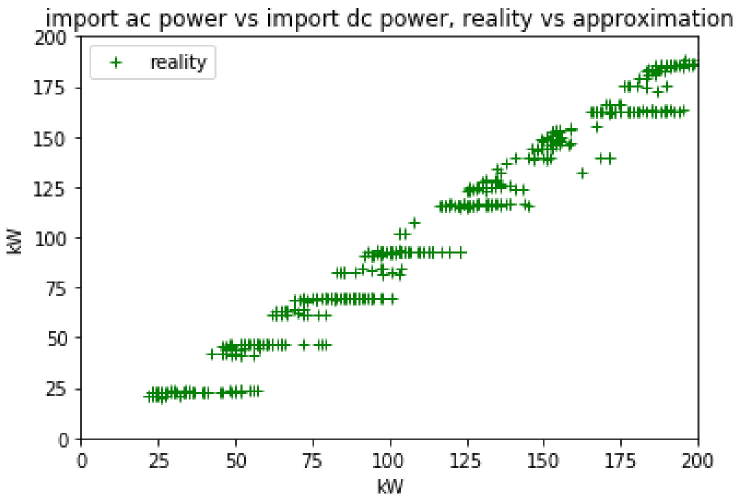


Figure 36 Import AC vs DC power, reality, zoomed in the low power region

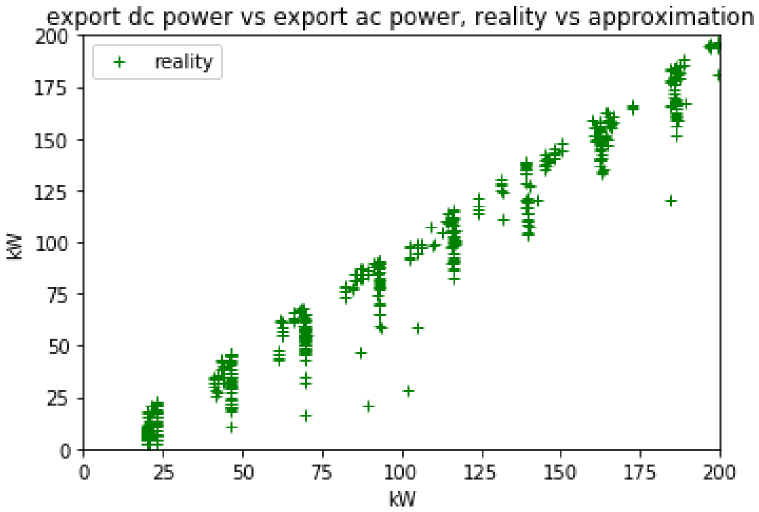


Figure 37 Export AC vs DC power, reality, zoomed in the low power region

The problem again in this case is that the dataset could not be filtered as deeply as before, otherwise there would be almost no data for the low power region, so the data are not very accurate and seem to be almost linear again, even though the expected behavior in the 0-100 W region would be different, and not only the expected one, but indeed the real one; to overcome this problem a new set of data, in which there is only one value of DC power matching one value of AC power was built, to be as close as possible to the real model, thus a nonlinear model, creating a function that matches the behavior of the battery in the low power transfer region.

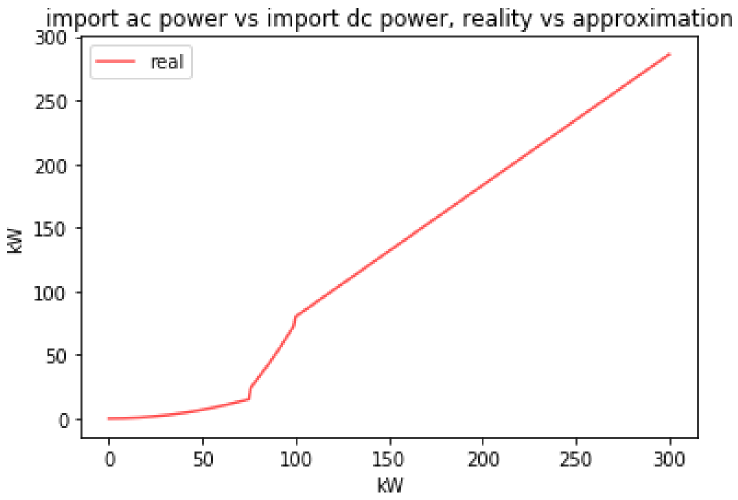


Figure 38 Import AC vs DC power, real function simulation

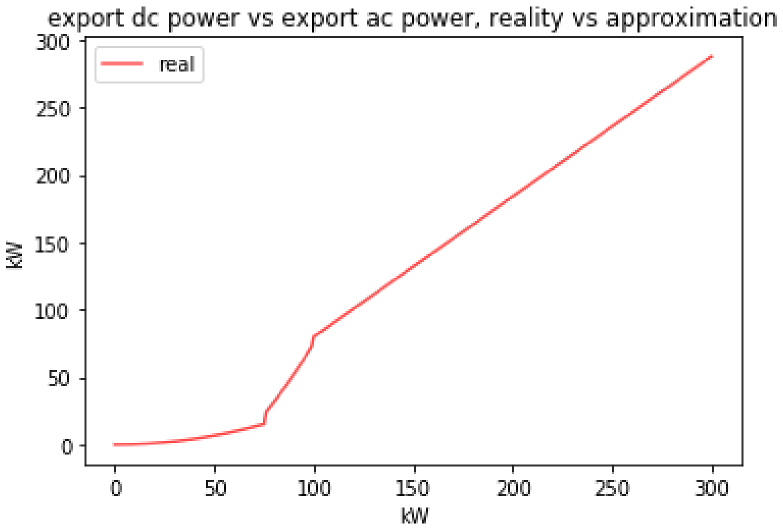


Figure 39 Export AC vs DC power, real function simulation

From these plots it is much easier to notice the nonlinear region from 0 to 100 W, both when exporting and when importing, and it is also noticeable the difference in the shape of the import power and the export power when the efficiency of the power conversion is taken into account. These plot are the result of a set of data which is not coming from a real battery, but of a function built on the behavior of a real battery, there should not be jumps in the function, the function in reality does not have jump, there is just an exponential increase on the efficiency value. At this point it was possible to start solving the problem of approximating the real battery power transfer with a piecewise linear function. To do this, again a linear programming problem was modelled and solved with PuLp, and also in this case there is no objective function but just a set of constraints.

`Program built to model the piecewise linear approximation of the power transfer efficiency`

1. `The minimization problem is declared`

```
2. Variables are declared, each variable has its boundaries, its name, its
   dimension (in this case a vector), and its type (four are integer numbers,
   the others real numbers)
3. The null objective function is added to the problem (an objective function
   must be added even if there is no need for it)
4. For loop over the length of the import power transfer efficiency vector

      The equation of the real behavior of the battery using the lambda
      formulation is added as a constraint like in the equations below

      The equation of the approximated behavior of the battery using the
      lambda formulation is added as a constraint like in the equations below

      A constraint is added to make sure that the sum of the lambdas is
      always equal to 1

      Four constraints are added in order to make sure that only two adjacent
      lambdas can be greater than one at the same instant in time

5. For loop over the length of the export power transfer efficiency vector

      The equation of the real behavior of the battery using the lambda
      formulation is added as a constraint like in the equations below

      The equation of the approximated behavior of the battery using the
      lambda formulation is added as a constraint like in the equations below

      A constraint is added to make sure that the sum of the lambdas is
      always equal to 1

      Four constraints are added in order to make sure that only two adjacent
      lambdas can be greater than one at the same instant in time

6. Problem is solved
7. Return flag value to check if the problem is properly solved
8. Return plots of the significant values
```

The way to make the piecewise linear approximation of the real function is the same as explained in the chapter about the linear programming and the linear programming tricks in particular, a weighted sum with a lambda formulation is the approach chosen. The function that needs to be approximated is the DC power against the AC power transfer, both when importing and exporting, function. Starting from the data it was possible to have a rough idea of how that function behaved and this way it was possible to build the approximated function. These are the formulas used for the weighted sum lambda formulation in the code

$$\lambda_1 * f(x_1) + \lambda_2 * f(x_2) + \lambda_3 * f(x_3) + \lambda_4 * f(x_4) = \bar{f}(x)$$

$$\lambda_1 * x_1 + \lambda_2 * x_2 + \lambda_3 * x_3 + \lambda_4 * x_4 = x$$

In order for these formulas to work properly it was important that only two adjacent lambdas are different from zero at the same time, and that the sum of the four lambdas is one. In this case the function was split in 3 different pieces, marked by 4 breakpoints, the initial point, the zero, two intermediate points where the slope of the real function significantly changes (in fig 21 and 22 this is clear) and the final point which is the maximum power transferred, that is different between import and export. The results are in the next plots.

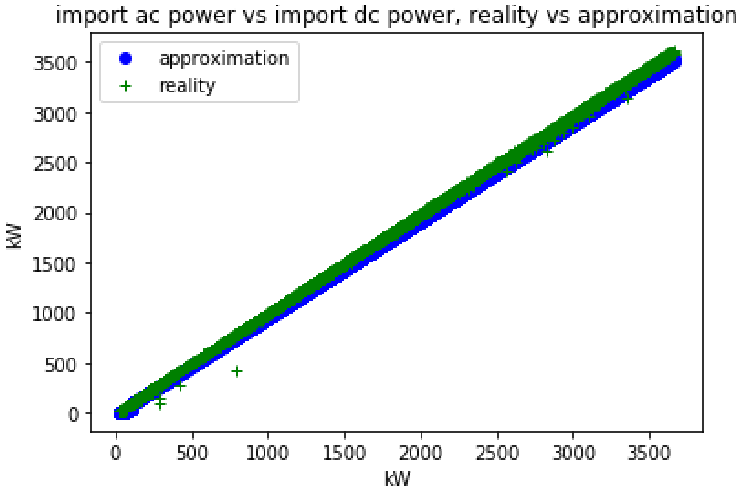


Figure 40 Import AC vs DC power, reality vs approximation

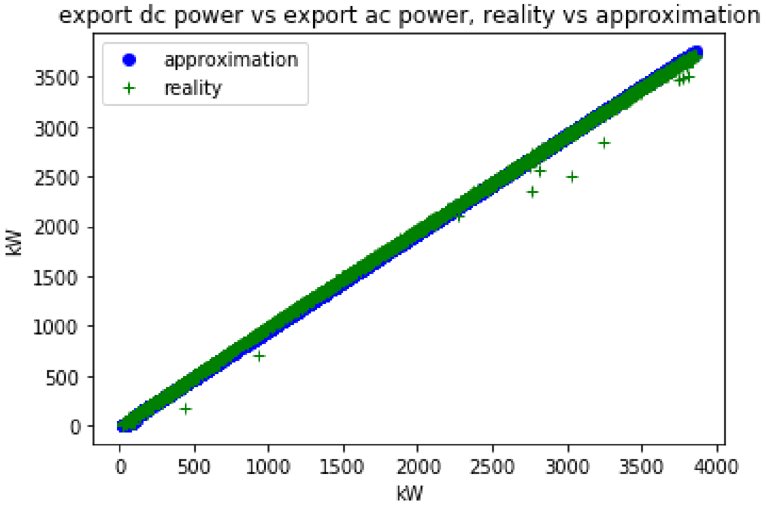


Figure 41 Export AC vs DC power, reality vs approximation

Just like before from these two plots it is not possible to notice the nonlinear area, but it is possible to see that the behavior of the real function is approximated well by the piecewise linear approximation made, over the whole range of power.

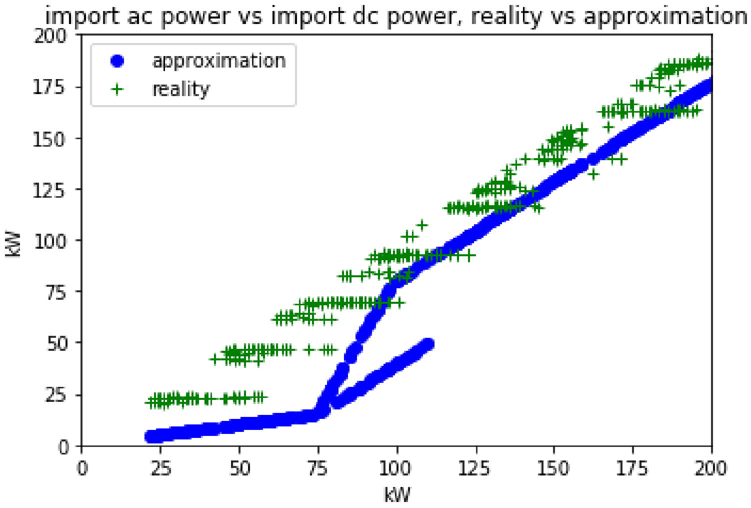


Figure 42 Import AC vs DC power, reality vs approximation, zoomed in the low power region

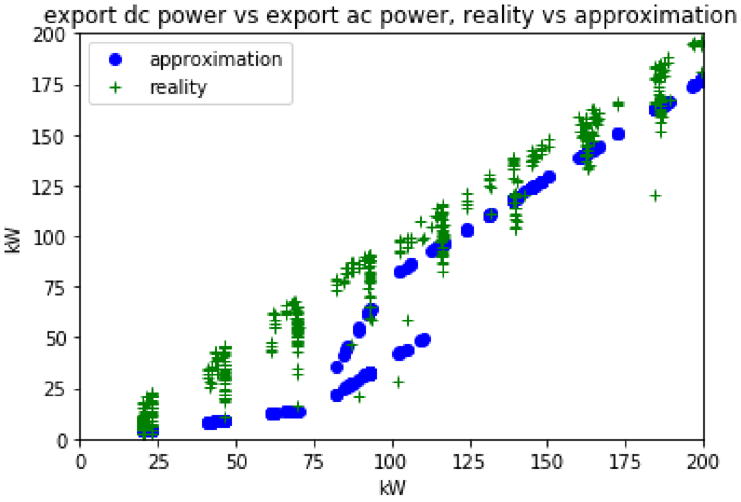


Figure 43 Export AC vs DC power, reality, zoomed in the low power region

Zooming in the area where the non-linearity is, it is possible now to see that the approximated function is following in a proper way the real function even when the non-linearities occur, though just like before, since the data are not the most accurate it is possible to notice some nonlinear behavior and the fact that in some values it seems like there are two values of DC (or AC, depending if importing or exporting) power matching one single value of AC (or DC, depending if importing or exporting) power, and this is not possible in a linear approximation. In order to make sure that the piecewise linear approximation worked properly and that it was ready to be implemented in the model of the battery built so far the same identical problem, was solved with the same approach and the same formulation also for the clean set of data that was built for this purpose. Obviously since the ranges of power are different, the values are as well different in the weighted sums lambda formulation in this code, but the model and the formula are still the same.

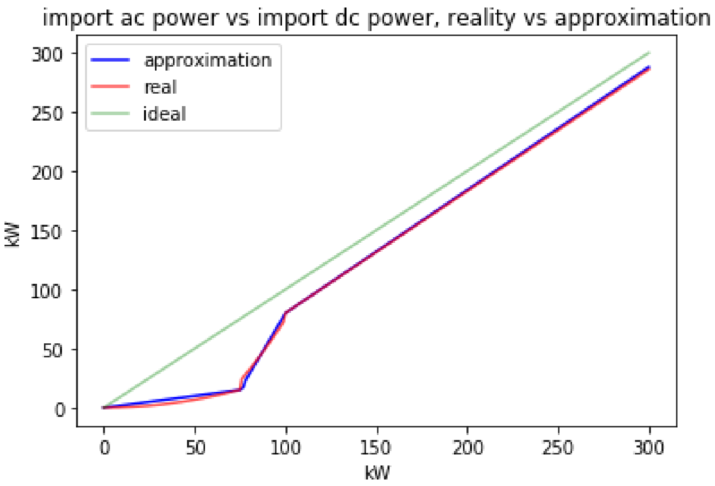


Figure 44 Import AC vs DC power, real function vs approximated function vs ideal power transfer simulation

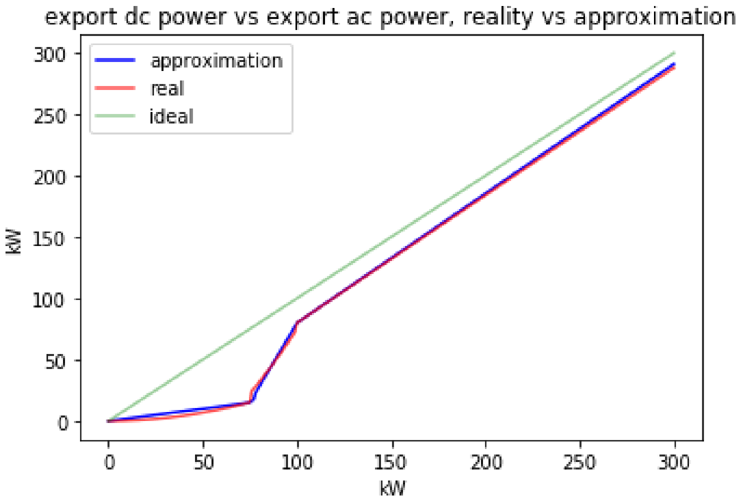


Figure 45 Export AC vs DC power, real function vs approximated function vs ideal power transfer simulation

It is now clear that the approximated function is following properly the real function over the whole range of power, and it is also clear how the efficiency evolves and grows from a very low value to almost ideal efficiency while the exchanged power grows, indeed the gap between the ideal, the real and the approximated function gets lower when the power exchanged gets higher. The fact that the ideal value is never reached or crossed (over efficient system, which cannot be real) and the evolution of the real and approximated function against the ideal one is another proof that the model is consistent, and that the result is exactly the expected one. At this moment the last thing left to do, in order to complete the model was to include these formulations in the model and to check how it changes the results.

3.9 A final constraint is added to the model, a comparison of three models, the decision on the best model to use for the final task

The ninth task is one of the most important ones, because it actually leads to the final model used to simulate the in the most accurate way the real system composed by a household with a PV and a battery, connected to the grid, that can import power from and export power to the grid, and has a certain load profile. Once this model is built it will be possible to solve the 55 problems regarding a real-life case of solar generation, load demand, import price and export price, and with a higher resolution, 15 minutes against 1 hour. And once those problems are solved it will be possible to solve the power flow of a standard IEEE network to see how the optimization of the load (DSR) affects the network, which is the goal of this dissertation. The model built up to task 5 will be now modified in order to include, first of all, a new constraint regarding the maximum amount of energy that is possible to discharge from the battery over a day in order to comply with the battery warranty, and this value is 6kWh, which is the rated kWh value for the battery as well.

Solving a model with battery and PV to minimize the energy price allowing both energy import and energy export with 90% efficiency and considering power conversion and power transfer losses, a set value for the state of charge and a maximum power discharge value allowed from the battery. The model now includes the maximum power transfer allowed from the battery considering its state of charge

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its dimension (in this case a vector), and its type (all of them are real numbers except six which are integer numbers)
3. The objective function is added to the program to minimize the cost of energy considering a flat tariff for energy import and a flat tariff for energy export
4. For loop over the timesteps

    A constraint for the overall system instantaneous power transfer is added

    A constraint for the instantaneous battery energy transfer and state of charge variation is added, including efficiency and losses

    Two constraints are added to solve the either/or condition that the power at each instant of time can be either imported or exported

    A constraint to state that the overall power transferred from the battery is the difference of charging and discharging at each timestep is added

    Two constraints are added to solve the either/or condition that the battery at each instant of time can be either charged or discharged

    A constraint to state that there are battery losses only when the state of charge of the battery is greater than 0 is added

    Two constraints to state that if the power is imported from the grid, and not from the battery the losses are only the power transfer losses and not the power conversion losses as well

    The set of constraints to limit the power transfer both when charging and when discharging the battery according to its state of charge is added

5. A constraint for the initial value of the state of charge to be 0 is added
6. A constraint to set the value of the state of charge to a fixed one at a defined timestep is added

```
7.  A constraint to make sure that the overall discharged energy from the
    battery in the whole day is lower than a certain value is added
8.  Problem is solved
9.  Store significant values on a file
10. Return flag value to check if the problem is properly solved
11. Return of the significant values
12. Return plots of the significant values
```

Since with the normal case, without editing the export price in order to make it profitable, the energy limit was not hit, the next plot, showing the solution to this problem is exactly the same as the one for the non-modified price model in task 5; indeed the final cost is still 0.78£, the total energy imported is still 6.33 kWh and the total energy discharged from the battery is still 2.51 kWh; another important value to remember at this point is the time taken to solve the problem which is 0.15 seconds.

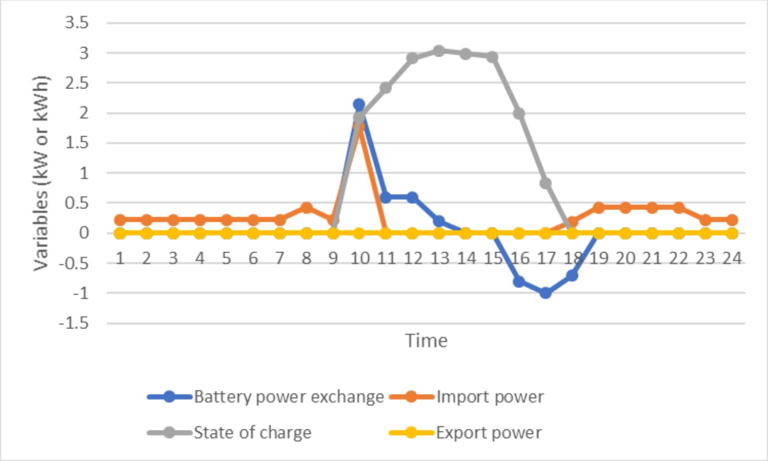


Figure 46 Results for the model with 90% efficiency

In order to check that the model and was actually responding properly to the new constraint and it was built properly as well, the same problem was solved with the increased price for energy export, just like in task 5. This time the difference is huge, because the power limit was hit before, but now the constraint is limiting the maximum discharged energy from the battery, and the final results are a cost of 0.73£, a total energy imported of 10.61kWh and a total energy discharged from the battery which is 4.464kWh while before it was 11.781kWh, this means that the constraint is working properly, even if the total discharged energy is not 6kWh and this happens because it would not be profitable to reach that limit. The plot for the solution to this problem is the following (notice that now the exported energy is not zero)

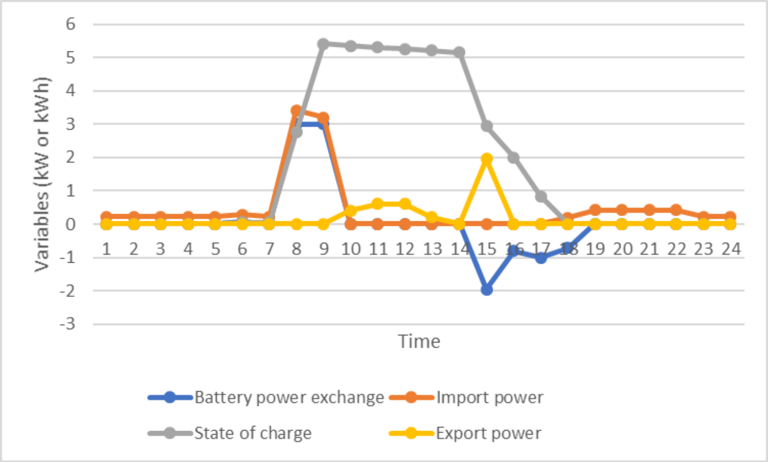


Figure 47 Results for the model with 90% efficiency and modified export price

Now that also this last constraint was tested, it was the time to include in the model the formulations made from task 6 to task 8, regarding the power conversion efficiency and the power transfer limits. Those formulations were made started from a real battery, with different ratings both in terms of energy storage and in terms of power output (and input), but this does not affect the validity of the formulations themselves because all the batteries behave in the same way but with different results due to the different ratings, so once the model is scaled to a 3kW and 6kWh battery it can be included in the models done so far, in order to increase the complexity of the model itself and make it as close as possible to a real model.

Solving a model with battery and PV to minimize the energy price allowing both energy import and energy export with piecewise linear approximation for the power conversion efficiency and considering power conversion and power transfer losses, a set value for the state of charge and a maximum power discharge value allowed from the battery. The model now includes the maximum power transfer allowed from the battery considering its state of charge

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its dimension (in this case a vector), and its type (all of them are real numbers except ten which are integer numbers)
3. The objective function is added to the program to minimize the cost of energy considering a flat tariff for energy import and a flat tariff for energy export
4. For loop over the timesteps

    A constraint for the overall system instantaneous power transfer is added

    A constraint for the instantaneous battery energy transfer and state of charge variation is added, including efficiency and losses

    Two constraints are added to solve the either/or condition that the power at each instant of time can be either imported or exported

    A constraint to state that the overall power transferred from the battery is the difference of charging and discharging at each timestep is added

Two constraints are added to solve the either/or condition that the battery at each instant of time can be either charged or discharged

A constraint to state that there are battery losses only when the state of charge of the battery is greater than 0 is added

Two constraints to state that if the power is imported from the grid, and not from the battery the losses are only the power transfer losses and not the power conversion losses as well

The set of constraints to limit the power transfer both when charging and when discharging the battery according to its state of charge is added

The set of constraints to model the piecewise linear approximation of the power transfer efficiency both when importing and when exporting is added

5. A constraint for the initial value of the state of charge to be 0 is added
6. A constraint to set the value of the state of charge to a fixed one at a defined timestep is added
7. A constraint to make sure that the overall discharged energy from the battery in the whole day is lower than a certain value is added
8. Problem is solved
9. Store significant values on a file
10. Return flag value to check if the problem is properly solved
11. Return of the significant values
12. Return plots of the significant values

This code is the code for the final model of the system and the addition to include the power conversion efficiency piecewise linear approximation makes this model become the most complex and accurate, as well as the closest to reality. There are a lot more variables in this model, and the main problem is that there a lot more integer (binary) variables as well, that are very heavy in terms of computational power, and time needed to solve the problem. The results are not too different from the problem without the addition of the real power conversion efficiency and of the power transfer limits, but are still noticeable, and this means that this model is much more accurate, and will become the base model to test against and compare the final model that will be built (in this task). The cost for the energy in this model is 0.71£, the total imported energy is 6kWh, the total discharged energy from the battery is 2.61kWh and the time taken to solve it is 0.37 seconds. The difference in the time might not seem as big as the difference in the results, but considering a much more complex system, with 4 times higher resolution will exponentially increase that difference, that will become huge when a one only model including 55 households will be built. The following is the plot for the solution to the problem using this model.

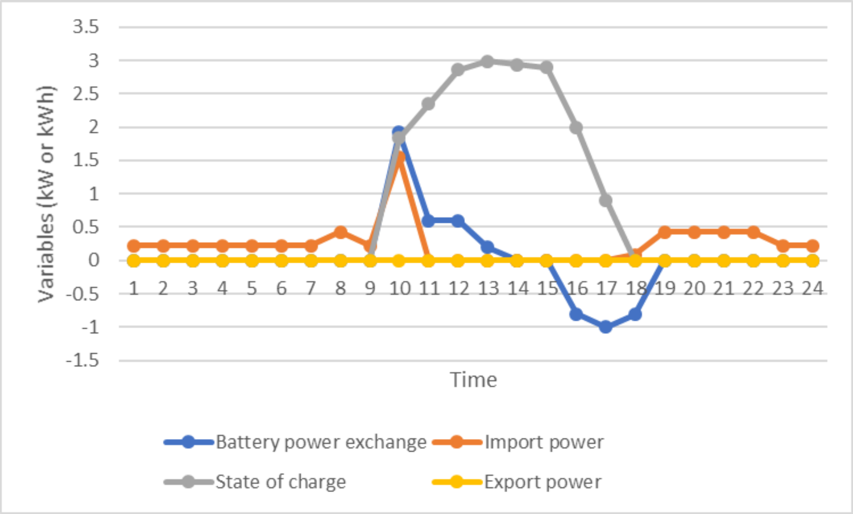


Figure 48 Results for model with approximated power conversion efficiency and power transfer limits

Another final model was built to check if it was better than the most accurate one, considering a small error not a problem when you get as a payback a much smaller computational time. Since the maximum power conversion efficiency both when importing and when exporting was reached with power transfer greater than 100W and it was 0.96 for the import power and 0.97 for the export power (where import and export means from the battery, not from the grid) the new model is considering a fixed value for the efficiency when exporting and when importing, equal to their maximum values, but it allows only power exchanges (with the battery) that are higher than 100W; the power exchanges limits according to the state of charge of the battery formulation was not changed.

Solving a model with battery and PV to minimize the energy price allowing both energy import and energy export with fixed values (one for import and one for export) for the power conversion efficiency and considering power conversion and power transfer losses, a set value for the state of charge and a maximum power discharge value allowed from the battery. The model now includes the maximum power transfer allowed from the battery considering its state of charge. The power transfer is now forced to be greater than 100W in order to make sure that the values set for the efficiency are right

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its dimension (in this case a vector), and its type (all of them are real numbers except eight which are integer numbers)
3. The objective function is added to the program to minimize the cost of energy considering a flat tariff for energy import and a flat tariff for energy export
4. For loop over the timesteps

    A constraint for the overall system instantaneous power transfer is added

    A constraint for the instantaneous battery energy transfer and state of charge variation is added, including efficiency and losses

    Two constraints are added to solve the either/or condition that the power at each instant of time can be either imported or exported

```
    A constraint to state that the overall power transferred from the battery is the difference of charging and discharging at each timestep is added

    Two constraints are added to solve the either/or condition that the battery at each instant of time can be either charged or discharged

    A constraint to state that there are battery losses only when the state of charge of the battery is greater than 0 is added

    Two constraints to state that if the power is imported from the grid, and not from the battery the losses are only the power transfer losses and not the power conversion losses as well

    The set of constraints to limit the power transfer both when charging and when discharging the battery according to its state of charge is added

    The set of constraint to make sure that the power transfer is higher than 100W is added

5. A constraint for the initial value of the state of charge to be 0 is added
6. A constraint to set the value of the state of charge to a fixed one at a defined timestep is added
7. A constraint to make sure that the overall discharged energy from the battery in the whole day is lower than a certain value is added
8. Problem is solved
9. Store significant values on a file
10. Return flag value to check if the problem is properly solved
11. Return of the significant values
12. Return plots of the significant values
```

The results for this problem are not much different from the results of the previous, more complex problem, indeed the total energy imported is 5.91 kWh (against 6kWh), the total energy discharged from the battery is 2.64kWh (against 2.61kWh), and the total cost is 0.69£ (against 0.71£); this means that this model is accurate enough and can be used to model the real system, especially if compared to the model with 90% fixed value for the efficiency and that did not take into account the maximum power transfer limits. Not only this model is accurate, it is also fast to solve, because it takes only 0.2 seconds against 0.37 seconds needed for solving the most complex model, meaning that this model runs almost 1.8 times faster.

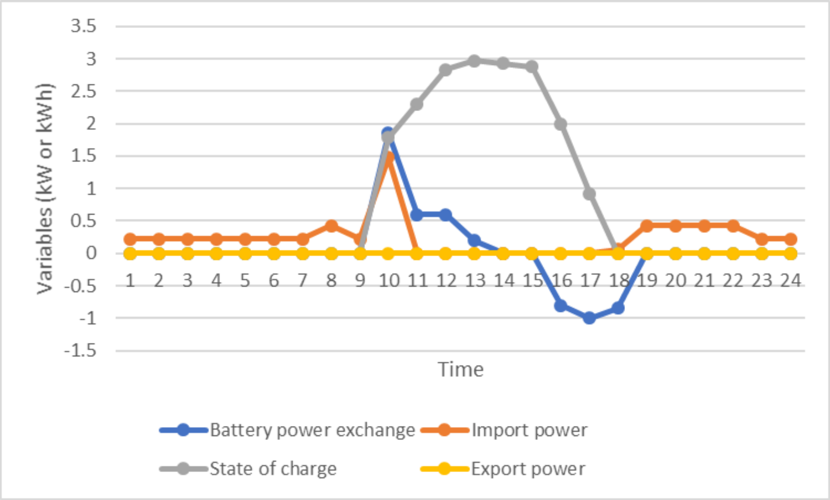


Figure 49 Results for the model with maximum values fixed for the efficiency, power exchanges greater than 100W and power transfer limits (final model)

This plot shows the results for this final model and it is almost impossible to spot any difference with the previous one, which is a proof that this last model is accurate. Considering the accuracy of this last model and the speed in terms of problem solving, this model is the chosen one to be used in the analysis of the real households, which will be described in task 10 (and partially in task 11), the data from the optimization problems solved in task 10 (and one only in task 11) will be then the input data of the power flow model in task 11, to check how the DSR affects an IEEE European LV standardized network.

3.10 Different control scenarios, individually optimized loads and results

This task was the main task in terms of optimization, is the heart of the work done here, because in this task data provided by the company about 55 real life British households, including their solar production and their load, were used as input to the model built so far in order to optimize each of the 55 systems individually for the best economic outcome in different scenarios. Not only the data for the households are real data from UK households, but as well the price for importing the energy and the price for exporting it are the real UK prices, with (for the import price) the real evolution of the price over the day. The first part was to build the vector for the prices, building one for the import price with time of use tariff, one for the export price and one for the import time but with a flat tariff, the data provided had either 1 hour or 30 minutes resolution, depending on which of the dataset, so there was the need for scaling these values to a 15 minutes resolution, because the target of the optimization is to optimize the system with a 15 minutes resolution.

```
Program made to scale the prices vectors to fifteen minutes resolution over a
day

1. Creating the empty vectors for the final prices
2. Reading thirty minutes resolution data for time of use tariff for import
   energy from a file
3. Scaling the time of use tariff for import energy with fifteen minutes
   resolution in a new vector
4. Building flat tariff for import energy 15 minutes resolution vector
5. Scaling the previously used flat tariff for export energy from one hour to
   fifteen minutes resolution and storing it into a new vector
```

```
6. Setting the new delta time to be fifteen minutes rather than one hour
```

This part was easy to do, and it was coded as showed above, the three tariffs, two for import and one for export, and the data for the real life import price are imported from a file, it is possible to notice at line 1 that the delta time to convert power into energy it is now no longer 1 but 1/4, so from now on it is very important to take care about it because it is no longer valid that energy = power because it is one hour resolution. At this point once the data for the prices were scaled it was the time to read the data for the solar production and for the loads of each of the houses and to scale them to 15 minutes resolution from the 1 minute resolution that was provided; this means taking 15 values and doing the average and so on up to the 1440th value to build a vector of 96 values with a 15 minutes resolution.

```
Program made to scale the load and the generation of each dwelling from one
minute resolution to fifteen minutes resolution

1. Reading the data from the file
2. Store the load and the generation in two different vectors
3. Initializing four temporary variables
4. Building the two empty vectors for load and generation
5. Initializing index variable = 0
6. While index variable is lower than a set value (end of the day – fifteen
   timesteps, 1440 – 15)

       For loop from the index value to the index value plus fifteen

               Store temporary load

               Store temporary generation

       Make average of load (temporary variable)

       Make average of generation (temporary variable)

       Store average values in the two vectors for generation and load

       Reset temporary variables

       Increase index, new index is equal to old one plus fifteen
```

With these few lines of code it was possible to build the vector with the input values for the constant of the solar production and the load of each system at each of 96 time steps in a 15 minutes resolution system, the division over 1000 was needed because the data were provided in W, while the problem is set up to work with kW. All of this had to be done 55 times to build the datasets for the input of the 55 systems. Once all the issues with the inputs were solved it was the time to work on the optimization problem, with different scenarios in order to be able to compare them and to check which optimization method was the one with the best economic outcome. Three different scenarios for the optimization were built plus a scenario with no optimization, where the whole load was powered directly from the grid, this last one, even if it was the easiest one and has no optimization, was very important because it became the base case against which it was possible to compare the optimization methods. The three scenarios built for the optimization are the typical control schemes that can be used to control a battery with a PV and a connection to the grid: a scenario in which the battery can be charged only from the PV, and it is not allowed then to get input power from the grid, and using a flat tariff for powering the load when the solar or the battery is not enough; a scenario in which the battery can import also from the grid, but still with the use of a flat tariff for power import; a final and more complex scenario in which the battery can be charged from the grid and the tariff for importing energy is a time of use tariff, allowing the battery in this case to be sensitive to the changes in the prices in the energy market. All the three scenarios are actually close one another, the only change between the time of use tariff scenario and the flat tariff where energy can be imported to charge the battery is actually the choosing of the tariff for the energy price, whilst the difference between the two flat tariff scenarios is the addition of a constraint forcing the battery to be charged only from the solar, only when the solar is higher than the load. Since each of the scenarios was meant to be run 55 times, a function for each scenario was coded so that it was faster to code the part in which each of the scenarios is run.

`Building a function to be called to solve the model with battery and PV to minimize the energy price allowing both energy import and energy export with fixed values (one for import and one for export) for the power conversion efficiency and considering power conversion and power transfer losses and a maximum power discharge value allowed from the battery. The model now includes the maximum power transfer allowed from the battery considering its state of charge. The power transfer is now forced to be greater than 100W in order to make sure that the values set for the efficiency are right. This function is set to be working both with a flat tariff for import energy and for a variable tariff, the tariff is one of the inputs`

1. `Function with 10 inputs returning 7 outputs`
   2. `The minimization problem is declared`
   3. `Variables are declared, each variable has its boundaries, its name, its dimension (in this case a vector), and its type (all of them are real numbers except eight which are integer numbers)`
   4. `The objective function is added to the program to minimize the cost of energy considering a flat or variable tariff for energy import and a flat tariff for energy export`
   5. `For loop over the timesteps`

      `A constraint for the overall system instantaneous power transfer is added`

      `A constraint for the instantaneous battery energy transfer and state of charge variation is added, including efficiency and losses`

      `Two constraints are added to solve the either/or condition that the power at each instant of time can be either imported or exported`

      `A constraint to state that the overall power transferred from the battery is the difference of charging and discharging at each timestep is added`

      `Two constraints are added to solve the either/or condition that the battery at each instant of time can be either charged or discharged`

      `A constraint to state that there are battery losses only when the state of charge of the battery is greater than 0 is added`

      `Two constraints to state that if the power is imported from the grid, and not from the battery the losses are only the power transfer losses and not the power conversion losses as well`

```
        The set of constraints to limit the power transfer both when
        charging and when discharging the battery according to its state of
        charge is added

        The set of constraint to make sure that the power transfer is higher
        than 100W is added

    6.  A constraint for the initial value of the state of charge to be 0 is
        added
    7.  A constraint to make sure that the overall discharged energy from the
        battery in the whole day is lower than a certain value is added
    8.  Problem is solved
    9.  Store significant values on a file
    10. Return flag value to check if the problem is properly solved
    11. Return of the significant values
    12. Return plots of the significant values
```

Without considering the whole building of the function, that is not necessary, the above code is the one used with the time of use tariff and for the flat tariff scenario with battery charging importing from the grid available, the only difference is the data that the function has set as default for the input when it comes to the “importprice” variable.

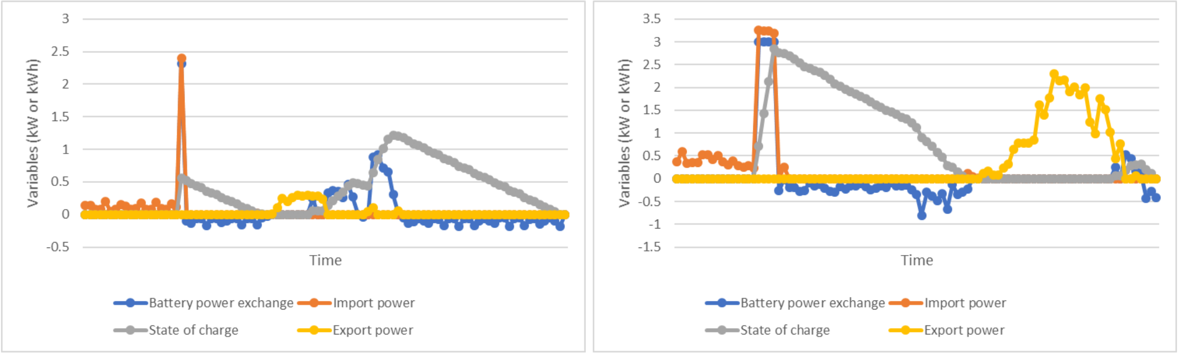
```
Building a function to be called to solve the model with battery and PV to
minimize the energy price allowing both energy import and energy export with
fixed values (one for import and one for export) for the power conversion
efficiency and considering power conversion and power transfer losses and a
maximum power discharge value allowed from the battery. The model now includes
the maximum power transfer allowed from the battery considering its state of
charge. The power transfer is now forced to be greater than 100W in order to
make sure that the values set for the efficiency are right. This function is
set to be working with a flat tariff for import energy

1. Function with 10 inputs returning 7 outputs
    2.  The minimization problem is declared
    3.  Variables are declared, each variable has its boundaries, its name, its
        dimension (in this case a vector), and its type (all of them are real
        numbers except nine which are integer numbers)
    4.  The objective function is added to the program to minimize the cost of
        energy considering a flat tariff for energy import and for energy
        export
    5.  For loop over the timesteps

            A constraint for the overall system instantaneous power transfer is
            added

            A constraint for the instantaneous battery energy transfer and state
            of charge variation is added, including efficiency and losses

            Two constraints are added to solve the either/or condition that the
            power at each instant of time can be either imported or exported

            A constraint to state that the overall power transferred from the
            battery is the difference of charging and discharging at each
            timestep is added

            Two constraints are added to solve the either/or condition that the
            battery at each instant of time can be either charged or discharged
```

```
          A constraint to state that there are battery losses only when the
          state of charge of the battery is greater than 0 is added

          Two constraints to state that if the power is imported from the
          grid, and not from the battery the losses are only the power
          transfer losses and not the power conversion losses as well

          The set of constraints to limit the power transfer both when
          charging and when discharging the battery according to its state of
          charge is added

          The set of constraint to make sure that the power transfer is higher
          than 100W is added

          The set of constraints to make sure that the battery charges only
          from the solar is added

    6.  A constraint for the initial value of the state of charge to be 0 is
        added
    7.  A constraint to make sure that the overall discharged energy from the
        battery in the whole day is lower than a certain value is added
    8.  Problem is solved
    9.  Store significant values on a file
    10. Return flag value to check if the problem is properly solved
    11. Return of the significant values
    12. Return plots of the significant values
```

This is the code for the scenario where the battery was allowed to be charged only from the solar, when the solar is greater than the demand, there is one extra decisional variable, and the main difference is that the power charging the battery is constrained to the solar power. The last scenario does not have any linear programming problem to be solved, it just reads the data from the load and sets the import power to be equal to the load at each time step, then it gets the cost for the import energy with a flat tariff, though as already mentioned before it is a very important scenario as it is the base case to compare against.

```
Program made to build the base case where no PV or battery is used at each
household

1. Storing import power in a variable
2. Building import energy vector multiplying the import power one times the
   delta time
3. Getting the energy price
4. Storing significant data into file
5. Plotting significant data
```

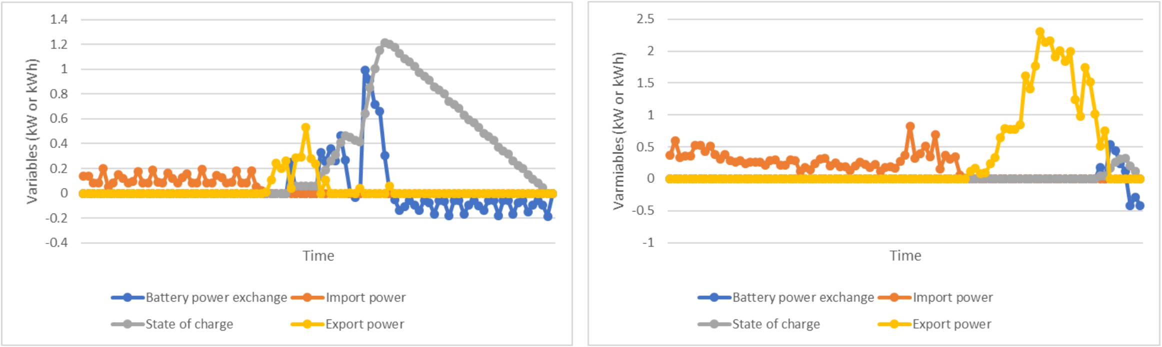
These few lines are enough to build the base case scenario, it actually is even a bit too simple because no losses for the import power were assumed, but it does not make any difference, because it will anyway be the worst case scenario by far, and the losses are very low. It would be useless to show the results for all the four scenarios and the 55 households, though the plots for one household and all the scenarios (base case, or actually load profile) included as well will now follow and be commented. The scenarios were run on two different days, the dataset included data from the 1st of January to the 31st of March, and to check the difference that the weather can make on a solar system, plus the seasonality of the solar energy the models were run of 1st of January and on the 31st of March and the difference in the results is very remarkable.



a) b)

Figure 50 Results for the 48th system, fully optimized with time of use tariff, 1st of Jan (a) and 31st of Mar (b)

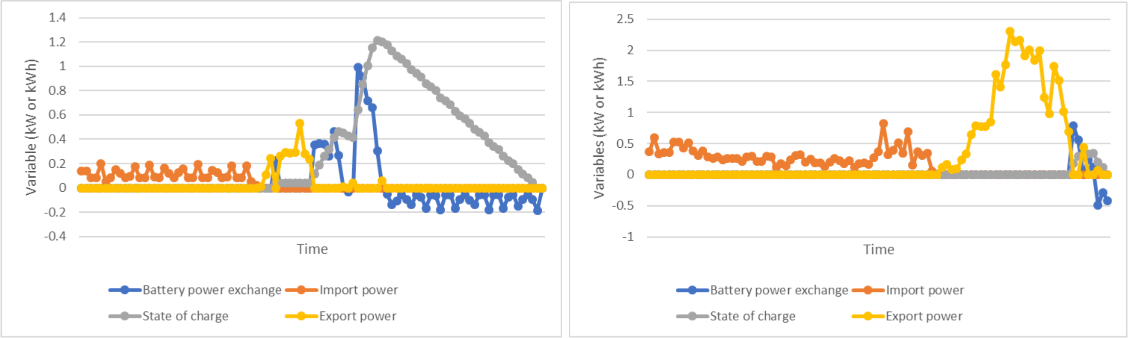
For this scenario the total energy imported is 1.16 kWh, the cost is 0.05£ and the energy discharged from the battery is 1.7 kWh on the 1st of January and the total energy imported is 4.88kWh, the cost is -0.05£ (gain) and the energy discharged from the battery is 2.59kWh on the 31st of March.



a) b)

Figure 51 Results for the 48th system, fully optimized with flat tariff, 1st of Jan (a) and 31st of Mar (b)

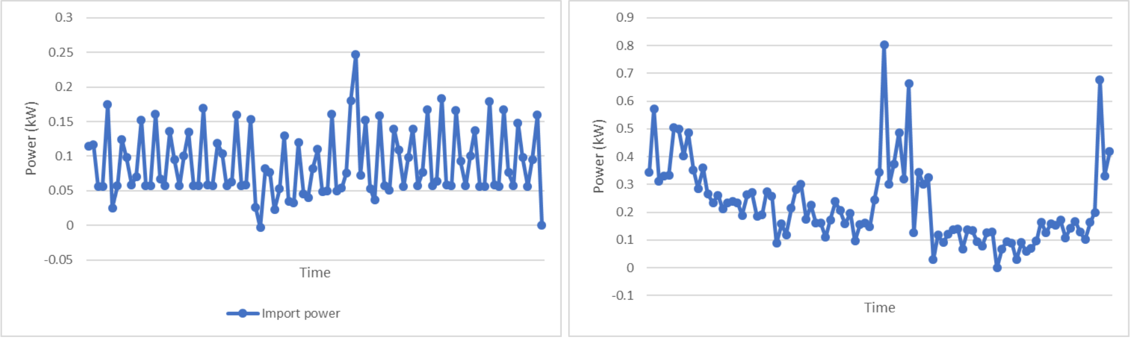
For this scenario the total energy imported is 1.04 kWh, the cost is 0.13£ and the energy discharged from the battery is 0.8 kWh on the 1st of January and the total energy imported is 4.44kWh, the cost is 0.28£ and the energy discharged from the battery is 0.28kWh on the 31st of March.



a) b)

Figure 52 Results for the 48th system, with battery charging only from solar, with flat tariff, 1st of Jan (a) and 31st of Mar (b)

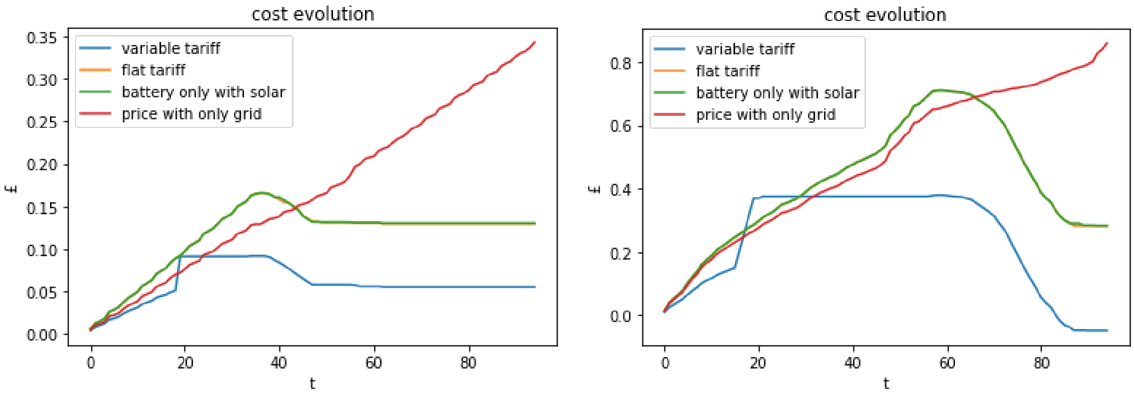
For this scenario the total energy imported is 1.04 kWh, the cost is 0.13£ and the energy discharged from the battery is 0.8 kWh on the 1st of January and the total energy imported is 4.44 kWh, the cost is 0.28£ and the energy discharged from the battery is 0.30kWh on the 31st of March.



a) b)

Figure 53 Results for the 48th system, without any battery or solar (base case, load profile), 1st of Jan (a) and 31st of Mar (b)

For this scenario the total energy imported is 2.14 kWh, the cost is 0.34£ and the energy discharged from the battery is obviously 0 kWh since there is no battery on the 1st of January and the total imported energy is 5.36kWh and the price is 0.86£ on the 31st of March. The first difference that is possible to spot is how the seasonality affects the DSR potential and the savings, and how it affects the demand as well: it is clear that for the fully optimized scenario the 31st of March results are much better in an economic point of view as compared to the ones on the 1st of January, actually the household rather than paying for energy gets an income because of the high export due to a much higher solar production; on the other hand it is possible to notice that the load itself is much higher, indeed in all the other scenarios the price for the energy is higher on the 31st of March rather than on the 1st of January, nevertheless the exported energy and the energy discharged from the battery are greater on March, this is because of the higher solar production, while the imported energy is as well higher on March, due to the higher load. It was as well expected that there is a big difference in the price when changing the tariff from a flat one to a time of use one, and not only in the price, actually it changes a lot also the optimization of the model, changing the shape of the evolution of the variables and as well the amount of imported energy (and the time where the import spike happens). This is important to remember because it will affect the network stability when running the power flows in the next task. The best scenario (use of time one) will have a spike in the loading (with regards to the grid) that is much higher than the peak load without the battery or any optimization at all. Obviously the other 54 cases will not evolve in the same way, nevertheless the spike in the demand for the time of use optimization scenario always happens in the early morning, roughly at the same time for all the systems, because of the low import price at that time, the difference in the price and in the energy import will be anyway comparable to this case for all the cases, meaning that the time of use scenario is by far the most demanding but as well the best one economic wise. The other two scenarios, just like in this case are always very similar, sometimes even identical, that happens because the price for the energy import is very high (and it is the same in both the scenarios) so even when it is possible to charge the battery from the grid and not only from the sun there is actually almost no difference in price, because the amount of energy needed to power up the load doesn't actually change, so even if it is possible to charge the battery from the grid, it is cheaper to charge the battery almost only when the solar production exceeds the demand, thus the two remaining scenarios (both with flat tariff, one allowing charging from the grid) give almost the same solution. These two scenarios are clearly the least demanding one from a grid energy import point of view, they never demand more than the actual load without any battery or optimization, and actually when the solar exceeds the demand the battery gets charged and then used to power the load, decreasing the demand from the grid. Obviously, the cost for these two scenarios are similar and higher than the cost with a use of time tariff. The base case, or load profile, or no battery scenario, is clearly the most expensive one, since it uses a flat tariff and has no chance to be optimized because there is no PV or battery. This next plot will help understanding the evolution in the cost over the day, with the final value being the actual value for the price of the import of energy per household, per scenario.



a) **b)**

Figure 54 Results for the 48th system, cost evolution over time, 1st of Jan (a) and 31st of Mar (b)

Without considering how the optimization affects the grid it is possible to conclude that the addition of a battery and a PV, regardless of the control system used and of the tariff for the energy import, will improve, economic wise, that single system cost for electrical energy, furthermore with a time of use tariff and a fully optimized control system, the economic impact is even bigger reducing the electricity bill in a very sensitive way. This is the first thing to consider to incentivize people to start using battery systems and PVs, but this does not mean that the fully optimized scenario is an improvement also for the network operator, because often the most economical solution is not a good solution power flow wise, and sometimes it is not even a feasible solution. To check whether a network with individually fully optimized households with a PV and a battery is a stable network a power flow analysis is required, which is what happens in the 11th task.

3.11 Power flow analysis and optimization of the system as a whole

Before analyzing the impact on a standard IEEE LV network of this system of domestic loads, the standard network model itself needed to be built on python and to be solved using pypower, and checked against the IEEE solution obtained using OpenDSS in order to make sure that the python model gives an accurate solution compared to the OpenDSS one. The main difference between the two model is that pypower will solve the three phase unbalanced system with a single phase equivalent, balanced model, this can cause difference in the results and inaccuracies, furthermore the solver itself is a different one, so it works differently, and the model must be coded in order to be solved, and there might be minor errors in the code. The IEEE network is a low voltage (416 V) network made up by 55 domestic loads, 907 buses, 906 lines, fed by one generator via a transformer (11 kV/416V); the base MVA used for solving the network is 0.8 that is also the MVA rating of the transformer. The code for modelling the network will not be provided because is a too long code (1 line per bus, one line per line, one line per generator) but still a figure showing the actual IEEE network will follow.

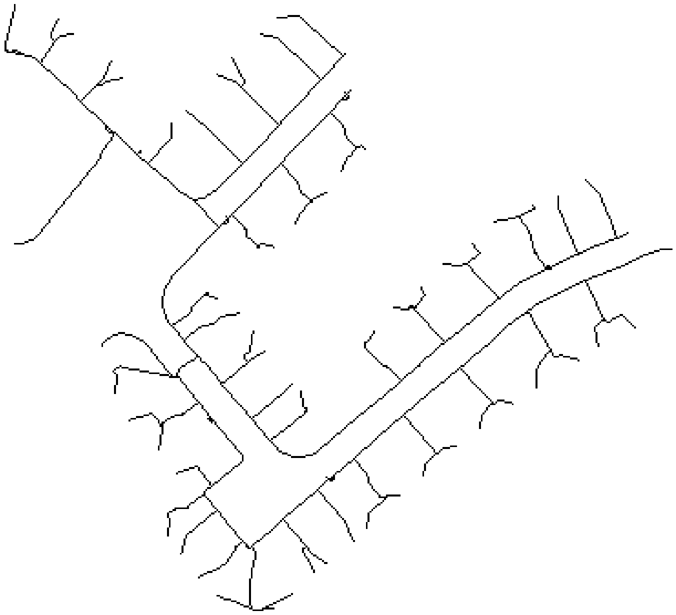


Figure 55 One-line diagram of the IEEE European LV Network test feeder [27]

For the work done each scenario was run 96 times, in a for loop, once per timestep, and the load was each time automatically changed in the model by the system, because the model was built in a way to be automatized. Once the IEEE original model was ready to be solved it was run and the solutions were then compared with the solutions provided by IEEE and obtained using a three-phase power flow model on openDSS, the following plots show the comparison and the results of the model run on the different solvers.

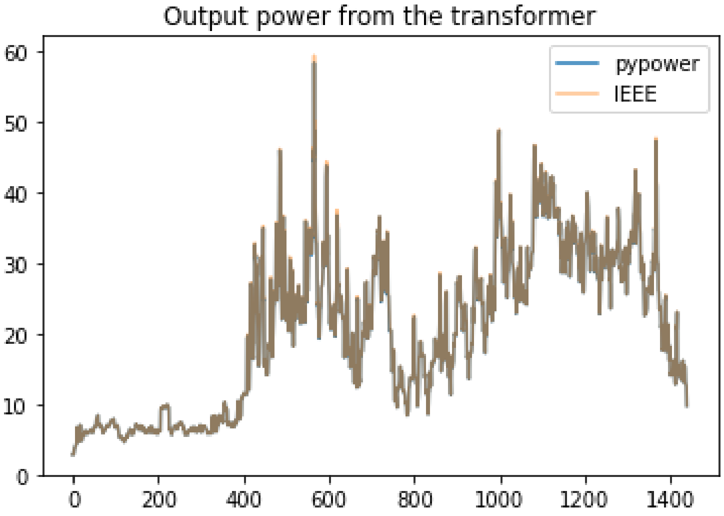


Figure 56 Bus 1 power output [kW] solvers comparison over time [minutes]

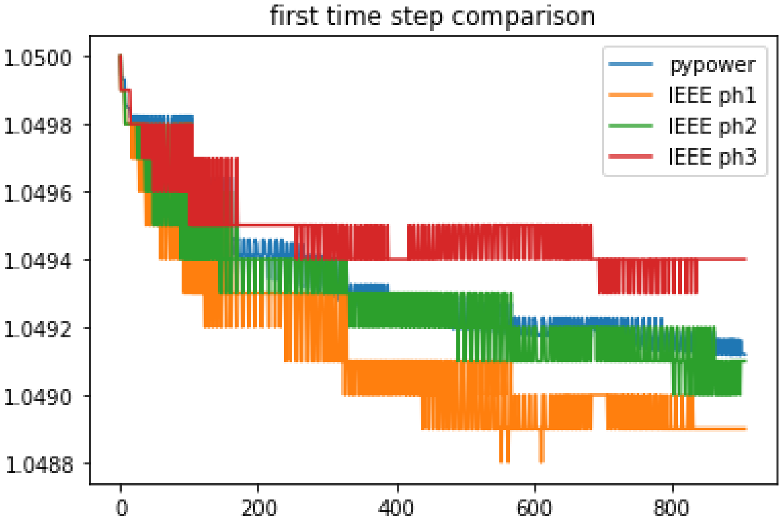


Figure 57 Pu bus voltage solvers comparison at first time step

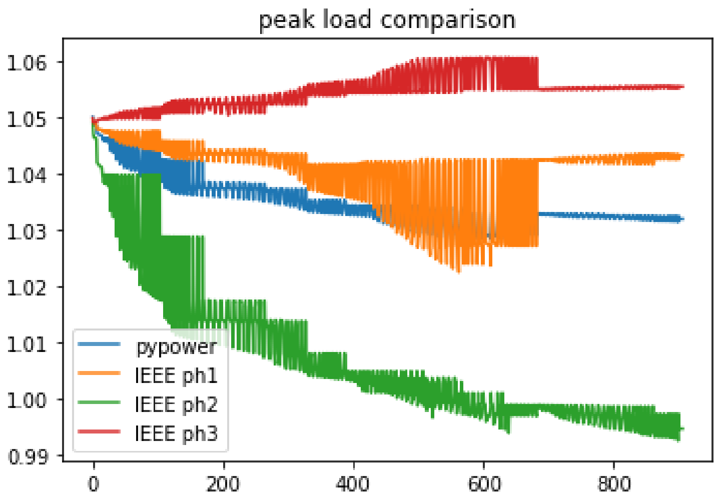


Figure 58 Pu bus voltage solvers comparison at peak load time step

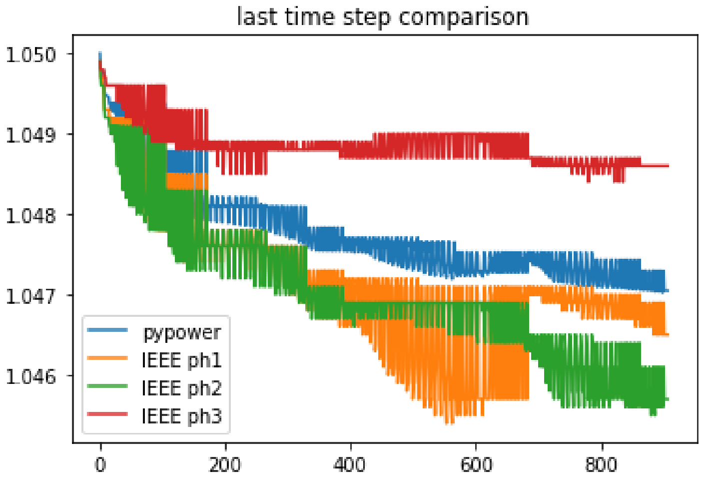
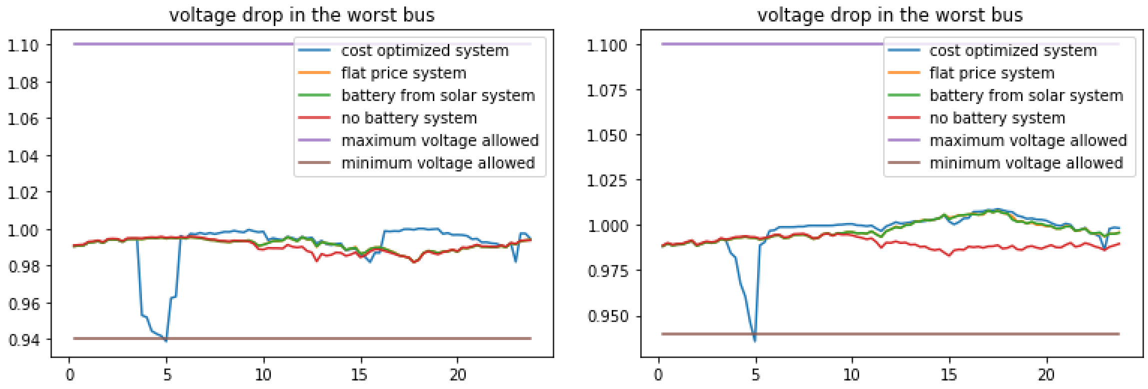


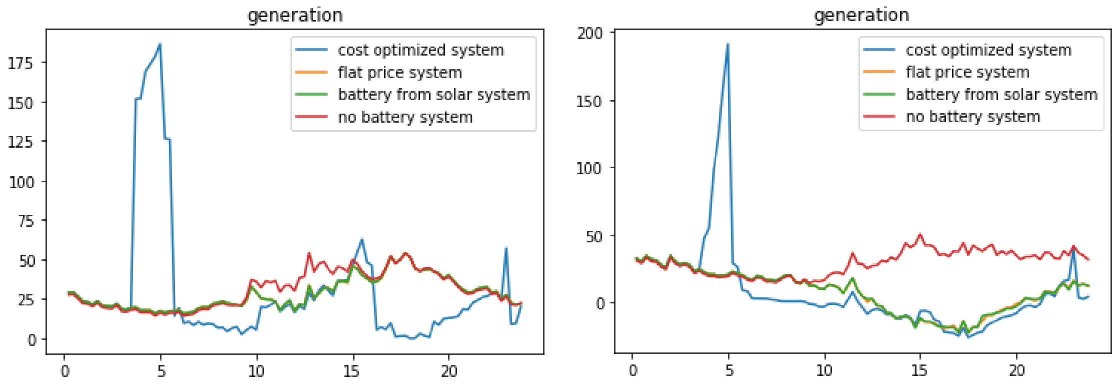
Figure 59 Pu bus voltage solvers comparison at the last time step

The first plots is a comparison of the power output of the transformer at each time, and it is clear that the two models match, meaning that the model built and solved by pypower is accurate, but that is not enough, because that is a solution taken from one bus only, and analyzed over the time, and furthermore that was the output of the bus feeding the whole network, so even if the network behaves differently, that bus would probably match anyway. The next three plots though are a proof that the model built and solved in pypower is an accurate model, because those three plots are plots of the per unit voltage in the whole network at different times, at the first one, at the peak load time, and at the last time step. Obviously the pypower solution does not match any of the phases of the IEEE model, but it is always close to the IEEE solution, and always in between the less and the most loaded phase, which is well expected, since the pypower model does not take into account the phases imbalances. Usually the per unit voltage is close to or lower than one, unless there is negative load on the network, in this case the voltage is roughly always above 1 per unit, and this is because the model was run with the generator voltage setpoint to be 1.05 per unit, only because the IEEE test was run that way. Now that model and solver using python and pypower is proved to be accurate and properly working, it is possible to move a further step towards the final solution: run a power flow of the network with the base case load and the optimized systems to check how the optimization affects the power flow. For this purpose, the resolution was again a fifteen minutes resolution and not a one-minute resolution, because the data for the input are fifteen minutes resolution data. The following plots are the result of the power flow analysis on the whole network, with the loading system provided by the modelling done in the previous task, taking into account all the scenarios, the no battery system is the base case, what it would happen if no PV or battery was installed on the network. The analysis include the per unit voltage drop analysis, plotting the data for the bus where the drop is the highest, the generator output power, and the losses over the whole network, an analysis on the overall costs for the whole system in each scenario, and for the worst voltage drop in each scenario is provided both considering the 1st of January case and considering the 31st of March case.



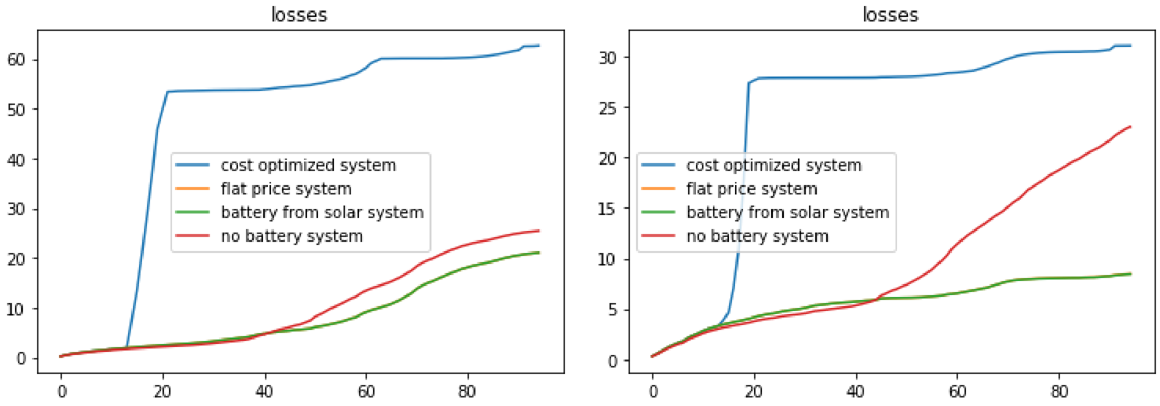
a) **b)**

Figure 60 Power flow results, pu voltage scenarios comparison over time at bus 900, 1st of Jan (a) and 31st of Mar (b)



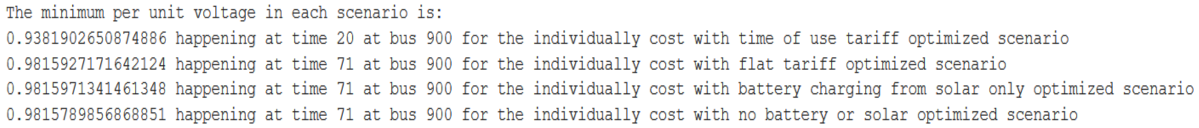
a) **b)**

Figure 61 Power flow results, kW power output of the generator, 1st of Jan (a) and 31st of Mar (b)



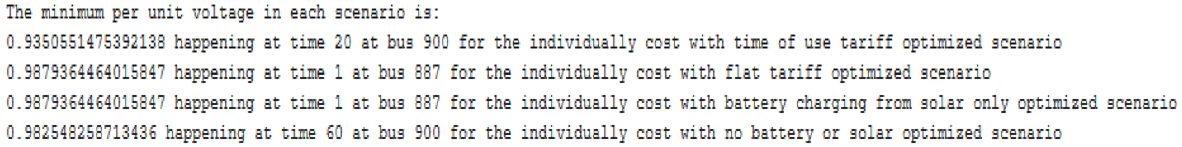
a) b)

Figure 62 Power flow results, kW losses over the whole system, 1st of Jan (a) and 31st of Mar (b)



```
The minimum per unit voltage in each scenario is:
0.9381902650874886 happening at time 20 at bus 900 for the individually cost with time of use tariff optimized scenario
0.9815927171642124 happening at time 71 at bus 900 for the individually cost with flat tariff optimized scenario
0.9815971341461348 happening at time 71 at bus 900 for the individually cost with battery charging from solar only optimized scenario
0.9815789856868851 happening at time 71 at bus 900 for the individually cost with no battery or solar optimized scenario
```

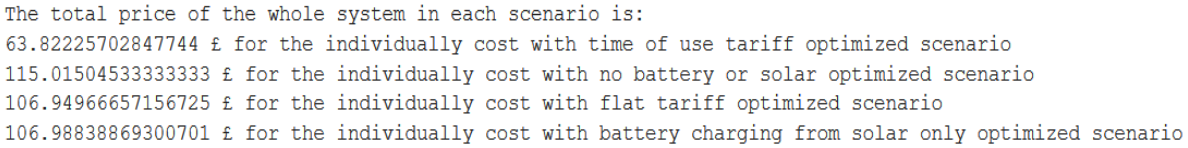
a)



```
The minimum per unit voltage in each scenario is:
0.9350551475392138 happening at time 20 at bus 900 for the individually cost with time of use tariff optimized scenario
0.9879364464015847 happening at time 1 at bus 887 for the individually cost with flat tariff optimized scenario
0.9879364464015847 happening at time 1 at bus 887 for the individually cost with battery charging from solar only optimized scenario
0.982548258713436 happening at time 60 at bus 900 for the individually cost with no battery or solar optimized scenario
```

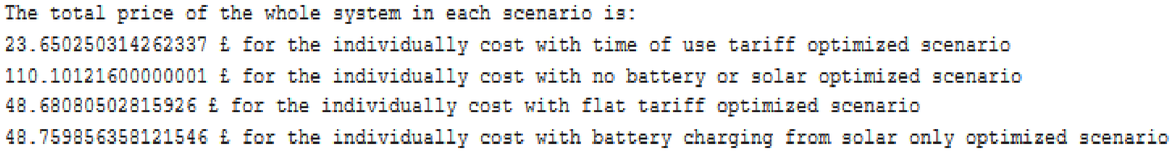
b)

Figure 63 Power flow results, pu bus voltage worst case per scenario, 1st of Jan (a) and 31st of Mar (b)



```
The total price of the whole system in each scenario is:
63.82225702847744 £ for the individually cost with time of use tariff optimized scenario
115.01504533333333 £ for the individually cost with no battery or solar optimized scenario
106.94966657156725 £ for the individually cost with flat tariff optimized scenario
106.98838869300701 £ for the individually cost with battery charging from solar only optimized scenario
```

a)



```
The total price of the whole system in each scenario is:
23.650250314262337 £ for the individually cost with time of use tariff optimized scenario
110.10121600000001 £ for the individually cost with no battery or solar optimized scenario
48.68080502815926 £ for the individually cost with flat tariff optimized scenario
48.759856358121546 £ for the individually cost with battery charging from solar only optimized scenario
```

b)

Figure 64 Power flow results, whole system time comparison per scenario, 1st of Jan (a) and 31st of Mar (b)

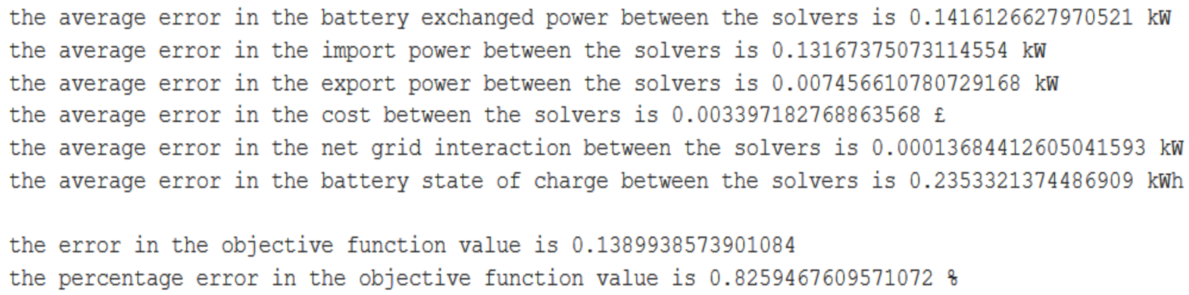
In the UK standard the minimum allowed per unit voltage on a system is 0.94pu and the maximum is 1.1pu, that is why on the first plot there are two constants value to check whether any of them is crossed, which would mean an unstable system, so that model could not be installed in real life. What it actually happens is that the individually cost optimized system actually crosses that boundary meaning that it is an unstable, thus infeasible, system. Economic wise as it is possible to see from the last picture, which is the output of the price comparison, the optimized scenario with the use of time tariff though is by far the best one, but as it is possible to see also from the figure which is the output of a comparison to check when and where the lowest voltage is reached, the voltage drops down to 0.938pu, which is lower than 0.94 (it is clear from the pu voltage plot). The optimized system is also more demanding in terms of generation and has bigger losses, which means that stronger lines are required, so considering all of these things together, even if it is optimal by an economic point of view it is not a good system so be used. To overcome this issue a model coordinating all the systems and solving them together to have a maximum power demand and to reduce the peak demand was built; this model including a cooperation is what actually flexibility and DSR can provide to a system. Before giving the results obviously a description of the model will follow. The model built for the coordination of the 55 loads in order to have a standard acceptable voltage drop on the whole network is not much different as compared to the model of the single household fully optimized, with time of use tariff. Comparing the 1st of January results against the 31st of March ones in the power flow analysis it is actually possible to notice that the difference is not as big as it was for the economic analysis, in the per unit voltage and in the losses the shapes are comparable, though if we check the values the losses are roughly the half in the 31st of March because more energy is produced and used locally, while generation wise, the shapes are roughly comparable, with some differences, there is only one big spike in the morning in the 31st of March results, while the 1st of January has other tinier spikes, and the most important difference is that in the afternoon in the 31st of March case the generation goes negative, meaning a net export power from the system to the grid, and the pu voltage obviously becomes greater than 1 (but never gets anywhere close the 1.1 pu limit). Considering the whole system cost, as it was possible to guess form the comparison made on task 10, the difference between the 1st of January and the 31st of March is huge. So overall it is possible to say that the potential of the DSR is much higher when the solar production is higher as well (regardless of the loads, that in this case were higher), both in an economic point of view and on a power flow point of view leading to a huge price and losses reduction, and as well a better voltage profile, even though the profile still reaches values out of the standard and needs to be rearranged in a cooperative optimized system.

Solving a model with 55 systems each having a battery and a PV to minimize the overall energy price allowing both energy import and energy export with fixed values (one for import and one for export) for the power conversion efficiency and considering power conversion and power transfer losses and a maximum power discharge value allowed from the battery. The model now includes the maximum power transfer allowed from the battery considering its state of charge. The power transfer is now forced to be greater than 100W in order to make sure that the values set for the efficiency are right. The limit of the overall maximum import power per timestep is added in order to cope with power flow undervoltage limits

1. The minimization problem is declared
2. Variables are declared, each variable has its boundaries, its name, its dimension (in this case a matrix of two dimension considering the timesteps and the number of systems), and its type (all of them are real numbers except eight which are integer numbers)

```
3. The objective function is added to the program to minimize the cost of energy considering a flat tariff for energy import and a flat tariff for energy export
4. For loop over the timesteps
       For loop over the number of systems
           A constraint for the overall system instantaneous power transfer is added
           A constraint for the instantaneous battery energy transfer and state of charge variation is added, including efficiency and losses
           Two constraints are added to solve the either/or condition that the power at each instant of time can be either imported or exported
           A constraint to state that the overall power transferred from the battery is the difference of charging and discharging at each timestep is added
           Two constraints are added to solve the either/or condition that the battery at each instant of time can be either charged or discharged
           A constraint to state that there are battery losses only when the state of charge of the battery is greater than 0 is added
           Two constraints to state that if the power is imported from the grid, and not from the battery the losses are only the power transfer losses and not the power conversion losses as well
           The set of constraints to limit the power transfer both when charging and when discharging the battery according to its state of charge is added
           The set of constraint to make sure that the power transfer is higher than 100W is added
5. For loop over the number of systems
       A constraint for the initial value of the state of charge to be 0 is added
       A constraint to make sure that the overall discharged energy from the battery in the whole day is lower than a certain value is added
6. For loop over the timesteps
       A constraint to limit the maximum power import from the grid for the whole system is added
7. Problem is solved with a time limit of 45 minutes for the program to run
8. Store significant values on a file
9. Return flag value to check if the problem is properly solved
10. Return of the significant values
11. Return plots of the significant values
```

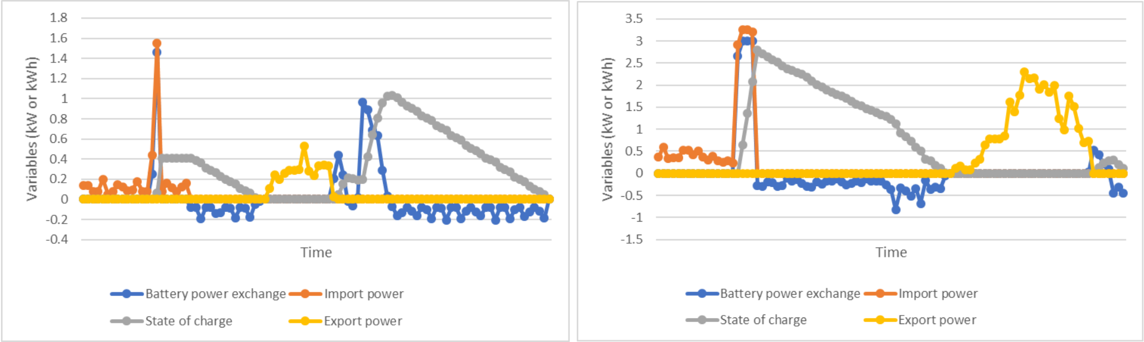
The main difference is that all the variables are no longer mono dimensional variables, but are bidimensional ones, in order to have both the information of each timestep and the information of each of the 55 households, the other difference is in the addition of the constraint in line 43/44 limiting the whole demanded power from the grid to be lower than or equal to 160kW at each time. Considering the complexity of the system, a solution was not reached after that the code was left running for days, so the same code was run also on a much faster solver (CPLEX) and after few minutes it reached a difference in the objective value obtained in two consecutive iterations that was lower than 1%, in order to make sure that also the solution obtained from the regular PuLp solver was accurate, the problem was left running for 45 minutes, forcing the solver to stop when the difference in the objective value obtained between two consecutive iterations reached was lower than 1%. At this point the two solutions were checked one against the other to make sure that the regular PuLp solver solution was accurate, and it was possible to use that one in the following steps left to finish the work. The actual check was done on a lower system of 22 households, but that does not affect the validity of the solution on a wider model, because the solver is still the same so the steps it does to reach a solution are still the same; the next figure shows the output of the code used to check the solutions of the solvers.



```
the average error in the battery exchanged power between the solvers is 0.1416126627970521 kW
the average error in the import power between the solvers is 0.13167375073114554 kW
the average error in the export power between the solvers is 0.007456610780729168 kW
the average error in the cost between the solvers is 0.003397182768863568 £
the average error in the net grid interaction between the solvers is 0.00013684412605041593 kW
the average error in the battery state of charge between the solvers is 0.2353321374486909 kWh

the error in the objective function value is 0.1389938573901084
the percentage error in the objective function value is 0.8259467609571072 %
```

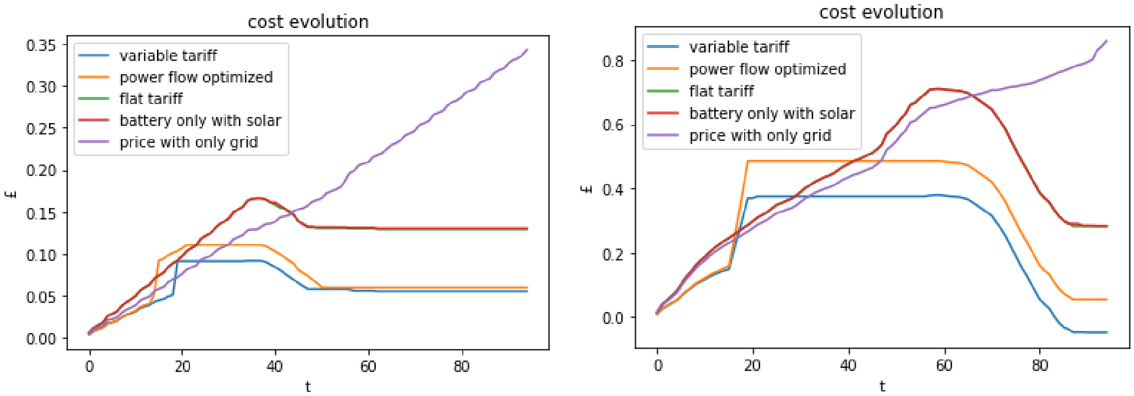
Figure 65 Results, comparison for coordinated system model

It is noticeable that all the errors are very small, but the most important value is the percentage error in the objective value reached, which is smaller than one per cent, that is the proof that the PuLp solve, even if it is slower, it reaches an accurate solution to the problem. The main issue at this point was to find the maximum power demand allowed per time step value, to do this a low limit was imposed the first time, and it was increased until the per unit voltage drop in the worst case was the closest possible to reach 0.94 per unit, without actually going lower than that value. Once that value was found and the coordinated system was solved, a solution of each of the households was available, in order to make a comparison with the data in task 10, the next figures are the outputs of the optimized problem for the 48th system and the cost evolution over that time for that specific system.



a) b)

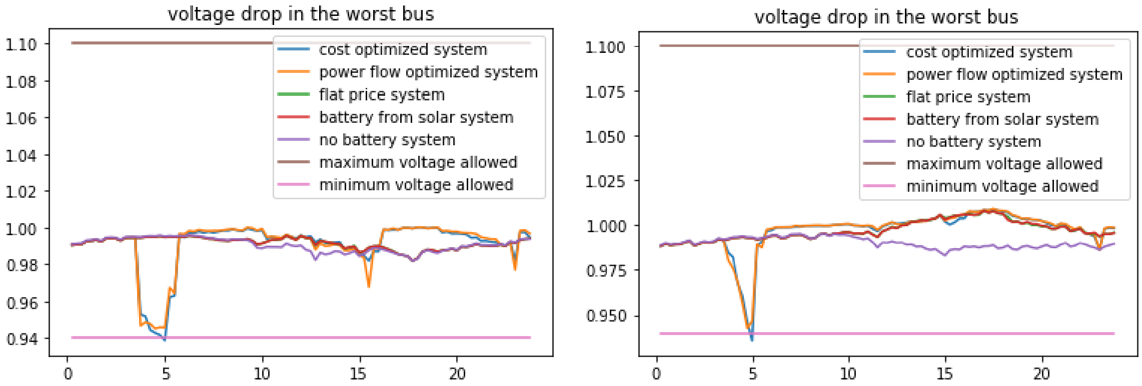
Figure 66 Results for the 48th system, coordinated against power flow optimized with time of use tariff, 1st of Jan (a) and 31st of Mar (b)



a) b)

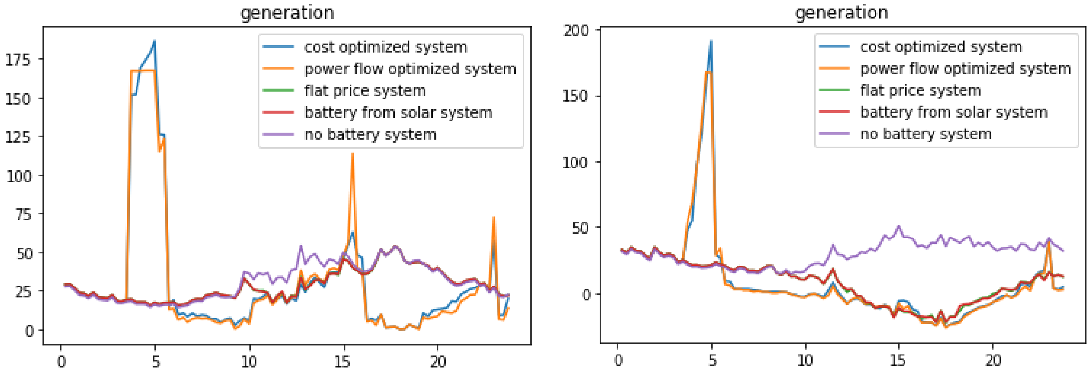
Figure 67 Results for the 48th system, cost evolution over time, 1st of Jan (a) and 31st of Mar (b)

It is possible to check that the peak demand now instead of being roughly 2.6 kW is roughly 1.6 kW for the 1st of January , while it keeps being close to 3.4 kW for the 31st of March, but even if it is not very noticeable, some minor changing in the loading curves happen, there are less spikes and as well the highest peak is reached for one timestep only rather than being constant for a while; so as expected there is a peak reduction, that on the other hand increases a bit the energy price, it is possible to notice that on the cost evolution figure, the cost actually raised from 0.05£ to 0.06£ for the 1st of January case, while it raised from -0.05£ to 0.05£ for the 31st of March case. The last thing left to do at this point is the power flow analysis of the coordinated system. The results are displayed in the following plots and figures.



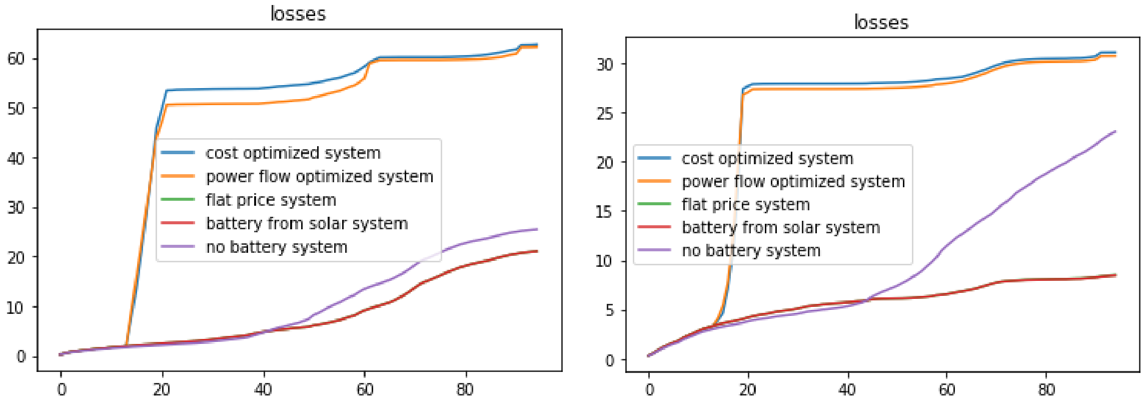
a) **b)**

Figure 68 Power flow results, pu voltage scenarios comparison over time at bus 900, 1st of Jan (a) and 31st of Mar (b)



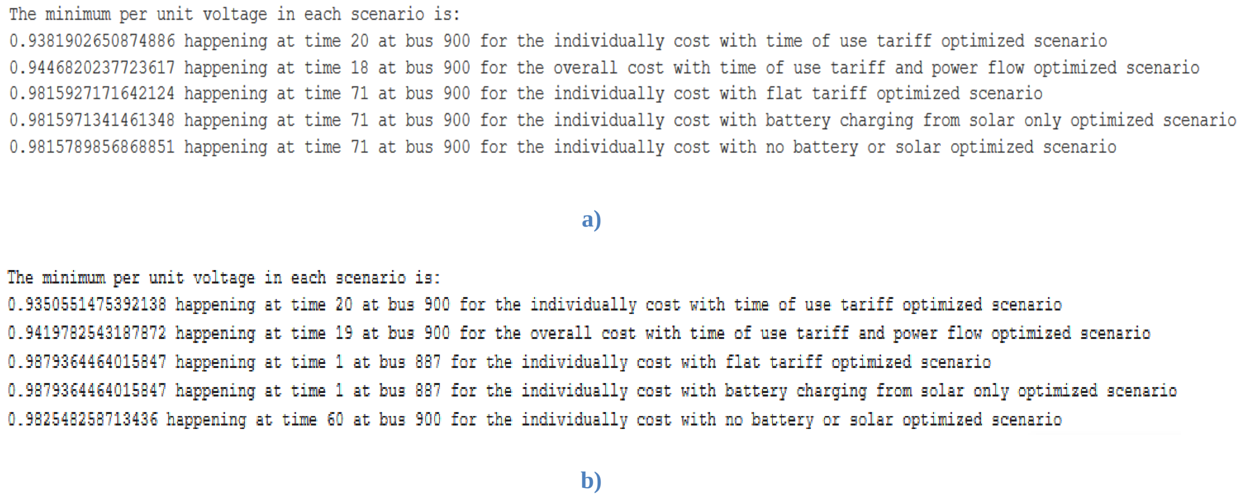
a) **b)**

Figure 69 Power flow results, kW power output of the generator, 1st of Jan (a) and 31st of Mar (b)



a) **b)**

Figure 70 Power flow results, kW losses over the whole system, 1st of Jan (a) and 31st of Mar (b)



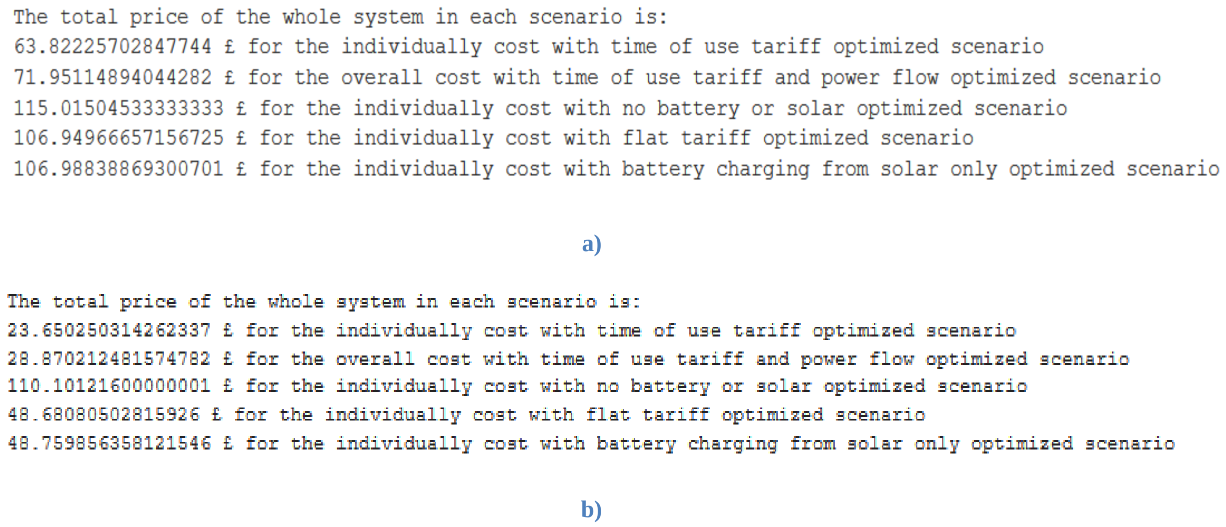
```
The minimum per unit voltage in each scenario is:
0.9381902650874886 happening at time 20 at bus 900 for the individually cost with time of use tariff optimized scenario
0.9446820237723617 happening at time 18 at bus 900 for the overall cost with time of use tariff and power flow optimized scenario
0.9815927171642124 happening at time 71 at bus 900 for the individually cost with flat tariff optimized scenario
0.9815971341461348 happening at time 71 at bus 900 for the individually cost with battery charging from solar only optimized scenario
0.9815789856868851 happening at time 71 at bus 900 for the individually cost with no battery or solar optimized scenario
```

a)

```
The minimum per unit voltage in each scenario is:
0.9350551475392138 happening at time 20 at bus 900 for the individually cost with time of use tariff optimized scenario
0.9419782543187872 happening at time 19 at bus 900 for the overall cost with time of use tariff and power flow optimized scenario
0.9879364464015847 happening at time 1 at bus 887 for the individually cost with flat tariff optimized scenario
0.9879364464015847 happening at time 1 at bus 887 for the individually cost with battery charging from solar only optimized scenario
0.982548258713436 happening at time 60 at bus 900 for the individually cost with no battery or solar optimized scenario
```

b)

Figure 71 Power flow results, pu bus voltage worst case per scenario, 1st of Jan (a) and 31st of Mar (b)



```
The total price of the whole system in each scenario is:
63.82225702847744 £ for the individually cost with time of use tariff optimized scenario
71.95114894044282 £ for the overall cost with time of use tariff and power flow optimized scenario
115.01504533333333 £ for the individually cost with no battery or solar optimized scenario
106.94966657156725 £ for the individually cost with flat tariff optimized scenario
106.98838869300701 £ for the individually cost with battery charging from solar only optimized scenario
```

a)

```
The total price of the whole system in each scenario is:
23.650250314262337 £ for the individually cost with time of use tariff optimized scenario
28.870212481574782 £ for the overall cost with time of use tariff and power flow optimized scenario
110.10121600000001 £ for the individually cost with no battery or solar optimized scenario
48.68080502815926 £ for the individually cost with flat tariff optimized scenario
48.759856358121546 £ for the individually cost with battery charging from solar only optimized scenario
```

b)

Figure 72 Power flow results, whole system time comparison per scenario, 1st of Jan (a) and 31st of Mar (b)

From these plots it is clear now that the per unit voltage never gets lower than 0.94, reaching the minimum value of 0.94 both in the 1st of January case and in the 31st of March one, which means that the coordinated system is accepted by standard, thus it is a feasible system. Considering the generation, it is noticeable how the peak power demand was reduced, indeed the spike is lower, but it keeps staying at high demand level for longer time, and there is another spike during the afternoon, by the point of view of the losses there is not a big difference, the two final values almost match, and the shapes are similar as well; that is anyways expected, because this new system is as close as possible to the individually optimized one, the main difference is that it prevents the peak demand to avoid an undervoltage in the network. On the economic side though this system costs a bit more to the users than the individually optimized one, with a total price of roughly 72£ against 64£ for the 1st of January case, while the price increases from roughly 24£ to roughly 29£ for the 31st of March case; this means that in order to have the users participate in coordination and DSR an incentive scheme is needed. The other general comments comparing the power flow results of 1st of January case against the 31st of March case were already provided before, when analyzing the results of the power flow analysis without the cooperative system scenario, those comments are still valid, and the differences keep being the same, even in the cooperative system scenario.

4 Chapter Four – Conclusions and future works

4.1 Conclusions

All the results obtained in the simulations were widely commented and discussed, yet summarizing all the work done to prove the impact of domestic DSR on an IEEE standard low voltage network, using real UK data, it is possible to say that:

- The difference economic wise between the results obtained on the 1st of January and on the 31st of March is remarkable, this not only can be a confirmation of the accuracy of the model, but also shows how the impact of the solar energy changes with the changing of the season, and even the loading level itself changes; it was easy to notice that the load was much higher, and the increase in the load was even bigger than the increase in the solar, nevertheless the economic impact on the single household was bigger on the 31st of March, leading to an actual negative price for some households using the fully optimized scenario, which means a gain rather than a payment, it is possible to think that in the summer the impact of using a PV and battery system would be even higher, leading to even lower prices because of the much higher solar production together with lower need for heat and lighting;
- The difference power flow wise of the impact of the DSR between the 1st of January and the 31st of March is remarkable as well: having a greater local production roughly halves the losses on the network, causing less stress to the component and giving them the chance to live longer, furthermore less losses lead to lower price for the energy as well; not only the losses were lower on the 31st of March, considering the voltage profile, even if it was similar, there were period in time during the afternoon when the pu voltage was actually higher than one, which means that the energy is being exported back to the grid and the production is greater than the load; on the generation point of view, since the loads were higher the peak power for the generator was higher on the 31st of March, but out of the peak time the values and the shapes are comparable between the two days, and considering that in the afternoon the generation goes negative for a while, meaning a net energy export from the system as a whole, back to the grid, the average value for the generation over the day is probably very close between the two days;
- A network that does not cooperate, but in which each household is provided with battery and solar system would cause instability in the grid due to under voltages, this happens because if each system is individually optimized for the best economic outcome, with a time of use tariff, all the systems will be importing power in their batteries at the same time, causing a huge spike in the demand, leading to a voltage drop below 0.94 per unit, that is the minimum voltage allowed in the UK;
- If there is cooperation in the demand, thus DSR, the network will be stable, because it will be possible to reduce the peak power demand, spreading in onto a wider time interval, but at a lower power level; this way the undervoltage is avoided, the losses are reduced and the peak demand, thus the peak power generation, is reduced as well, moreover the voltage profile and the power demand profile are flatter, even though there is a second spike in the demand that didn’t happen with the individually optimized loads;
- The cooperative system overall price over a day is slightly higher than the individually optimized price, both the prices are anyway much lower than the price paid with a flat tariff, both with the aid of a PV and a battery, and especially when the house is not provided with battery and PV;

- In order to have people participating in DSR an incentive scheme must be planned, this way the price difference between the cooperating system and the “selfish” individually optimized system, in terms of price for the energy would be reduced or it could even happen that a cooperating system would produce a lower price than an individually optimized one.
- Nevertheless the impact on the network of the batteries and PV is huge, in each and any of the scenarios, even when not cooperating, this means that incentives should be done also on the prices of PV and battery systems to allow people buying more of these devices and adding flexibility and DSR possibilities to the network;
- Last but not least, the impact of the cooperative DSR optimization on the network is huge, and the goal of all the countries and the energy market players should be to facilitate and incentivize the penetration of DSR in their power grid.

4.2 Future works

The endpoint of this study can be used in many ways as a starting point for future developments on this work, and here are some examples:

- When new housing will be built and they need grid connection, it is possible to save money on the connections, leading to a lower price on electricity fees and for building, thus buying the new houses; using DSR for peak saving decreases the capacity needs of the new network, which affects the generation power needs and the material needed for the wiring as well;
- A company can sell DSR services to a DSO operating in an area where a weak and unstable network is; this is similar to the service offered for the new housing, but in this case the peak saving will not affect the construction of a new network, it will indeed improve the behavior of an already existing network; the DSO can benefit from DSR because it can reduce the peak demand and the loading in general, so that the network will not suffer from overloading;
- A better model can be built to improve the analysis, using a full tri-phase power flow rather than a single-phase equivalent; this way the results would me more accurate and realistic, and a better analysis with more accurate level can be used a starting point for some economic assessment for instance;
- The test could be run on different and more realistic network, eventually with data from a real network to assess the impact of DSR on it;
- An economic assessment could be done starting from this dissertation in order to find the right incentives to help the DSR deployment, considering all the possible savings that people can have, but also the costs to install the system and the better income they might have if they could control their system without coordination; all of this can be put together to find the right price for electricity or discounts to provide to people willing to take part to DSR;
- The model of the household can be improved, to increase the DSR potentiality including other than the battery and the PV also the appliances in the house that the user is happy to give up control on; appliances such as electric heaters or air conditioners, clothes dryers or washing machines, are loads that with the use of smart devices are detectable, and each and any of the steps of their working cycle can be detected as well with the latest models that are now manufactured; this means that these loads can be delayed in time to provide DSR if the owner is happy to give up on the control of the loads, and the models of these loads can be included in the linear programming problem formulation. This way all domestic DSR

potential could be untapped. The data for a set of appliances is provided in an IEEE paper (Load Profiles of Selected Major Household Appliances and Their Demand Response Opportunities) [26], that explains how the appliances work, their operational cycles, their DSR potential according to the kind of load, how it can be time shifted or interrupted, and gives a rough idea on how the models for the appliances can be built;

- Last but not least, on this test there was a generator always plugged in powering the network, some tests could be done to check if the network has the possibility to be powered without the use of a generator, and how long it could be plugged off before it collapses, this way the planning for maintenance operations can be optimized as well.

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