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Programa de Doctorado de Materiales

GENERATION AND CLASSIFICATION OF ADS SOLUTIONS AND THEIR CFT INTERPRETATION

TESIS DOCTORAL

Jesús Montero Aragón

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TESIS DOCTORAL

Directora de tesis:
Yolanda Lozano Gómez

RESUMEN DEL CONTENIDO DE TESIS DOCTORAL

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RESUMEN (en español)

En esta tesis se tratará el papel de T-dualidad no abeliana como técnica para generar soluciones en supergravedad de tipo II. Nos centraremos en el problema abierto de proporcionar a los nuevos *backgrounds* AdS, de forma consistente, teorías de campos súperconformes, las cuales pueden ser vistas como puntos fijos del flujo de renormalización de *quivers* lineales de rango creciente. Se propondrá así mismo una interesante relación entre estos *quivers* y la descripción mediante T-dualidad abeliana de las teorías originales en términos de *quivers* circulares.

Se revisará por otra parte el potencial de T-dualidad no abeliana para motivar, sondar o poner a prueba clasificaciones de soluciones supersimétricas. Encontraremos nuevos *backgrounds* para distintos espacios AdS y cantidades de supersimetría preservada, incluyendo una solución N=2 AdS4 explícita en teoría M con flujo puramente magnético, la cual resulta relevante para nuevas dualidades de SCFT en tres dimensiones. Nuestros esfuerzos nos conducirán así mismo a nuevos candidatos AdS3xS2 para límites *near-horizon* de agujeros negros en dimensiones altas, motivando así una extensión de la clase conocida de estas geometrías.

Por último, se intentará conseguir una clasificación más amplia de soluciones Minkowski supersimétricas de tipo II en dimensiones bajas, en las cuales se permitirá que la simetría R se encuentre manifiesta geométricamente. Además de recuperar diversas geometrías *near-horizon* AdS, pondremos de manifiesto la unicidad de la geometría N=4 AdS4xS3 de tipo II. Por otra parte, se descubrirá una nueva clase de soluciones Mink3, que incluye un *background* puramente NS. Se espera que estos resultados resulten de interés para las compactaciones con flujo y la extensión de holografía a teorías no conformes, aparte de la clasificación de geometrías AdS en dimensiones más altas.

RESUMEN (en Inglés)

In this thesis we will discuss non-Abelian T-duality as a solution generating technique in type II supergravity. We will focus on the open problem of providing newly generated AdS brackgrounds with consistent dual superconformal field theories, which can be seen as renormalization group fixed points of linear quivers of increasing rank. An interesting relation between these quivers and the Abelian T-dual description of the original theories in terms of circular quivers will be pointed out.

We will review the potential of non-Abelian T-duality to motivate, probe or challenge classifications of supersymmetric solutions. We will find new backgrounds for different AdS spaces and amounts of preserved supersymmetry, including an explicit N=2 AdS4 solution in M-theory with purely magnetic flux, of relevance for new dualities of three-dimensional SCFTs. Our endeavours will also lead us to new AdS3xS2 candidates to higher-dimensional black hole near-horizon limits, while motivating an extension of the known class of such geometries.

Finally, a broader classification of type II supersymmetric lower-dimensional Minkowski solutions will be attempted, in which the R-symmetry is allowed to be realized geometrically. Apart from recovering several known AdS near-horizon geometries, we will report on the uniqueness of the type II N=4 AdS4xS3 geometry. Besides, a new class of Mink3 solutions will be uncovered, which includes a pure NS background. These results are expected to be of interest for flux compactifications and the extension of holography to non-conformal theories, other than the classification of AdS geometries in higher dimensions.

Generación y clasificación de soluciones AdS y su interpretación como CFT

Dedicado a mis abuelos, por su apoyo y cariño a lo largo de toda mi vida.

Abstract

In this thesis we will discuss non-Abelian T-duality as a solution generating technique in type II supergravity. We will focus on the open problem of providing newly generated AdS backgrounds with consistent dual superconformal field theories, which can be seen as renormalization group fixed points of linear quivers of increasing rank. An interesting relation between these quivers and the Abelian T-dual description of the original theories in terms of circular quivers will be pointed out.

We will review the potential of non-Abelian T-duality to motivate, probe or challenge classifications of supersymmetric solutions. We will find new backgrounds for different AdS spaces and amounts of preserved supersymmetry, including an explicit $\mathcal{N} = 2$ AdS_4 solution in M-theory with purely magnetic flux, of relevance for new dualities of three-dimensional SCFTs. Our endeavours will also lead us to new $\mathrm{AdS}_3 \times S^2$ candidates to higher-dimensional black hole near-horizon limits, while motivating an extension of the known class of such geometries.

Finally, a broader classification of type II supersymmetric lower-dimensional Minkowski solutions will be attempted, in which the R-symmetry is allowed to be realized geometrically. Apart from recovering several known AdS near-horizon geometries, we will report on the uniqueness of the type II $\mathcal{N} = 4$ $\mathrm{AdS}_4 \times S^3$ geometry. Besides, a new class of Mink_3 solutions will be uncovered, which includes a pure NS background. These results are expected to be of interest for flux compactifications and the extension of holography to non-conformal theories, other than the classification of AdS geometries in higher dimensions.

Resumen

En esta tesis se tratará el papel de T-dualidad no abeliana como técnica para generar soluciones en supergravedad de tipo II. Nos centraremos en el problema abierto de proporcionar a los nuevos *backgrounds* AdS, de forma consistente, teorías de campos súperconformes, las cuales pueden ser vistas como puntos fijos del flujo de renormalización de *quivers* lineales de rango creciente. Se propondrá así mismo una interesante relación entre estos *quivers* y la descripción mediante T-dualidad abeliana de las teorías originales en términos de *quivers* circulares.

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Por último, se intentará conseguir una clasificación más amplia de soluciones Minkowski supersimétricas de tipo II en dimensiones bajas, en las cuales se permitirá que la simetría R se encuentre manifiesta geométricamente. Además de recuperar diversas geometrías *near-horizon* AdS, pondremos de manifiesto la unicidad de la geometría $\mathcal{N} = 4$ $\mathrm{AdS}_4 \times S^3$ de tipo II. Por otra parte, se descubrirá una nueva clase de soluciones Mink_3, que incluye un *background* puramente NS. Se espera que estos resultados resulten de interés para las compactaciones con flujo y la extensión de holografía a teorías no conformes, aparte de la clasificación de geometrías AdS en dimensiones más altas.

Contents

Preface

This thesis has been submitted to the University of Oviedo, as a partial fulfillment of the requirements to obtain the PhD degree. The work presented here has been developed during the years 2014-2018 under the supervision of Prof. Yolanda Lozano in the High Energy Physics Theory Group at the Department of Physics of the same university.

Thesis objectives

In this work we intend to study and classify solutions of type II supergravity, providing a holographic interpretation thereof. In particular we aim to:

- Using non-Abelian T-duality, generate AdS solutions in type II supergravity that are hard to reach employing other methods available in the literature.
- Regarding the existence, or not, of a CFT associated with the new AdS solutions, provide an interpretation to the non-compact direction arising under non-Abelian T-duality.
- Improve our understanding of this transformation, in order to eventually elucidate whether it can be thought of as a string theory symmetry or not.
- Using pure spinors and G-structure formalism, propose an extension of the known classes of supersymmetric solutions, motivated by the new backgrounds found using non-Abelian T-duality.
- Extend to lower dimensions the existing results on the classification of supersymmetric type II geometries with a Minkowski external space.

The structure of this thesis, in the collection-of-papers format, is as follows: We start with the introduction in section 1, followed by a review the worldsheet formalism of non-Abelian T-duality in section 2, that includes the transformation rules for both the NS-NS and RR-sectors. A copy of the papers on which this thesis is based is given in section 3. Next, a technical summary of the results is provided in section 4, followed by a more general discussion and conclusions in section 5. Last, a list of references for this text is supplied, apart from the ones included in the copies of the articles.

We remark that, following requirements of the PhD programme, the abstract and conclusions are written both in English and Spanish.

Acknowledgements

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I am also happy to thank many research institutions and scientists around Europe and the US for their hospitality during several sojourns, workshops and seminars. The list includes (not exhaustively) the Physics departments of the following universities: Milano-Bicocca, Stony Brook, Swansea, KU Leuven, SISSA, King's College London, Oxford, IPhT CEA/Saclay, IPN Lyon, etc.

A number of friends coloured my experience in Oviedo, helping me to realize what lies beyond Physics and work, and allowing me to enjoy the wonderful landscape and possibilities of Asturias, "*paraíso natural*". This includes the cheerful and sporty people of the *Grupo de Montaña Uniovi*, a number of its members I also joined for numerous friendly running sessions and races. Being honest, these people mostly contributed to my taste of *sidra*, *cachopo* and *espcichas*. I thank Alessandro Pini for getting me in touch with these people. Not less friendly or active were to me the fellow PhD students of the Geology Depart. at Oviedo, who I couldn't have met without the warm reception of my friend Pablo Valenzuela.

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Finally, thank you, Cris, for being *always* by my side…

Impact factors

The impact factors of the scientific journals where the following articles have been published are included below. Source: Journal Citation Reports.

1. N. T. Macpherson, J. Montero, D. Prins, *"$Mink_3 \times S^3$ solutions of type II supergravity"*, Nuclear Physics B (2018).

2. G. Itsios, Y. Lozano, J. Montero, C. Núñez, *"The AdS_5 non-Abelian T-dual of Klebanov-Witten as a $\mathcal{N}=1$ linear quiver from M5-branes"*, Journal of High Energy Physics **09**, 038 (2017).

3. Y. Lozano, N. T. Macpherson, J. Montero, C. Núñez, *"Three-dimensional $\mathcal{N}=4$ linear quivers and non-Abelian T-duals"*, Journal of High Energy Physics **11**, 133 (2016).

4. O. Kelekci, Y. Lozano, J. Montero, E. Ó. Colgáin and M. Park, *"Large superconformal near-horizons from M-theory"*, Physical Review D **93**, 086010 (2016).

5. Y. Lozano, N T. Macpherson, J Montero, *"$\mathcal{N}=2$ supersymmetric AdS4 solution in M-theory with purely magnetic flux"*, Journal of High Energy Physics **10**, 4 (2015).

6. Y. Lozano, N. T. Macpherson, J. Montero, E. Ó. Colgáin, *"New $AdS_3 \times S^2$ T-duals with $\mathcal{N}=(0,4)$ supersymmetry"*, Journal of High Energy Physics **8**, 121 (2015).

Journal	**Year**	**Impact Factor**	**Area**
Nuclear Physics B	2016	3.678	Physics, Particles and Fields
Physical Review D	2016	4.557	Astronomy, Astrophysics, Physics, Particles and Fields
JHEP	2016	6.063	Physics, Particles and Fields

1. Introduction

We start providing some context and motivation for the use of non-Abelian T-duality to generate AdS solutions of supergravity, reviewing the main advantages and issues of this technique, with emphasis on the holographic interpretation of the newly generated solutions. We will continue this introduction in section 1.2, where the focus will change to the classification of the novel AdS geometries, still in the holographic setting. We will end up motivating the use of Killing spinor techniques to build up classes of non-AdS solutions.

1.1 Non-Abelian T-duality and holography

In recent years, the gauge/gravity or holographic correspondence has proven to be a powerful spin-off of String Theory, permitting the study of strongly-coupled quantum field theories, even when a Lagrangian description is not available, by means of a dual weakly-coupled semiclassical theory of gravity. This duality was originally formulated as an AdS/CFT correspondence [1], relating a string theory on an Anti de-Sitter (AdS) space to a quantum conformal field theory (CFT) on flat spacetime. In particular, the Maldacena conjecture connected type IIB supergravity (the low-energy limit of string theory) on $AdS_5 \times S^5$ to the strong coupling limit of $\mathcal{N} = 4$ super Yang-Mills (SYM) in four dimensions.

By means of the AdS/CFT correspondence, several dualities arising in string theory have become common tools in the field theoretical understanding of CFTs, translating the action e.g. of T- or S-duality relating different AdS vacua to the dual field theories.

Many interesting applications of holography, including the study of QCD phenomena and condensed matter systems, require models to be scale-dependent and show minimal or no preserved supersymmetry, as this has not been observed yet. This urged for an extension of the AdS/CFT correspondence to non-conformal theories, giving rise to a more general gauge/gravity duality.

In recent years, the systematic study of supersymmetric solutions with non-trivial fluxes in the low-energy limit of string and M-theory has proven relevant in the context of holography as well as in phenomenologically relevant string compactifications. The geometries of interest are usually a warped product of either a Minkowski or AdS external space with an internal manifold which, in the latter case, is taken to be compact in order to allow for a holographic interpretation. Indeed, many AdS/CFT pairs have been constructed from solutions generated in supergravity.

In this thesis we will focus on the use of non-Abelian T-duality (NATD) as a solution generating technique in type II supergravity. We will uncover new supersymmetric solutions in both type IIA/IIB, starting from seed solutions of type IIB/IIA. These are chosen carefully: they have a well known holographic dual field theory, but also they are expected to lead us to new backgrounds, allowing us e.g. to provide explicit examples to known classifications, when these are hard to reach using other methods. Furthermore, if we start with a 10D gravity solution with AdS external space and apply NATD to the isometries of the internal space, the AdS factor will be preserved and we may wonder what kind of dual CFT could be associated to the new solution. Based on computations relying on the backgrounds fields (i.e. the metric, dilaton, Kalb-Ramond two-form and RR fluxes), we may try to deduce some properties of this putative CFT.

The T-duality formalism[1] relies on a non-linear sigma-model (NLSM) describing the propagation of a string in 10D target space, for which an isometry group G is assumed. For Abelian T-duality, $G = U(1)$, otherwise G is a non-Abelian isometry group. In order to get the dual action, the theory is minimally coupled to G, gauging the isometry but enforcing a flat connection with the addition to the action of a Lagrange multiplier term. Integrating out this G-connection, one is led to the dual NLSM, describing the propagation of a string in a different target space. For the usual $U(1)$ T-duality, the Buscher rules [2] allow us to relate algebraically the metric and B-field between the original 10D background and its T-dual, reading them off from the coefficients of the embedding scalars. An important point is that the coordinate for the original isometric direction is replaced, after integrating out the G-connection, by the corresponding Lagrange multiplier, which lives in the Lie algebra of G.

The transformation of the RR-sector under T-duality was derived in [3], matching the field contents of both reductions to nine dimensions of two T-dual theories in type II. This permitted to apply T-duality to an initial solution with RR fluxes turned on and get a T-dual background which still was a type II supergravity solution.

[1] We introduce the worldsheet formalism of non-Abelian T-duality in section 2.

The generalization of Abelian T-duality (ATD) to non-Abelian isometries at the level of the string sigma model was known since the nineties [4, 5], allowing for the transformation of the NS-NS sector of a given 10D target space. However, it has not been until this decade that the work of [6] taught us how to transform the RR sector accordingly, extending the construction made for the Abelian case by [7]. This relied on the spinor representation of T-duality, which can be derived from the Lorentz transformation relating the vielbeins for left and right movers of the dual target space. The transformation rules for the RR sector then follow writing the fluxes as bispinors.

The discovery of [6] led to the recent works uncovering a plethora of new backgrounds, for different AdS dimensionalities and amounts of preserved supersymmetry. In fact, NATD permits to reach solutions that are difficult to be found using other methods, and without e.g. needing to tackle with the PDEs of some class of solutions. A prototypical example is the explicit AdS_6 solution in type IIB supergravity[2] found in [9] applying NATD to the type IIA Brandhuber-Oz solution [10], which permitted to get hints of the possible dual 5D fixed point theories [11]. A rich class of solutions constructed using NATD included AdS_5 backgrounds with a Sasaki-Einstein internal space. These geometries were extensively investigated [12, 13, 14], supplying a rich phenomenology for the possible SCFTs and RG flows of the new supersymmetric backgrounds. However, a full description of the dual field theories (e.g. including a superpotential) was typically missing in these examples.

Despite its power as a solution generator, NATD presents several, long standing questions that hinder the holographic interpretation of the new backgrounds, see e.g. [15, 16]. Even if the sigma model computation of the NS-NS sector of the new background is straight-forward, the procedure is not known to be invertible, unlike its Abelian counterpart, and it fails to provide all the global information about the new geometry, this latter issue being one of the main points we deal with in this thesis.

In the ATD case, it was possible to derive a condition to recover the $U(1)$ group manifold from the Lagrange multiplier living in its Lie algebra. This was achieved requiring the cancellation of holonomies of the $U(1)$ gauge connection around non-trivial cycles of the string worldsheet (whose number n is given by the power $\mathcal{O}(g_s^n)$ of the string coupling constant perturbative series). This killed two birds with one stone: on the one hand, it showed ATD maps circles to circles (even if the radius thereof is inverted), and on the other hand, it proved that it was a symmetry to all orders in perturbative string theory.

[2] Another example of an explicit AdS_6 IIB solution being the Abelian T-dual of Brandhuber-Oz, found in [8].

However, in the non-Abelian case, non-commutativity of the gauge connection gives rise to terms apparently impeding the cancellation of the unwanted holonomies. As an immediate consequence, NATD can only be proven to be a transformation between string NLSMs at tree level in string perturbation theory (i.e. for spherical worldsheets). As a by-product, we also have lost the argument that allowed in the Abelian case to keep a compact space after dualization: all that is left as dual coordinates by NATD live in the Lie algebra of the isometry group, which is obviously non-compact by its construction as a vector space. In addition to this, regarding corrections in the string length, the conformal symmetry of the string NLSM is only proven to survive the NATD transformation at first order in α' (as opposed to its Abelian counterpart, shown to work at all orders). These issues have an overall and clear output:

NATD is not proven to be a string theory symmetry, but just a transformation, or map, between type II supergravity solutions.

It is precisely this fact what allows us to claim that the backgrounds generated under its action can be associated to different vacua of string theory, and are not just physically equivalent realizations of the same vacuum, as opposed to its Abelian counterpart [17, 18].

As a drawback, we currently lack a well-defined representation of NATD on the field theory side. The putative CFT duals to the new backgrounds are only guaranteed to exist in the strong coupling regime, as the NATD transformation is not known to survive α' or $1/N$ corrections. Still, we may infer some properties of this would-be CFT from the new background, and even *define* a quiver gauge theory, from which the CFT would arise as an RG flow fixed point. This will be our approach in this thesis.

We will focus on NATD applied with respect to an $SU(2)$ isometry group acting without isotropy[3], and dualize along the directions of its group manifold realization in 10D target space: a (possibly squashed) 3-sphere. Note, however, that other incarnations of NATD as a solution generator are known in the literature, see e.g. [19] or [20]. Our choice of NATD is known to provide a map between SUGRA solutions [21] [4] and usually allows for some (if not all) SUSY to be preserved, choosing appropriately the dualization directions[5], whereas NATD on coset spaces leads in known examples to non-

[3]The isometry group is chosen such that there are no fixed points under its action.

[4]NATD on non-semisimple groups has been shown to lead to solutions of generalized supergravity for some Bianchi spaces, see [22] and the references therein.

[5]Upon dualization on a $SU(2)$ isometry, it was shown in [21] that the vanishing of Kosmann spinorial Lie derivative for the Killing spinors along the isometry directions in

supersymmetric solutions [19]. Besides, an $SU(2)$ isometry is the simplest generalization of the Abelian case. The use of a larger isometry group would require a more constrained initial background, allowing less room for possible seed solutions.

From the discussion above, we expect the group manifold coordinates of $SU(2)$, e.g. Euler angles realizing the 3-sphere in target space, to get mapped to elements of the uncompact $\mathfrak{su}(2)$. Still, an adequate parametrization of the dual coordinates may be required in order to realize any left-over global symmetries. Indeed, upon dualization on a round 3-sphere with $SO(4)$ isometry, an $SU(2)$ subgroup is expected to survive in the dual background[6]. Using spherical coordinates to parametrize the NAT-dual directions, this symmetry is realized explicitly at the level of the metric as a warped $M_3 \simeq \mathbf{R}^+ \times S^2$ space.

Regarding compactness, we seem to get a 3-sphere substituted for this M_3 space. Usually, one can run along its "radial" direction $\rho \in \mathbf{R}^+$ without finding any hint of the 10D supergravity description breaking down (e.g. singularities, large curvature or dilaton). In the simplest cases, the metric on M_3 clearly interpolates between $\mathbf{R}^3$ for small ρ, and a cylinder for large ρ, thus describing a space with the shape of an infinite cigar. At least in this cases, we could claim the global properties of the new background are perfectly determined, M_3 being uncompact and thus having the new background an internal space of infinite volume.

However, if one is interested in the holographic interpretation of the newly found AdS solution, an infinite internal space would lead to an inconsistent dual SCFT, with an infinite holographic central charge or free energy. We would then need to discard this solution as suitable for holography. Instead, in this thesis we settle for an alternative perspective: the fact that the dual coordinates live in a Lie algebra, and no group manifold can be assigned to them from the information provided by NATD alone, is just a hint that, apart from the residual isometries[7], *the global properties of the dual manifold cannot be inferred from the transformation itself.* In this second approach, the internal space is thought to be compact, yet globally unknown.

Assuming the new solutions have indeed a compact internal space, compatible with a would-be consistent dual field theory, but not fully determined,

the original background was a necessary and also sufficient condition for SUSY to be preserved.

[6]For a full $SO(4) \approx SU(2) \times SU(2)$ isometry, the string worldsheet theory is a principal chiral model exhibiting the to-be-dualized $SU(2)$ isometry, but also an additional spectator $SU(2)$, which can be realized as a residual isometry after NATD in appropriate coordinates.

[7]Any global symmetries already present in the original target space, and independent of the dualization directions, will be also manifest in the new background.

gives rise to one of the most important open questions in the study of NATD: *How* do we get the piece of extra information needed to determine the new AdS/CFT pair? We hope to make a contribution towards a definite answer to this question in this work.

Our approach in [23, 24], see sections 3.4 and 3.5, followed the ideas developed in [25], also extended and applied in other recent works [26, 27, 28, 29].

In [25] the non-Abelian T-dual of $\text{AdS}_5 \times S^5$ already found in the seminal paper [6] is investigated, using its embedding in the $\mathcal{N} = 2$ Gaiotto-Maldacena geometries [30], in order to regularize the field theory and *complete* the background. GM geometries are the gravity duals of theories living in the worldvolume of D4-NS5 intersections. The embedding of the NATD background in this class provided a strong support to the brane set-up proposed for the solution (let us recall that the mapping of brane intersections under NATD is not known). In particular, they manage to use the detailed AdS/CFT dictionary known for this class of solutions to propose a consistent 4D $\mathcal{N} = 2$ long quiver for which a *complete* gravity dual solution (i.e. with finite internal volume) can be determined. Then, the NATD gravity solution arises as an asymptotic, local realization of this completed solution[8].

The first extension of these ideas was done in [23], in which a solution generated with NATD is embedded in the class [32, 33] of type IIB $\mathcal{N} = 4$ $\text{AdS}_4 \times S^2 \times S^2$ backgrounds. A consistent field theory proposal was achieved for the new background employing the holographic dictionary of [34] between these geometries and the $T_\rho^{\hat{\rho}}(SU(N))$ Gaiotto-Witten theories [35] living on the worldvolume of a D3-D5-NS5 system[9].

Later, it was claimed in [24] that the NATD solution obtained from the gravity dual of the Klebanov-Witten model [36] can be seen as the near-horizon limit of a D4-NS5-NS5' orthogonal intersection, whose worldvolume dynamics is given at the conformal fixed point by the $\mathcal{N} = 1$ mass deformation of the $\mathcal{N} = 2$ theories proposed in [25]. This is an extension to the non-Abelian case of the relation between the Abelian T-duals of the $\mathcal{N} = 2$ $\text{AdS}_5 \times S^5/\mathbf{Z}_2$ and the $\mathcal{N} = 1$ Klebanov-Witten solutions [37].

More generally, the rough idea is to use the AdS/CFT correspondence to provide the missing information from string theory about the NATD gravity solutions. A (consistent) field theory proposal is employed to complete the background, even possibly smoothing out its singularities. In the works enclosed in this thesis, our approach to this problem is the following:

[8] Indeed, the completed gravity solution arises as a regular superposition of Maldacena-Núñez geometries [31], which yields the (singular) NATD solution in a certain limit.

[9] A brief introduction to these theories and their holographic duals is given in [23], included in section 3.4 .

- For the new AdS solutions generated using NATD, we systematically study their properties, computing the quantized charges associated to RR-fluxes, the holographic central charge/free energy and providing an analysis of the preserved supersymmetry.
- We propose a Hanany-Witten brane set-up based on the quantized charges.
- We use insights from four- and three-dimensional SCFTs with well-known gravity duals to propose a consistent linear quiver, regularizing the former brane set-up by a motivated addition of flavour branes. It will be conjectured that this field theory is dual to a regular background with finite internal space, from which the non-Abelian T-dual solution arises in a certain limit.

Other than generating and holographically interpreting supergravity solutions generated with NATD, it is also interesting to consider how the new solutions relate to known classes of supersymmetric geometries. In some cases, this might even lead to the extension of such classifications. Applied techniques can then be utilized to construct other classes of geometries, even if not directly related to NATD backgrounds.

1.2 Classification of the new solutions

Killing spinor techniques, such as the pure spinor formalism and G-structures [38, 39], have been extensively used in the literature to find supersymmetric classes of AdS solutions of supergravity with different amounts of preserved supersymmetry, even if originally devised for $\mathcal{N}=1$ backgrounds[10]. Using a decomposition Ansatz for the Killing spinors into external and internal components, the dilatino and gravitino variations are reduced to constrains on the internal manifold. The Killing spinor equations can then be rephrased, in terms of polyforms, as conditions for a certain reduction of the structure group of the corresponding internal manifold.

Typical examples in the NATD literature are gravity backgrounds with RR-fluxes dual to $\mathcal{N}=1$ theories (see e.g. [41, 42]), in which internal spinors defined in a 4- or 6-dimensional manifold give rise to a $SU(3)$-structure for that space. This is the simplest generalization of Calabi-Yau manifolds, permitting string compactifications for backgrounds supported with either Kalb-Ramond or RR-fluxes, in which the non-vanishing torsion classes (e.g. the

[10] String compactifications of flux backgrounds, some of them of phenomenological interest, were a motivation for these methods (see [40] for a review).

non-closure of the holomorphic 3-form) deforming the Calabi-Yau are sourced by the non-trivial fluxes.

The effect of NATD on the Killing spinors is well-known[11]. The RR-fluxes transform under the spinor representation of NATD [6], which can be derived from the Lorentz transformation relating the frame of left and right movers. Applying the aforementioned decomposition to the Killing spinors, NATD has the effect of rotating one internal spinor w.r.t. the other, typically reducing the structure group [44, 42]. This general pattern helps identifying the class of solutions the new background should belong to.

The G-structure formalism was used in [43] to embed a new $\mathcal{N} = 2$ AdS_4 background, generated with NATD, in 11D. This solution has no electric flux and should therefore fit into the classification of [45, 46]. This solution is the second possible candidate to provide a holographic realization of the 3d-3d correspondence between three-dimensional cycles wrapped by M5-branes, and the 3D $\mathcal{N} = 2$ CFTs resulting from the twisted compactification of the corresponding 6D $(2, 0)$ theory. The first explicit solution within this class was the uplift [47, 48] from 7D gauged supergravity of the Pernici-Sezgin solution [49], constructed in the 80s.

Considering $\mathrm{AdS}_3 \times S^2$ solutions dual to 2D $\mathcal{N} = (0, 4)$ SCFTs, a celebrated example is the $\mathrm{AdS}_3 \times S^2 \times CY_3$ near-horizon of the Maldacena-Strominger-Witten 4D black-hole [50], which results from M-theory compactified on $CY_3 \times S^1$. The dual $\mathcal{N} = (0, 4)$ CFT, arising from M5-branes wrapping a 4-cycle inside the CY_3, allows for the counting of black hole microstates (the Bekenstein-Hawking entropy and subleading corrections to it) in terms of the central charge of the CFT [51, 52]. This solution has a small superconformal symmetry[12], with an $SU(2)$ R-symmetry, but other realizations of the algebra are known to exist, what suggested that a larger class of such black hole near-horizons should be available. Addressing this point, $\mathcal{N} = (0, 4)$ $\mathrm{AdS}_3 \times S^2$ solutions with $SU(2) \times SU(2)$ R-symmetry, compatible with a *large* superconformal algebra, are found in [54]. These lie outside the known class of solutions engineered in [55] and therefore motivated its extension to include the new backgrounds. This was done in [56], where the aforementioned decomposition method was applied to the Killing spinors, in order to get the most general Ansatz compatible with a $SU(2)$-structure internal manifold[13].

[11] For a review of $SU(3)$ and $SU(2)$ structures on six-dimensional internal manifolds, and details of the effect of NATD on them, see the appendices of [43], section 3.2, and the references therein.

[12] The different possible 2D $\mathcal{N} = 4$ superconformal algebras are introduced e.g. in [53].

[13] The minimal $SU(2)$ R-symmetry of these solutions was assumed to be realized geometrically as the $SU(2)$ isometry from the S^2 factor in the $\mathrm{AdS}_3 \times S^2$ Ansatz. The large

An important example of allowing the R-symmetry to be manifest as an isometry for the bosonic supergravity fields is the comprehensive classification work done for type IIB $\mathrm{AdS}_4 \times S^2 \times S^2$ solutions [32, 33]. This was motivated by the search for holographic duals to three dimensional $\mathcal{N} = 4$ CFTs arising from 4D SYM with a planar defect. The dual SCFTs, fixed points of the $T_{\rho}^{\hat{\rho}}(SU(N))$ theories of Gaiotto and Witten [35], admit a global $SO(2,3) \times SO(4)$ symmetry, with the $SO(4) \simeq SO(3) \times SO(3)$ R-symmetry holographically related to an isometry of the bulk geometry. This demanded that the internal manifold should be written as $S^2 \times S^2 \times \Sigma_2$ with a Riemann surface Σ_2, which was in fact highly restricted, even allowing to explicitly solve the Killing spinor equations without resorting to the geometrical techniques outlined above. As a result, a full classification of type IIB $\mathcal{N} = 4$ $\mathrm{AdS}_4 \times S^2 \times S^2 \times \Sigma_2$ backgrounds, arising in the near-horizon limit of D3-D5-NS5 brane intersections, was given in terms of a pair of harmonic functions defined on the Riemann surface Σ_2, which was either an infinite strip or an annulus. This permitted a field theory interpretation in terms of, respectively, linear or circular quiver realizations of the Gaiotto-Witten theories [34, 57].

As we mentioned before, this is the class of AdS/CFT pairs used in [23], see section 3.4, to embed a new type IIB solution generated with NATD from the $\mathcal{N} = 4$ $\mathrm{AdS}_4 \times S^3 \times S^2$ geometry, arising in the near-horizon limit of a D2-D6 brane system[14]. This non-Abelian T-dual background could be associated this way to a regularised dual linear quiver. Interestingly, the Abelian T-dual of this same IIA seed solution was already recovered in [57] in a certain limit of a circular brane set-up, dual to a circular-quiver version of the Gaiotto-Witten theories.

Turning our attention to gravity backgrounds beyond conformal solutions, which can be phenomenologically relevant to flux compactifications, a large number of attempts have been made to construct/classify solutions with external Minkowski space and minimal supersymmetry, see [59, 60, 38, 61] or [62, 63, 64, 65] for recent results. The relatively low amount of (super)isometries entails significant difficulties to solve the reduced Killing spinor equations. Progress was made in [66] for backgrounds containing a Mink_4 factor, assuming $\mathcal{N} = 2$ supersymmetry and a geometrically-realised

superconformal symmetry can then arise when an extra $SU(2)_R$ factor emerges from the Killing spinor bilinears derived in the analysis.

[14]This geometry can be obtained from the $\mathcal{N} = 8$ $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ solution of 11D supergravity, upon reduction to type IIA on the Hopf fibre (where the $\mathbf{Z}_k$ orbifold is applied) of one of the two 3-spheres inside the S^7. This is in contrast to the IIA $\mathcal{N} = 6$ gravity dual of ABJM [58], also associated to a D2-D6 system, and arising from the reduction of the same 11D solution, but on the $\mathbf{Z}_k$-orbifolded circle $S^1 \hookrightarrow S^7 \to \mathbf{CP}^3$.

$SU(2)$ R-symmetry, which allowed to recover AdS solutions[15].

Inspired by this example, we tackled the problem of classifying type II solutions with a $\mathrm{Mink}_3 \times S^3$ factor in [67], in which the three-sphere allows for the geometrization of a would-be $SO(4)$ R-symmetry, instead of the $S^2 \times S^2$ factor assumed in [32] and subsequent works. In fact, our initial goal was to find gravity duals of three-dimensional $\mathcal{N} = 4$ SCFTs by examining $\mathrm{AdS}_4 \times S^3$ type II backgrounds, but this approach seemed too restrictive and allowing for a more general Mink_3 factor turned out to be more convenient.

Besides, it is worth highlighting that even if the assumption of the S^3 factor will lead to solutions with $\mathcal{N} > 1$, our classification could still be of interest for phenomenological applications, as we expect to be able to reduce supersymmetry through a deformation of the 3-sphere.

The structure of this thesis, in the collection-of-papers format, is as follows: In section 2, we review the worldsheet formalism of non-Abelian T-duality, deriving the transformation rules for both the NS-NS and RR-sectors. A copy of the papers on which this thesis is based is given in section 3. Next, a technical summary of the results is provided in section 4, followed by a more general discussion and conclusions in section 5. Last, a list of references for this text is supplied, apart from the ones included in the copies of the articles.

We remark that, following requirements of the PhD programme, the abstract and conclusions of this thesis are written both in English and Spanish.

[15]Even if optional for supersymmetry, the R-symmetry is part of the superconformal algebra required for the AdS factor. Remark that a 10D solution with a Mink_d factor can be regarded as a AdS_p solution with $p > d$, in particular as AdS_{d+1}:

$$ds^2 = e^{2A} ds^2(Mink_d) + ds^2_{10-d} = e^{2B}\left(e^{2\rho} ds^2(Mink_d) + d\rho^2\right) + ds^2_{9-d}\,,$$

where A and B are warp factors depending only on the internal manifolds M_{10-d} and M_{9-d}, respectively.

2. Worldsheet formalism of non-Abelian T-duality

We present below a brief introduction to the formulation of non-Abelian T-duality (NATD) for the string sigma model. This follows closely and summarizes the detailed review given in [12], complemented by [41] and [68].

2.1 Transformation of the NS-NS sector

Consider a 10D target space with an $SU(2)$ isometry and metric written as:

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + 2G_{\mu i}(x)dx^\mu L^i + G_{ij}(x)L^i L^j \ , \tag{2.1}$$

where $\mu = 1, 2, \ldots, 7$ run over the transverse-space directions and $L^i = -i\mathrm{Tr}(t^i g^{-1}dg)$, for $i = 1, 2, 3$ are the Maurer-Cartan forms, with t^i the $SU(2)$ generators. The group element $g \in SU(2)$ depends on the target space isometry directions, realizing the $SU(2)$ group manifold. We can choose to parametrize g such that the corresponding coordinates are the Euler angles:

$$g = e^{\frac{i}{2}\tau_3\,\phi}\, e^{\frac{i}{2}\tau_2\,\theta}\, e^{\frac{i}{2}\tau_3\,\psi} \ , \tag{2.2}$$

where $\theta \in [0, \pi]$, $\psi \in [0, 4\pi]$, $\phi \in [0, 2\pi]$ and the $\tau_i = \sqrt{2}\,t_i$ are the Pauli matrices, as $SU(2)$ generators. On the other hand, the Kalb-Ramond field[1] can be written as:

$$B = B_{\mu\nu}(x)dx^\mu \wedge dx^\nu + B_{\mu i}(x)dx^\mu \wedge L^i + \frac{1}{2}B_{ij}(x)L^i \wedge L^j \ . \tag{2.3}$$

Remark that all the metric and B field coefficients do only depend, by construction, on the transversal directions, same as for the dilaton $\Phi(x)$. The only fields depending on the $SU(2)$ directions are the L^i forms.

[1] For the sake of generality, we take an unspecified B in this exposition. Remark, however, that $B = 0$ for the seed solution in all examples considered in this thesis.

The Lagrangian density for the NLSM describing the propagation of a string in the above 10D target space is given by:

$$\mathcal{L} = Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + Q_{\mu i}\partial_+ X^\mu L^i_- + Q_{i\mu} L^i_+ \partial_- X^\mu + E_{ij} L^i_+ L^j_- \ , \tag{2.4}$$

where we have introduced the embedding scalars $\{X^\mu\}$ for the transverse directions[2], $L^i_\pm = -i\mathrm{Tr}(t^i g^{-1}\partial_\pm g)$ and the coefficients

$$\begin{aligned} &Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu} \ , \qquad Q_{\mu i} = G_{\mu i} + B_{\mu i} \ , \qquad Q_{i\mu} = G_{i\mu} + B_{i\mu} \ , \\ &E_{ij} = G_{ij} + B_{ij} \ , \end{aligned} \tag{2.5}$$

from which metric and B-field are read-off. In order to perform the NATD procedure, we minimally couple the theory to the isometry group $SU(2)$, substituting partial for covariant derivatives,

$$\partial_\pm g \longrightarrow D_\pm g = \partial_\pm g - A_\pm g \ , \tag{2.6}$$

where $A_\pm$ is the $SU(2)$ connection. We enforce it to be flat by adding the Lagrange multiplier term $-i\mathrm{Tr}(vF_\pm)$, where $F_\pm = \partial_+ A_- - \partial_- A_+ - [A_+, A_-]$ is the curvature for $A_\pm$. The resulting action is then invariant under

$$g \to h^{-1} g \ , \quad v \to h^{-1} v h \ , \qquad A_\pm \to h^{-1} A_\pm h - h^{-1}\partial_\pm h \ , \tag{2.7}$$

for $h \in SU(2)$. If we now integrate out the $SU(2)$ connection (i.e. find its equations of motion and substitute it back into the action) the dual action will be reached. However, the new theory will still depend on the spectator fields X^μ, the Euler angles and also the Lagrange multipliers v_i. In order to preserve the number of d.o.f. , we gauge fix the $SU(2)$ isometry by setting $g = \mathbb{I}$, i.e. in eq. (2.2) we take $\theta = \psi = \phi = 0$. With this choice of gauge, the original coordinates get substituted by the Lagrange multipliers v_i, so that the dual sigma model does only depend on them and on the spectator fields X^μ.

We remark that other gauge fixings are possible (even mixing original and dual coordinates), that might be useful e.g. in realizing some residual isometries. The different choices are locally related by diffeomorphisms in the NATD target space, as shown in section 2.2.3 of [12]. A general gauge fixing is also discussed in appendix B of that paper.

After the $g = \mathbb{I}$ gauge fixing and a partial integration on the Lagrange multiplier term, the EOMs of the gauge connection yield:

$$\begin{aligned} A^i_+ &= iM^{-1}_{ji}(\partial_+ v_j + Q_{\mu j}\partial_+ X^\mu) \ , \\ A^i_- &= -iM^{-1}_{ij}(\partial_- v_j - Q_{j\mu}\partial_- X^\mu) \ , \end{aligned} \tag{2.8}$$

[2] These are spectator fields under NATD.

where we introduced the matrix $M = E + f$, with $f_{ij} = f_{ij}{}^k v_k$ and $f_{ij}{}^k$ the structure constants of $SU(2)$. Substituting back $A^i_\pm$ into the gauged action, we get the dual NLSM:

$$\widehat{\mathcal{L}} = Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + (\partial_+ v_i + \partial_+ X^\mu Q_{\mu i}) M^{-1}_{ij} (\partial_- v_j - Q_{j\mu}\partial_- X^\mu) \ . \tag{2.9}$$

The metric and B-field of the new target space can now be read-off as the symmetric and anstisymmetic part of the above coefficients w.r.t. the kinetic terms of the new embedding scalars $\{X^\mu, v^i\}$. This yields a generalization of the Buscher rules [2] to the non-Abelian case:

$$\begin{aligned}
&\widehat{Q}_{\mu\nu} = Q_{\mu\nu} - Q_{\mu i} M^{-1}_{ij} Q_{j\nu} \ , \qquad \widehat{E}_{ij} = M^{-1}_{ij} \ , \\
&\widehat{Q}_{\mu i} = Q_{\mu j} M^{-1}_{ji} \ , \qquad \widehat{Q}_{i\mu} = -M^{-1}_{ij} Q_{j\mu} \ .
\end{aligned} \tag{2.10}$$

From the path integral derivation of the former, one sees the dilaton suffers a 1-loop shift:

$$\widehat{\Phi}(x, v) = \Phi(x) - \frac{1}{2}\log(\det M) \, . \tag{2.11}$$

Correspondingly, the transformation of the worldsheet derivatives is

$$\begin{aligned}
L^i_+ &= -(M^{-1})_{ji} \left(\partial_+ v_j + Q_{\mu j}\partial_+ X^\mu\right) \ , \qquad (2.12) \\
L^i_- &= (M^{-1})_{ij} (\partial_- v_j - Q_{j\mu}\partial_- X^\mu) \, ,
\end{aligned}$$

while $\partial_\pm X^\mu$ remain invariant. This represents a canonical transformation between the pair of T-dual sigma models [69, 70].

For the examples considered in this thesis $B = 0$ in the seed solution, so that $Q = G$ and the above (2.10) simplifies to:

$$\begin{aligned}
&\hat{G}_{\mu\nu} = G_{\mu\nu} - M^{-1}_{ij} G_{\mu i} G_{\nu j} \, , \qquad \hat{G}_{ij} = \frac{1}{2} M^{-1}_{(ij)}, \qquad \hat{G}_{i\mu} = -\frac{1}{2} M^{-1}_{[ij]} G_{j\mu} \\
&\hat{B}_{i\mu} = -\frac{1}{2} M^{-1}_{(ij)} G_{j\mu} \, , \qquad \hat{B}_{ij} = \frac{1}{2} M^{-1}_{[ij]} \ ,
\end{aligned} \tag{2.13}$$

where now the matrix M reduces to $M_{ij} = G_{ij} + \sqrt{2}\,\epsilon_{ijk} v_k$. Remark that, in general, Kalb-Ramond 2-form components are generated for the dual background, even if the original seed solution had none.

2.2 Transformation of spinors and the RR sector

In order to determine how the RR sector transforms under NATD, we first need to derive the action on the spinors of the Lorentz transformation relating left and right movers. We start rewriting the metric of the original target space in terms of the frame fields

$$ds^2 = \eta_{AB} e^A e^B + e^a e^a \,, \tag{2.14}$$

where the vielbein is chosen to be

$$e^A = e^A_\mu dx^\mu \;, \qquad e^a = \kappa^a{}_j L^j + \lambda^a_\mu dx^\mu \;, \tag{2.15}$$

with $A = 1, 2, \ldots, 7$ and $a = 1, 2, 3$. Left and right movers transform differently under NATD, as given in (2.12), leading to two different sets of frame fields after the transformation:

$$\begin{aligned} \hat{e}_+ &= -\kappa M^{-T}(dv + Q^T dX) + \lambda dX \,, \\ \hat{e}_- &= \kappa M^{-1}(dv - Q dX) + \lambda dX \,. \end{aligned} \tag{2.16}$$

However, these frame fields still describe the same geometry, and must therefore be related by a Lorentz transformation, $\hat{e}_+ = \Lambda\, \hat{e}_-$, which can be found to be

$$\Lambda = -\kappa M^{-T} M \kappa^{-1} = -\kappa^{-T} M M^{-T} \kappa^T \,. \tag{2.17}$$

The action Ω induced by this Lorentz transformation on spinors is then given by:

$$\Omega^{-1} \Gamma^a \Omega = \Lambda^a{}_b \Gamma^b \,. \tag{2.18}$$

For NATD performed on a freely acting $SU(2)$ isometry, Ω reads[3]:

$$\Omega = \Gamma_{11} \frac{-\Gamma_{123} + \zeta_a \Gamma^a}{\sqrt{1 + \zeta_a \zeta^a}} \;, \tag{2.19}$$

where Γ_{11} is the product of all ten gamma matrices, with $(\Gamma_{11})^2 = \mathbb{I}$, and $\zeta^a = \kappa^a{}_i v^i / (\det \kappa)$ (for $B_{ij} = 0$). Let us remark that Ω leaves invariant the gamma matrices Γ^A corresponding to the transverse directions.

We are now ready to introduce the action of NATD on the RR sector. Employing the democratic formalism [71], RR fields are combined with their Hodge duals[4] to form the following bispinors:

$$P = \frac{e^\Phi}{2} \sum_{n=0}^{4} \not{F}_{2n+1} \;, \qquad \widehat{P} = \frac{e^{\widehat{\Phi}}}{2} \sum_{n=0}^{5} \widehat{\not{F}}_{2n} \;, \tag{2.20}$$

[3] A general form of Ω for an isometry group acting without isotropy is given in [19].

[4] Higher p-forms are related to the lower ones by $F_p = (-1)^{\left[\frac{p}{2}\right]} \star F_{10-p}$, with the Hodge dual given by:

$$(\star F_p)_{\mu_{p+1} \cdot \mu_D} = \frac{1}{p!} \sqrt{|g|}\; \epsilon_{\mu_1 \ldots \mu_D} F_p^{\mu_1 \cdots \mu_p}$$

where $\not{F}_p = \frac{1}{p!}\Gamma_{\mu_1 \ldots m_p} F_p^{\mu_1 \mu_2 \ldots m_p}$ using the Clifford map. Remark that F_{2n+1} odd-degree fluxes correspond to type IIB and F_{2n} even-degree fluxes to type IIA. The action of NATD on the RR sector, from a type IIB to a IIA solution, is then given by:

$$\widehat{P} = P \cdot \Omega^{-1} , \tag{2.21}$$

where Ω is given in eq. (2.19). If starting from a type IIA to a IIB solution instead, the role of P and $\widehat{P}$ is exchanged. Thanks to this relation, given the RR sector of a seed solution, we can determine the fluxes needed for the T-dual background to be also a solution of type II supergravity.

In general, fluxes along the duality directions are mapped as $F_p \rightarrow \hat{F}_{p-1}, \hat{F}_{p-3}$ under NATD, while the map works the other way round for fluxes not along the duality directions, $F_p \rightarrow \hat{F}_{p+1}, \hat{F}_{p+3}$. This is a generalization of the Abelian case, in which $F_p \rightarrow \hat{F}_{p\pm1}$, and underlies the change in the dimensionality of the D-branes under NATD, mapping p-branes to $(p \pm 1)$ and $(p \pm 3)$-branes[5]. A review of the "recipes" relating the original and T-dual fluxes for the simple case of NATD on a round S^3 can be found in the appendix B of our [23], see section 3.4, where also the Abelian case [3] is reviewed. The more general prescription for NATD applied on a squashed S^3 is treated in the appendices of [12, 68].

Knowledge of the spinor representation Ω of NATD also allows us to transform the ten-dimensional MW Killing spinors, say ϵ_1 and ϵ_2, of a type II supergravity solution to get the spinors $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ associated to the T-dual solution:

$$\hat{\epsilon}_1 = \epsilon_1 , \qquad \hat{\epsilon}_2 = \Omega \, \epsilon_2 .$$

In the examples considered, where NATD is always applied to the internal space, the above transformation reduces to a rotation of one of the Killing spinors w.r.t. the other. This has the effect of reducing the structure group of the internal space. A fast review of the transformation of pure spinors and G-structures under NATD is given in the appendices of the enclosed work [43], in section 3.2. An extended introduction can be also found in [41].

[5]Remark, however, that this does not imply that the number of independent charges is doubled under NATD. Sometimes, some of the would-be branes are found not to be BPS, see e.g. [54], included in section 3.1. In other examples, some branes dielectrically expand into others of higher dimensionality, and are interpreted to actually describe the same d.o.f.s. This is discussed in the results section 4 .

3. Articles

We present below a copy of the papers on which this thesis is based.

Contents

3.1 New $AdS_3 \times S^2$ T-duals with $\mathcal{N} = (0, 4)$ supersymmetry

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New $AdS_3 \times S^2$ T-duals with $\mathcal{N} = (0, 4)$ supersymmetry

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Abstract: It is well known that Hopf-fibre T-duality and uplift takes the D1-D5 near-horizon into a class of $AdS_3 \times S^2$ geometries in 11D where the internal space is a Calabi-Yau three-fold. Moreover, supersymmetry dictates that Calabi-Yau is the only permissible SU(3)-structure manifold. Generalising this duality chain to non-Abelian isometries, a strong parallel exists, resulting in the first explicit example of a class of $AdS_3 \times S^2$ geometries with SU(2)-structure. Furthermore, the non-Abelian T-dual of $AdS_3 \times S^3 \times S^3 \times S^1$ results in a new supersymmetric $AdS_3 \times S^2$ geometry, which falls outside of all known classifications. We explore the basic properties of the holographic duals associated to the new backgrounds. We compute the central charges and show that they are compatible with a large $\mathcal{N} = 4$ superconformal algebra in the infra-red.

Keywords: Supersymmetry and Duality, AdS-CFT Correspondence

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Contents

1 Introduction

It is not surprising that supersymmetric $AdS_3 \times S^2$ solutions to 11D supergravity [1, 2] bear a striking resemblance to their $AdS_5 \times S^2$ counterparts [3]; obvious cosmetic differences, such as supersymmetry and G-structures,[1] are ultimately tied to dimensionality. In common, we note that both spacetimes possess manifest SU(2) isometries, dual to the R-symmetries of

[1]Killing spinors, or supersymmetry variations, transform as a doublet under the SU(2) R-symmetry and are tensored with the Killing spinors of AdS_{d+1}, which have $2^{\frac{d}{2}}$ complex components.

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the respective 2D $\mathcal{N} = (0,4)$ [4, 5] and 4D $\mathcal{N} = 2$ [6, 7] SCFTs, and that supersymmetric geometries are in one-to-one correspondence with second-order PDEs. For the $\frac{1}{2}$-BPS bubbling geometries of Lin, Lunin and Maldacena (LLM), one famously encounters the 3D continuous Toda equation [3], while a similar local analysis in [2] has revealed a 5D analogue for $\frac{1}{4}$-BPS geometries:

$$y\,\partial_y\left(y^{-1}\partial_y J\right) = \mathrm{d}_4\left(J\cdot \mathrm{d}_4 \mathrm{sech}^2\zeta\right), \tag{1.1}$$

where the internal space exhibits SU(2)-structure.[2] Above ζ is a scalar depending on the 5D coordinates (y, x_i), J is the Kähler-form of the 4D base and d_4 denotes the pull-back of the derivative to the base. The 4D base corresponds to an *almost Calabi-Yau* two-fold [8].

Finding explicit supersymmetric geometries is thus equivalent, at least locally, to solving these PDEs. Despite the difficulties, we have witnessed a growing number of $AdS_5\times S^2$ geometries, and associated Toda solutions; starting with early constructions from gauged supergravity [9], through examples found directly in 11D [10],[3] recently a large number of solutions have been constructed by exploiting an added isometry and a connection to electrostatics [7, 12–14]. More recently, exotic solutions without an electrostatic, or with only an emergent electrostatic description have been found [15, 16]. Relevant to this current work, it is noteworthy that the SU(2) non-Abelian T-dual of $AdS_5 \times S^5$ also corresponds to a solution in this class [17].

In contrast, little is known about solutions to (1.1). Given the current literature, if we eliminate geometries exhibiting more supersymmetry, which one can disguise as $AdS_3\times S^2$ (see section 4 of [2]), there is no known $\frac{1}{4}$-BPS geometry that solves (1.1). In this paper, after uplift to 11D, we identify the non-Abelian T-dual of $AdS_3\times S^3\times CY_2$ [17] as the first example in this class. Admittedly, this example solves (1.1) in the most trivial way, since $\partial_y J = \mathrm{d}_4\zeta = 0$. That being said, it should be borne in mind that the linear supersymmetry conditions are satisfied non-trivially. It is worth appreciating an obvious parallel to Abelian T-duality, where the uplifted geometry is an example of an SU(3)-structure manifold, namely Calabi-Yau.

Before proceeding, we touch upon the generality of (1.1). It is not clear if all supersymmetric $\frac{1}{4}$-BPS $AdS_3\times S^2$ solutions in 11D with SU(2)-structure satisfy (1.1). Indeed, the analysis of LLM made the simplifying assumption that there are no $AdS_5\times S^2$ geometries with purely magnetic flux. Similarly, [2] precluded both purely electric and magnetic fluxes [2], a choice that is supported by AdS-limits of wrapped M5-brane geometries [1, 18]. For LLM, it can be explicitly shown that extra fluxes are inconsistent with supersymmetry [19][4] and an attempt at a more general analysis for $AdS_3\times S^2$ geometries appeared in [22], which derives the supersymmetry conditions in all generality, but unfortunately fails to constrain the fluxes greatly. Using these conditions, one can show that the existence of a single chiral spinor internally, corresponding to SU(3)-structure,

[2]SU(2)-structure in 6D is equivalent to two canonical SU(3)-structures.

[3]The 11D solution can be dimensionally reduced and T-dualised, where it becomes a quotient of $AdS_5\times S^5$. This provides no contradiction with a no-go result for $\frac{1}{2}$-BPS AdS_5 in IIB ref. [11].

[4]Generalising the Killing spinor ansatz [20] allows one to also describe maximally supersymmetric 11D solutions or $\frac{1}{2}$-BPS pp-waves, such as [21].

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implies Calabi-Yau.[5] For SU(2)-structure manifolds, we note that the non-Abelian T-dual of $AdS_3 \times S^3 \times CY_2$ fits neatly into the classification of [2]. In contrast, the non-Abelian T-dual of $AdS_3 \times S^3 \times S^3 \times S^1$ preserves the same supersymmetry, $\mathcal{N} = (0,4)$ in 2D, yet falls outside this class, thus motivating future work to extract the more general class [24].

Non-Abelian T-duality has revealed itself as a powerful tool to construct explicit AdS solutions that seemed unreachable by other means. In this work we present some further examples. Interesting solutions generated this way[6] are the only explicit AdS_6 solution to Type IIB supergravity constructed in [37][7] and the recent $\mathcal{N} = 2$ AdS_4 solution to M-theory with purely magnetic flux constructed in [42], which provides the only such explicit solution besides the Pernici-Sezgin background derived in the eighties [43]. Both these solutions may play an important role as gravity duals of, respectively, 5d fixed point theories arising from Type IIB brane configurations, probably from 7-branes as in [44] (see [45]), and of 3d SCFTs arising from M5-branes wrapped on 3d manifolds in the context of the 3d-3d correspondence [46]. In turn, the new AdS_3 backgrounds that we construct in this paper may provide the holographic duals of new 2D large $\mathcal{N} = (0,4)$ field theories arising from D-brane intersections. Other AdS_3 backgrounds dual to $\mathcal{N} = (0,2)$ 2D field theories haven been constructed recently in [28] (see also [47]) by compactifying on a 2D manifold the Klebanov-Witten background, combined with Abelian and non-Abelian T-dualities.

An essential difference with respect to its Abelian counterpart, is that non-Abelian T-duality has not been proved to be a symmetry of String Theory. In the context of the AdS/CFT correspondence one could thus expect new AdS backgrounds from known ones with very different dual CFTs. Furthermore, these CFTs are only guaranteed to exist in the strong coupling regime, since there is no reason to expect that the transformation should survive α' or $1/N$ corrections.

Even if the understanding of the CFT interpretation of the transformation is today very preliminary, some results point indeed in these directions. The non-Abelian T-dual of the $AdS_5 \times S^5$ background constructed in [17] has been shown for instance to belong to the family of $\mathcal{N} = 2$ Gaiotto-Maldacena geometries [7], proposed as duals of the, intrinsically strongly coupled, T_N Gaiotto theories [6]. Similarly, the non-Abelian T-dual of the $AdS_5 \times T^{1,1}$ background [48] gives rise to an AdS_5 background [49] that belongs to the general class of $\mathcal{N} = 1$ solutions in [50, 51], whose dual CFTs generalize the so-called Sicilian quivers of [52], and are the $\mathcal{N} = 1$ analogues of the $\mathcal{N} = 2$ solutions in [6].

[5] A small caveat here is that one of the 6D spinors $\epsilon_\pm$ was assumed to be chiral, however the supersymmetry constraints on scalar bilinears are strong enough to ensure $\epsilon_- = -i\epsilon_+$. The Calabi-Yau conditions $\mathrm{d}J = \mathrm{d}\Omega = 0$ then follow. We thank D. Tsimpis for raising this loop-hole.

[6] See also [25–28] for further AdS solutions and [29–36] for a more varied sample of the NAT duality literature.

[7] Supersymmetry imposes severe constraints to the existence of AdS_6 solutions in ten and eleven dimensions [38, 39]. Prior to [37] the only known explicit solution to Type II supergravities was the Brandhuber and Oz background [41], which was shown to be the only possible such solution in (massive) IIA in [38]. Later [39] proved the non-existence of AdS_6 solutions in M-theory and derived the PDEs that such solutions must satisfy in Type IIB (see also [40]), to which the example in [37], constructed from the Brandhuber and Oz solution via non-Abelian T-duality, provides the only known explicit solution (besides the Abelian T-dual).

Some works have tried to explore in more depth the CFT realization of AdS backgrounds generated through non-Abelian T-duality in different dimensions [25, 45]–[28]. Its interplay with supersymmetry [53] and phenomenological properties of the dual CFTs, such as the type of branes generating the geometry, the behavior of universal quantities such as the free energy, or the entanglement entropy, the realization of baryon vertices, instantons, giant gravitons, are by now quite systematized (see [54]). Very recently, we have witnessed as well an exciting and novel application in the exchange of particles with vortices [55]. In this paper we will analyze some of these properties in the 2D holographic duals to the new AdS_3 backgrounds that we generate. We will see that they fit in the general picture observed in other dimensions.

Perhaps the most puzzling obstacle towards a precise CFT interpretation of non-Abelian T-duality is the fact that even if the group used to construct the non-Abelian T-dual background is compact, the original coordinates transforming under this group are replaced in the dual by coordinates living in its Lie algebra. Non-compact internal directions are thus generated, which are hard to interpret in the CFT. We will also encounter this problem for the backgrounds generated in this paper.

The paper is organized as follows. In section 2 we present the first explicit example of an $AdS_3 \times S^2$ geometry belonging to the general class of solutions [2]. This is constructed by uplifting the non-Abelian T-dual of $AdS_3 \times S^3 \times CY_2$ derived in [17] to 11D. In section 3 we recall the basic properties of the $AdS_3 \times S^3 \times S^3 \times S^1$ background that will be the basis of the new solutions that we present in sections 4, 5 and 6. In section 4 we construct the non-Abelian T-dual of this background with respect to a freely acting SU(2) on one of the S^3. By exploring the solution we derive some properties of the associated dual CFT such as the central charge and the type of color and flavor branes from which it may arise. We suggest a possible explicit realization in terms of intersecting branes. In section 5 we construct one further solution through Abelian T-duality plus uplift to 11D from the previous one and show that it provides an explicit example of an $AdS_3 \times S^2$ geometry in 11D belonging to a new class that is beyond the ansatz in [2]. In section 6 we present a new $AdS_3 \times S^2 \times S^2$ solution to Type IIB obtained by further dualizing the solution in section 3 with respect to a freely acting SU(2) on the remaining S^3. By analyzing the same brane configurations we argue that the field theory dual shares some common properties with the CFT dual to the original $AdS_3 \times S^3 \times S^3 \times S^1$ background but in a less symmetric fashion. In section 7 we analyze in detail the supersymmetries preserved by the different solutions that we construct. We show that the solutions constructed through non-Abelian T-duality from the $AdS_3 \times S^3 \times S^3 \times S^1$ background exhibit large $\mathcal{N} = (0, 4)$ supersymmetry. This is supported by the analysis of the central charges performed in sections 4 and 6. Section 8 contains our conclusions. Finally, in the appendix we study in detail the effect of Hopf-fibre T-duality in the $AdS_3 \times S^3 \times S^3 \times S^1$ background to further support our claims in the text concerning the isometry supergroup of our solutions.

2 $AdS_3 \times S^2$ geometries in 11D with SU(2)-structure

In this section we demonstrate that the non-Abelian T-dual of the D1-D5 near-horizon, a solution that was originally written down in [17], uplifts to 11D, where it provides the

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first explicit example of a $\frac{1}{4}$-BPS $AdS_3 \times S^2$ geometry with an internal SU(2)-structure manifold. We recall that this class has appeared in a series of classifications [1, 2, 18, 22], yet until now, not a single explicit example in this class was known. It is indeed pleasing to recognise that the chain of dualities that generates this new example is no more than a simple non-Abelian generalisation of a well-known mapping from the $AdS_3 \times S^3 \times CY_2$ geometry of Type IIB supergravity into the 11D supergravity class $AdS_3 \times S^2 \times CY_3$.[8] It is worth noting that until relatively recently [17] (also [49]), the workings of this new mapping, which is made possible through non-Abelian T-duality, were also unknown.

We begin by reviewing the classification of ref. [22], which has an advantage over other approaches [1], since it uses local techniques and is thus guaranteed to capture all supersymmetric solutions. Moreover, this work also extends the ansatz of ref. [2] and dispenses with the need for an analytic continuation from $S^3 \times S^2$ to $AdS_3 \times S^2$. Based on symmetries, the general form for a supersymmetric spacetime of this type may be expressed as

$$\begin{aligned} \mathrm{d}s^2_{11} &= e^{2\lambda}\left[\frac{1}{m^2}\mathrm{d}s^2(AdS_3) + e^{2A}\mathrm{d}s^2(S^2) + \mathrm{d}s^2_6\right], \\ G_4 &= \mathrm{Vol}(AdS_3)\wedge\mathcal{A} + \mathrm{Vol}(S^2)\wedge\mathcal{H} + \mathcal{G}, \end{aligned} \tag{2.1}$$

where λ, A denote warp-factors depending on the coordinates of the 6D internal space and $\mathcal{A}, \mathcal{H}$ and $\mathcal{G}$ correspond to one, two and four-forms, respectively, with legs on the internal space. The constant m denotes the inverse radius of AdS_3. The supersymmetry conditions, which are given in terms of differential conditions on spinor bilinears, further built from two a priori independent 6D spinors $\epsilon_\pm$, can be found in [22].

Setting $\mathcal{A} = \mathcal{G} = 0$, one finds that only a particular linear combination, $\tilde{\epsilon} = \epsilon_+ \pm i\gamma_7\epsilon_-$ appears in the effective 6D Killing spinor equations, allowing one to recover the work of [2]. In this simplifying case one can show that the internal space must be of the form [2, 22]

$$\mathrm{d}s^2_6 = g_{ij}\mathrm{d}x^i\mathrm{d}x^j + e^{-6\lambda}\sec^2\zeta\mathrm{d}y^2 + \cos^2\zeta(\mathrm{d}\psi + P)^2 \tag{2.2}$$

with P a one-form connection on the 4D base with metric g_{ij}. The SU(2)-structure is then specified by 2 one-forms, $K^1 \equiv \cos\zeta(\mathrm{d}\psi + P)$, $K^2 \equiv e^{-3\lambda}\sec\zeta\mathrm{d}y$, the Kähler-form, J, and the complex two-form, Ω, on the base.

The remaining two-form appearing in the field strength, G_4, is fully determined by supersymmetry,

$$\begin{aligned} \mathcal{H} &= -yJ - \frac{1}{2m}\partial_y(y\sin^2\zeta)\mathrm{d}y\wedge(\mathrm{d}\psi + P) \\ &\quad -\frac{y}{m}\cos\zeta\sin\zeta\mathrm{d}_4\zeta\wedge(\mathrm{d}\psi + P) + \frac{y\cos^2\zeta}{2m}\mathrm{d}P. \end{aligned} \tag{2.3}$$

[8]See appendix B of ref. [56] for a concrete realisation of the (Abelian) duality chain.

The above class of geometries is subject to the supersymmetry conditions:

$$\begin{aligned} 2my &= e^{3\lambda}\sin\zeta, \\ e^{A} &= \frac{\sin\zeta}{2m}, \\ \mathrm{d}(e^{3\lambda}\cos\zeta\Omega) &= 0, \\ 2m\mathrm{d}(e^{3\lambda+2A}J) &= \mathrm{d}_4P\wedge\mathrm{d}y. \end{aligned} \tag{2.4}$$

Details of how (1.1) is implied by these conditions can be found in [2].

In order to identify a solution in this class, we start by recalling the non-Abelian T-dual of $AdS_3\times S^3\times T^4$ [17], which provides a solution to massive IIA supergravity,

$$\begin{aligned} \mathrm{d}s^2_{IIA} &= \mathrm{d}s^2(AdS_3)+\mathrm{d}\rho^2+\frac{\rho^2}{1+\rho^2}\mathrm{d}s^2(S^2)+\mathrm{d}s^2(T^4), \\ B_2 &= \frac{\rho^3}{1+\rho^2}\mathrm{vol}(S^2), \qquad \Phi = -\frac{1}{2}\ln(1+\rho^2), \\ m &= 1, \qquad F_2 = \frac{\rho^3}{1+\rho^2}\mathrm{vol}(S^2), \\ F_4 &= \mathrm{vol}(AdS_3)\wedge\rho\mathrm{d}\rho+\mathrm{vol}(T^4), \end{aligned} \tag{2.5}$$

where following [17], we have suppressed factors associated to radii for simplicity. As a consequence, the AdS_3 metric is normalised so that $R_{\mu\nu}=-\frac{1}{2}g_{\mu\nu}$, whereas S^2 is canonically normalised to unit radius.

We next perform two T-dualities along the T^4, the coordinates of which we label, $x_1,\dots x_4$. Performing T-dualities with respect to x_1 and x_2, we can replace the Romans' mass, $m=1$, with higher-dimensional forms, while leaving the NS sector unaltered. In addition to the NS two-form, the geometry is then supported by the following potentials from the RR sector,

$$\begin{aligned} C_1 &= \frac{1}{2}(x_1\mathrm{d}x_2-x_2\mathrm{d}x_1+x_3\mathrm{d}x_4-x_4\mathrm{d}x_3), \\ C_3 &= \frac{\rho^3}{1+\rho^2}\mathrm{vol}(S^2)\wedge C_1. \end{aligned} \tag{2.6}$$

We note that $\mathrm{d}C_1=J$, where J is the Kähler form on T^4 and the Bianchi for F_4, namely $\mathrm{d}F_4=H_3\wedge F_2$ is satisfied in a trivial way since $F_4=\mathrm{d}C_3=B_2\wedge J$. We can now uplift the solution on a circle to 11D by considering the standard Kaluza-Klein ansatz,

$$\begin{aligned} \mathrm{d}s^2_{11} &= (1+\rho^2)^{\frac{1}{3}}\left[\mathrm{d}s^2(AdS_3)+\frac{\rho^2}{1+\rho^2}\mathrm{d}s^2(S^2)+\mathrm{d}\rho^2+\mathrm{d}s^2(T^4)\right]+(1+\rho^2)^{-\frac{2}{3}}\mathrm{D}z^2, \\ G_4 &= \mathrm{vol}(S^2)\left[\frac{\rho^3}{1+\rho^2}J+\frac{\rho^2(\rho^2+3)}{(1+\rho^2)^2}\mathrm{d}\rho\wedge\mathrm{D}z\right], \end{aligned} \tag{2.7}$$

where we have defined $\mathrm{D}z\equiv\mathrm{d}z+C_1$.

Adopting $m = 2$, so that normalisations for AdS_3 agree, and up to an overall sign in $\mathcal{H}$, which can be accommodated through the sign flip $\rho \leftrightarrow -\rho$, we find that the supersymmetry conditions (2.4) are satisfied once one identifies accordingly

$$\begin{aligned} &y = \rho, \qquad e^{\lambda} = (1+\rho^2)^{\frac{1}{6}}, \qquad e^{A} = \frac{\rho}{(1+\rho^2)^{\frac{1}{2}}}, \qquad P = C_1, \\ &J = \mathrm{d}x_1 \wedge \mathrm{d}x_2 + \mathrm{d}x_3 \wedge \mathrm{d}x_4, \\ &\Omega = (\mathrm{d}x_1 + i\mathrm{d}x_2) \wedge (\mathrm{d}x_3 + i\mathrm{d}x_4). \end{aligned} \tag{2.8}$$

Thus the non-Abelian T-dual plus 11D uplift of the D1-D5 near horizon fits in the classifications [1, 2, 18, 22]. It is easy to see that one can replace T^4 with K3 and the construction still holds. It is also easy to see that the above solution can be derived on the assumption that the base is Calabi-Yau and that λ, ζ only depend on y. Indeed, this is a requirement of the 6D SU(2)-structure manifold to be a complex manifold [22]. In this case, the supersymmetry conditions imply $e^{3\lambda}\cos\zeta$ is a constant. We can then solve for λ, ζ and A giving us the above solution.

Another interesting feature of the 11D solution is that in performing the classification exercise using Killing spinor bilinears [2, 22], one finds a U(1) isometry that emerges from the analysis for free. Often this U(1) corresponds to an R-symmetry, for example [3, 23], but in this setting, the relevant superconformal symmetry in 2D either corresponds to small superconformal symmetry with R-symmetry SU(2), or large superconformal symmetry with R-symmetry SU(2) × SU(2). There appears to be no place for a U(1) R-symmetry and it is an interesting feature of solutions fitting into the class of [2] that the U(1) is the M-theory circle and the Killing spinors are uncharged with respect to this direction.[9]

In the rest of this paper, we study non-Abelian T-duals of another well-known $\frac{1}{2}$-BPS AdS_3 solution with $\mathcal{N} = (4,4)$ supersymmetry, namely $AdS_3 \times S^3 \times S^3 \times S^1$, where we will find a new supersymmetric solution that does not fit into the class in [2].

3 The $AdS_3 \times S^3 \times S^3 \times S^1$ background with pure RR flux

In this section we recall the basic properties of the $AdS_3 \times S^3 \times S^3 \times S^1$ background [57]–[58], which will be the basis of our study in the following sections.

The $AdS_3 \times S^3 \times S^3 \times S^1$ background is a half-BPS solution of Type II string theory supported by NS5-brane and string flux. In this paper we will be interested in its realization in Type IIB where it is supported by D5 and D1-brane fluxes [59]. This description arises after compactifying on a circle the $AdS_3 \times S^3 \times S^3 \times \mathbb{R}$ near horizon geometry of a D1-D5-D5' system where the two stacks of D5-branes are orthogonal and intersect only along the line of the D1-branes [57, 60, 61]. How to implement the S^1 compactification has remained unclear (see [59]), and it has only been argued recently [62] that the $\mathbb{R}$ instead of the S^1

[9] In 11D one can identify the two projection conditions to verify that supersymmetry is not enhanced. From CY_2 directions, we inherit $\Gamma^{6789}\eta = -\eta$, the rotation on the 11D spinor becomes $\eta = \exp[-\frac{1}{2}\tan^{-1}\left(\frac{1}{\rho}\right)\Gamma^{x\xi z}]\tilde{\eta}$. One finds the additional projector, $\Gamma^{\rho z 67}\tilde{\eta} = -\tilde{\eta}$, thus confirming that the 11D solution is indeed $\frac{1}{4}$-BPS.

factor arising in the near horizon limit could just be an artefact of the smearing of the D1-branes on the transverse directions prior to taking the limit. This reference has also provided the explicit $\mathcal{N} = (4,4)$ CFT realization conjectured in [58, 59, 63] for the field theory dual. This CFT arises as the infrared fixed point of the $\mathcal{N} = (0,4)$ gauge theory living on the D1-D5-D5' intersecting D-branes.

The $AdS_3 \times S^3_+ \times S^3_- \times \mathbb{R}$ metric is given by

$$ds^2_{IIB} = L^2 \mathrm{d}s^2(AdS_3) + R_+^2 \mathrm{d}s^2(S^3_+) + R_-^2 \mathrm{d}s^2(S^3_-) + \mathrm{d}x^2 \tag{3.1}$$

with

$$\mathrm{d}s^2(AdS_3) = r^2(-\mathrm{d}t^2 + \mathrm{d}x_1^2) + \frac{\mathrm{d}r^2}{r^2} \tag{3.2}$$

in Poincaré coordinates. Plus, the background is supported by a single non trivial RR flux

$$F_3 = 2L^2 \mathrm{Vol}(AdS_3) + 2R_+^2 \mathrm{Vol}(S^3_+) + 2R_-^2 \mathrm{Vol}(S^3_-), \tag{3.3}$$

with Hodge dual

$$F_7 = \Big\{ 2L^3 \mathrm{Vol}(AdS_3) \wedge \Big(-\frac{R_+^3}{R_-} \mathrm{Vol}(S^3_+) + \frac{R_-^3}{R_+} \mathrm{Vol}(S^3_-) \Big) + \frac{2R_+^3 R_-^3}{L} \mathrm{Vol}(S^3_+) \wedge \mathrm{Vol}(S^3_-) \Big\} \wedge \mathrm{d}x \tag{3.4}$$

We take $g_s = 1$ such that the dilaton is zero and Einstein's equations are satisfied only when

$$\frac{1}{L^2} = \frac{1}{R_+^2} + \frac{1}{R_-^2}. \tag{3.5}$$

This background has a large invariance under $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$ spatial rotations. Of these $\mathrm{SU}(2)_R^+ \times \mathrm{SU}(2)_R^-$ correspond to the R-symmetry group of the $\mathcal{N} = (0,4)$ field theory living at the D1-D5-D5' intersection, and $\mathrm{SU}(2)_L^+ \times \mathrm{SU}(2)_L^-$ to a global symmetry. The field theory has gauge group $\mathrm{U}(N_1)$, with N_1 the number of D1-branes, and a global symmetry $\mathrm{SU}(N_5^+) \times \mathrm{SU}(N_5^-)$, with N_5^+ and N_5^- the number of D5 and D5' branes. The two R-symmetries give rise to two current algebras at levels depending on the background charges, and to a large $\mathcal{N} = (4,4)$ superconformal symmetry in the infra-red [58, 59, 62, 63].

The study of the supergravity solution allows to derive properties of the dual field theory that we will be able to mimic after the non-Abelian T-duality transformation. In the next subsections we analyze the quantized charges, some brane configurations such as baryon vertices and 't Hooft monopoles, and the central charge associated to the $AdS_3 \times S^3 \times S^3 \times S^1$ background.

3.1 Quantized charges

The F_7 and F_3 fluxes generate D1 and D5-brane charges given by:

$$N_1 = \frac{1}{2\pi\kappa_{10}^2 T_1} \int (-F_7) = \frac{R_+^3 R_-^3 \delta x}{8L\pi^2}\,, \tag{3.6}$$

where δx is the length of the x-direction interval, which should be chosen such that N_1 is quantized, and

$$N_5^+ = \frac{1}{2\pi\kappa_{10}^2 T_5}\int_{S^3_-}(-F_3) = R_-^2\,, \qquad N_5^- = \frac{1}{2\pi\kappa_{10}^2 T_5}\int_{S^3_+}(-F_3) = R_+^2\,, \tag{3.7}$$

which should also be quantized. Accordingly, one can find D1 and D5 BPS solutions. The D1 are extended along the $\{t, x_1\}$ directions and couple to the potential

$$C_2 = L^2 r^2 \mathrm{d}t \wedge \mathrm{d}x_1\,. \tag{3.8}$$

Changing coordinates to

$$\begin{cases} r = r_+ r_- \\ x = \dfrac{R_+^2}{\sqrt{R_+^2 + R_-^2}}\log r_+ - \dfrac{R_-^2}{\sqrt{R_+^2 + R_-^2}}\log r_-\,, \end{cases} \tag{3.9}$$

the metric becomes the near horizon limit of the intersecting D1-D5-D5' configuration [57, 60, 61]:

$$\begin{aligned} N_5^+\ D5 :&\quad 012345 \\ N_5^-\ D5' :&\quad 016789 \\ N_1\,D1 :&\quad 01 \end{aligned} \tag{3.10}$$

with $\mathrm{d}x_2^2 + \cdots + \mathrm{d}x_5^2 = \mathrm{d}r_+^2 + r_+^2\mathrm{d}s^2(S^3_+)$, $\mathrm{d}x_6^2 + \cdots + \mathrm{d}x_9^2 = \mathrm{d}r_-^2 + r_-^2\mathrm{d}s^2(S^3_-)$, with the D1-branes smeared on these directions:

$$\mathrm{d}s^2_{IIB} = L^2 r_+^2 r_-^2(-\mathrm{d}t^2 + \mathrm{d}x_1^2) + R_+^2\frac{\mathrm{d}r_+^2}{r_+^2} + R_-^2\frac{\mathrm{d}r_-^2}{r_-^2} + R_+^2\mathrm{d}s^2(S^3_+) + R_-^2\mathrm{d}s^2(S^3_-)\,. \tag{3.11}$$

The BPS D5-branes are then found lying on the (t, x_1, r_+, S^3_+), (t, x_1, r_-, S^3_-) directions.

The 2D $\mathcal{N} = (0,4)$ gauge theory living on the worldvolume of the D1-branes and intersecting D5-branes has been identified recently in [62]. A key role is played by the chiral fermions of the D5-D5' strings that lie at the intersection. Quite remarkably the central charge of the $\mathcal{N} = (4,4)$ CFT to which this theory flows in the infra-red has been shown to coincide with the central charge of the supergravity solution, that we review in subsection 3.4.

3.2 Instantons

The previous configuration of D5, D5' branes joined in a single manifold, where the D1-branes lie, admits a Higgs branch where the D1-branes are realized as instantons in the D5-branes [58]. One can indeed compute the quadratic fluctuations of the D5-branes to obtain the effective YM coupling:

$$S^{\mathrm{D5}}_{\mathrm{fluc}} = -\int\frac{1}{g^2_{\mathrm{D5}}}F^2_{\mu\nu} \qquad \text{with} \qquad \frac{1}{g^2_{\mathrm{D5}}} = \frac{L^2 r_+^2 r_-^2}{4(2\pi)^3} \tag{3.12}$$

and check that the DBI action of the D1-branes satisfies

$$S^{\mathrm{D1}}_{\mathrm{DBI}} = -\int\frac{16\pi^2}{g^2_{\mathrm{D5}}} \tag{3.13}$$

as expected for an instantonic brane.

3.3 Baryon vertices and 't Hooft monopoles

A D7-brane wrapped on $S^3_+ \times S^3_- \times S^1$ realizes a baryon vertex in the $AdS_3 \times S^3_+ \times S^3_- \times S^1$ geometry, since it develops a tadpole of N_1 units, as it is inferred from its CS action:

$$S^{\rm D7}_{\rm CS} = 2\pi\, T_7 \int C_6 \wedge F = -2\pi\, T_7 \int_{S^3_+ \times S^3_- \times S^1} F_7 \int {\rm d}t A_t = -N_1 \int {\rm d}t A_t \;, \tag{3.14}$$

where δx is taken to satisfy that N_1 is an integer as in (3.6).

Similarly, there are two t'Hooft monopoles associated to the ranks of the two flavor groups that are realized in the $AdS_3 \times S^3_+ \times S^3_- \times S^1$ background as D3-branes wrapping the $S^3_\pm$. The corresponding Chern-Simons terms show that these branes have tadpoles of $N_5^\mp$ units that should be cancelled with the addition of these numbers of fundamental strings:

$$S^{\rm D3^\pm}_{\rm CS} = -2\pi T_3 \int_{S^3_\pm} F_3 \int {\rm d}t A_t = N_5^\mp \int {\rm d}t A_t \;. \tag{3.15}$$

3.4 Central charge

The central charge associated to the $AdS_3 \times S^3_+ \times S^3_- \times S^1$ background can be computed using the Brown-Henneaux formula [64], giving [58, 59]:

$$c = 2N_1 \,\frac{N_5^+ N_5^-}{N_5^+ + N_5^-} \;. \tag{3.16}$$

This expression agrees with the central charge for a large $\mathcal{N} = (4,4)$ CFT with affine $\mathrm{SU}(2)^\pm$ current algebras at levels $k^\pm$: $c = 2k^+k^-/(k^+ + k^-)$ [65], with $k^\pm = N_1 N_5^\pm$. A strong check of the validity of the $\mathcal{N} = (0,4)$ field theory in the D1-branes proposed in [62] is that it correctly reproduces (3.16) at the infrared fixed point (see also [58]).

4 Non-Abelian T-dual $AdS_3 \times S^3 \times S^2$ solution in IIA

In this section we dualize the $AdS_3 \times S^3_+ \times S^3_- \times S^1$ solution with respect to the $\mathrm{SU}(2)^-_L$ acting on the S^3_-. This dualization was reported in [53] to produce a new AdS_3 solution preserving 16 supercharges. As we shall demonstrate in section 7 [53] overlooked an extra implied condition and the preserved supersymmetry is in fact 8 supercharges. The solution thus preserves large $\mathcal{N} = (0,4)$ supersymmetry in 2D. In this section we present a detailed study of the geometry and infer some properties about the field theory interpretation of this solution.

4.1 Background

Applying the general rules in [66] (see also [53]) we find a dual metric

$${\rm d}s^2_{IIA} = L^2 {\rm d}s^2(AdS_3) + R_+^2 {\rm d}s^2(S^3_+) + \frac{4}{R_-^2}\left({\rm d}\rho^2 + \frac{R_-^6\rho^2}{64\Delta}\left({\rm d}\chi^2 + \sin^2\chi {\rm d}\xi^2\right)\right) + {\rm d}x^2, \tag{4.1}$$

where

$$\Delta = \frac{R_-^6 + 16R_-^2\rho^2}{64}. \tag{4.2}$$

The dual dilaton is given by

$$e^{-2\Phi} = \Delta, \tag{4.3}$$

while the NS 2-form is simply

$$B_2 = \frac{R_-^2 \rho^3}{4\Delta} \text{Vol}(S^2) \tag{4.4}$$

where S^2 refers to the 2-sphere parametrised by $0 \le \chi \le \pi,\ 0 \le \xi < 2\pi$ in (4.1).

The dual RR-sector is given by

$$\begin{aligned}
m &= \frac{R_-^2}{4}, \\
\hat{F}_2 &= 0, \\
\hat{F}_4 &= -\frac{R_-^3}{4LR_+}\big(L^4 \text{Vol}(AdS_3) + R_+^4 \text{Vol}(S_+^3)\big) \wedge \mathrm{d}x \\
&\qquad\qquad\qquad +2\rho\big(L^2 \text{Vol}(AdS_3) + R_+^2 \text{Vol}(S_+^3)\big) \wedge \mathrm{d}\rho, \\
\hat{F}_6 &= -2L^2\rho^2 \text{Vol}(AdS_3) \wedge \text{Vol}(S^2) \wedge \mathrm{d}\rho - 2R_+^2 \rho^2 \text{Vol}(S_+^3) \wedge \text{Vol}(S^2) \wedge \mathrm{d}\rho, \\
\hat{F}_8 &= -\frac{2L^3 R_+^3 \rho}{R_-} \text{Vol}(AdS_3) \wedge \text{Vol}(S_+^3) \wedge \mathrm{d}x \wedge \mathrm{d}\rho, \\
\hat{F}_{10} &= \frac{2L^3 R_+^3 \rho^2}{R_-} \text{Vol}(AdS_3) \wedge \text{Vol}(S_+^3) \wedge \text{Vol}(S^2) \wedge \mathrm{d}x \wedge \mathrm{d}\rho.
\end{aligned}$$

Here $\hat{F} = Fe^{-B_2}$ and $F_p = \mathrm{d}C_{p-1} - H_3 \wedge C_{p-3}$. Page charges will be computed from these $\hat{F}$ according to $\mathrm{d} * \hat{F} = *j^{\text{Page}}$.

Applying the results in [66] this background is guaranteed to satisfy the (massive) IIA supergravity equations of motion. Given that the S^3 on which we have dualized has constant radius the non-Abelian T-dual solution is also automatically non-singular. An open problem though is the range of the new coordinate ρ, which as a result of the non-Abelian T-duality transformation lives in $\mathbb{R}^+$.

The generation of non-compact directions under non-Abelian T-duality is indeed a generic feature that does not occur under its Abelian counterpart. In the last case the extension of the transformation beyond tree level in string perturbation theory determines uniquely the global properties of the, in principle non-compact, coordinate that replaces the dualized U(1) direction. How to extend non-Abelian T-duality beyond tree level is however a long standing open problem (see [67] for more details), and as a result we are lacking a general mechanism that allows to compactify the new coordinates. For freely acting SU(2) examples we need to account in particular for the presence of the non-compact ρ-direction in the dual internal geometry, which poses a problem to its CFT interpretation, where we can expect operators with continuous conformal dimensions. Note that in the $AdS_3 \times S^2 \times S^1$ duals under consideration in this paper one cannot hope that the same mechanism that should be at work for compactifying the $\mathbb{R}$ factor arising in the original $AdS_3 \times S^3 \times S^3 \times \mathbb{R}$ geometry should be applicable. As argued in [62], the $\mathbb{R}$ instead of the

S^1 factor arising in the near horizon limit could be due to the smearing of the D1-branes on the transverse directions prior to taking the limit, and could presumably be avoided with a supergravity solution describing localized branes. This is not directly applicable to our situation because ρ is not an isometric direction.

Previous approaches in the recent non-Abelian T-duality literature have tried to infer global properties through imposing consistency to the dual CFT [45, 54]. We will also follow this approach in this paper. We should start noticing that the new AdS_3 metric described by (4.1) is perfectly regular for all $\rho \in [0, \infty)$, with the 3d space replacing the S^3_- in the original background becoming $\mathbb{R}^3$ for small ρ and $\mathbb{R} \times S^2$ for large ρ. As shown in [45, 54] the definition of large gauge transformations in the dual geometry can give however non-trivial information about its global properties.

4.2 Large gauge transformations

The relevance of large gauge transformations is linked to the existence of non-trivial 2-cycles in the geometry, where

$$\frac{1}{4\pi^2}\left|\int_{2-\text{cycle}} B_2\right| \in [0,1)\,. \tag{4.5}$$

In our non-singular metric we can only guarantee the existence of a non-trivial S^2 for large ρ. For finite ρ and given the absence of any global information, we will resort to the most general situation in which the cycle remains non-trivial and we need to care about large gauge transformations. We will see that consistency of the CFT in this most general situation will lead to the condition of vanishing large gauge transformations, which is compatible with the original situation in which the two-cycle may in fact be trivial at finite ρ.

Assuming the existence of a non-trivial two-cycle at finite ρ, the ρ dependence of B_2 in (4.4) implies that large gauge transformations must be defined such that (4.5) is satisfied as we move in this direction. This implies that for $\rho \in [\rho_n, \rho_{n+1}]$ with ρ_n determined by $16\rho_n^3/(R_-^4 + 16\rho_n^2) = n\pi$, B_2 must be given by

$$B_2 = \left(\frac{R_-^2\rho^3}{4\Delta} - n\pi\right)\text{Vol}(S^2)\,. \tag{4.6}$$

The fluxes from which the Page charges are computed then change in the different intervals to

$$\begin{aligned}\hat{F}_2 &\to \hat{F}_2 + n\pi\,\hat{F}_0\,\text{Vol}(S^2)\\ \hat{F}_6 &\to \hat{F}_6 + n\pi\,\hat{F}_4 \wedge \text{Vol}(S^2)\,,\end{aligned} \tag{4.7}$$

which will affect the values of the Page charges that we compute next.

4.3 Quantized charges

The transformation of the RR fluxes under non-Abelian T-duality implies that the D1 color branes of the original background transform into D2-branes extended on $\{t, x_1, \rho\}$ and D4-branes on $\{t, x_1, \rho, S^2\}$. Analogously, the D5 flavor branes wrapped on the S^3_- are mapped into D2 and D4 branes wrapped on $\{t, x_1, r_-\}$ and $\{t, x_1, r_-, S^2\}$ respectively, and the D5 transverse to the S^3_- are transformed into D6 and D8 branes wrapped on $\{t, x_1, r_+, S^3_+, \rho\}$ and $\{t, x_1, r_+, S^3_+, \rho, S^2\}$, respectively. We show in this section that there are quantized charges in the non-Abelian T-dual background that can be associated to these branes.

4.3.1 Color branes

It is possible to define N_2 and N_4 quantized charges in the dual background that should be associated to D2 and D4 color branes:

$$N_4 = \frac{1}{2\pi\kappa_{10}^2 T_4} \int_{S^3_+ \times S^1} \hat{F}_4 = \frac{R_+^3 R_-^3}{16\pi L}\, \delta x \,, \tag{4.8}$$

$$N_2 = \frac{1}{2\pi\kappa_{10}^2 T_2} \int_{S^3_+ \times S^2 \times S^1} \hat{F}_6 = n N_4 \,, \tag{4.9}$$

where n is the parameter labeling large gauge transformations. (4.9) is the value of the D2 charge in the $\rho \in [\rho_n, \rho_{n+1}]$ interval. Note that, as it seems to be quite generic under non-Abelian T-duality, the condition imposed on the geometry by (4.8) is different, and in fact incompatible, from the one that the original background satisfied, given by (3.6). A re-quantization must thus be done in the new background.

Let us now analyze the condition (4.9). We first see that for zero n the charge associated to the D2-branes vanishes. Second, as we change interval, N_2 undergoes a transformation, $N_2 \to N_2 - N_4$, that is very reminiscent of Seiberg duality [68]. This was proposed in [54] as a way to relate the CFTs dual to the solution as we move in ρ. As stressed in [28][10] this cannot be however the full story since there is a change in the number of degrees of freedom as we move in ρ. This is explicit in the holographic free energies. The precise realization in the CFT of the running of ρ remains at the very heart of our full understanding of the interplay between non-Abelian T-duality and AdS/CFT. We hope we will be able to report progress in this direction in future publications.

For the particular background considered in this paper it is only possible to find BPS color and flavor branes when $n = 0$. In particular, color branes are D4-branes wrapped on the $\{t, x_1, \rho, S^2\}$ directions. Thus, we will take the view that ρ is restricted to the fundamental region $[0, \rho_1]$, with ρ_1 satisfying $16\rho_1^3/(R_-^4 + 16\rho_1^2) = \pi$. Choosing to end the geometry at a regular point presents however other problems, now for the geometry, where extra localized sources should be included. It was proposed in [28] that at these transition points new gauge groups would be added to the CFT through an "unhiggsing" mechanism not associated to an energy scale. Given that this mechanism relies in the existence of large gauge transformations it does not seem applicable to our background. A full understanding of the "unhiggsing" mechanism and its precise realization in the absence of an energy scale remains as an interesting open problem.

4.3.2 Flavor branes

Let us now examine flavor branes in the dual background. We find the following quantized charges in the dual background that should be associated to flavor branes:

$$N_8^f = 2\pi F_0 = \frac{\pi}{2} R_-^2 \,, \qquad N_6^f = \frac{1}{2\pi\kappa_{10}^2 T_6} \int_{S^2} \hat{F}_2 = n N_8^f \,, \tag{4.10}$$

$$N_4^f = \frac{1}{2\pi\kappa_{10}^2 T_4} \int_{S^3_+} \int_{\rho_n}^{\rho_{n+1}} \mathrm{d}\rho\, \hat{F}_4 \,, \qquad N_2^f = \frac{1}{2\pi\kappa_{10}^2 T_2} \left| \int_{S^3_+ \times S^2} \int_{\rho_n}^{\rho_{n+1}} \mathrm{d}\rho\, \hat{F}_6 \right| . \tag{4.11}$$

[10]We would also like to acknowledge fruitful conversations with D. Rodríguez-Gómez on this issue.

Here we have made explicit the interval on which the ρ direction has to be integrated and we have not restricted ourselves to vanishing large gauge transformations.

The first two charges in (4.10) correspond to the D8 and D6 flavor branes that originate on the N_5^+ D5-branes of the original background. Thus, our expectation is to find BPS D8 wrapped on $\{t, x_1, r_+, S^3_+, \rho, S^2\}$ and BPS D6 wrapped on $\{t, x_1, r_+, S^3_+, \rho\}$. However, as for the color branes, we also find that the D6 are never BPS unless $R_- = 0$ and that the D8 (anti-D8 in our conventions) are BPS only in the absence of large gauge transformations. This is again suggestive of a dual background where large gauge transformations are not possible. In the absence of these the D5 flavor branes give rise to just D8 flavor branes in the dual background.

The D5' branes of the original background give rise in turn to D4-branes wrapped on $\{t, x_1, r_-, S^2\}$ and D2-branes wrapped on $\{t, x_1, r_-\}$, which turn out to be BPS only when located at $\rho = 0$. In this position however both the DBI and CS actions of the D4 vanish, leaving just D2-branes as candidate flavor branes.

4.3.3 A possible brane intersection?

Summarizing, we have found that there are only BPS color and flavor branes in the absence of large gauge transformations, in which case there is only one color or flavor brane in the non-Abelian T-dual background associated to each color or flavor brane of the original theory. Note that this is essentially different from previous examples in the literature (for instance [45, 54]) where both types of color and flavor branes were guaranteed to exist for all n. We argue in the conclusions that this could be explained by the absence of non-trivial 2-cycles in our particular dual geometry.[11]

We have shown that the D1-branes are replaced by D4-branes wrapped on $\{t, x_1, \rho, S^2\}$ and the D5 and D5' flavor branes are replaced by D8-branes wrapped on $\{t, x_1, r_+, S^3_+, \rho, S^2\}$ and D2-branes wrapped on $\{t, x_1, r_-\}$, respectively. This is summarized pictorially as

$$\begin{tikzcd}[row sep=small] & \cancel{D2} \\ D1 \arrow[ur] \arrow[dr] & \\ & D4 \end{tikzcd} \qquad \begin{tikzcd}[row sep=small] & \cancel{D6} \\ D5 \arrow[ur] \arrow[dr] & \\ & D8 \end{tikzcd} \qquad \begin{tikzcd}[row sep=small] & \cancel{D4} \\ D5' \arrow[ur] \arrow[dr] & \\ & D2 \end{tikzcd}$$

Here we have also indicated the brane that turns out not to occur as a BPS configuration even if expected a priori from the analysis of the fluxes.

Note that precisely a $D1 \to D4$, $D5 \to D8$, $D5' \to D2$ map is what one would have obtained after (Abelian) T-dualizing the D1, D5, D5' system along three directions transverse to the D1 and the D5 and longitudinal to the D5'. This suggests a dual geometry coming out as the near horizon limit of the brane intersection:

$$\begin{array}{ll} N_8^f\ D8: & 012345789 \\ N_2^f\ D2: & 016 \\ N_4\ D4: & 01789 \end{array} \tag{4.12}$$

[11]In the examples in [45, 54] non-trivial S^2 were guaranteed to exist due to the presence of singularities.

In this brane intersection the $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$ symmetry of the original field theory is replaced by a $\mathrm{SO}(4)^+ \times \mathrm{SU}(2)$ symmetry. Of this, $\mathrm{SU}(2)^+_R \times \mathrm{SU}(2)_R$ would correspond to the R-symmetry group of a large $\mathcal{N} = (0,4)$ field theory living at the intersection, and the remaining $\mathrm{SU}(2)^+_L$ to a global symmetry. This is consistent with the central charge computation in subsection 4.6 and with the supersymmetry analysis in section 7 (see also the appendix). The field theory would moreover have gauge group $\mathrm{U}(N_4)$ and a global symmetry $\mathrm{SU}(N_8^f) \times \mathrm{SU}(N_2^f)$. Some field theory configurations that we present next are compatible with this brane realization.

4.4 Instantons

A very similar calculation to the one in subsection 3.2 shows that the D4 color branes can be realized as instantons in the D8 flavor branes. In this case

$$S^{\mathrm{D8}}_{\mathrm{fluc}} = -\int \frac{1}{g^2_{\mathrm{D8}}} F^2_{\mu\nu} \qquad \text{with} \qquad \frac{1}{g^2_{\mathrm{D8}}} = \frac{L^2 r_+^2 r_-^2 \rho^2}{(2\pi)^6} \tag{4.13}$$

and the DBI action of the D4-branes satisfies

$$S^{\mathrm{D4}}_{\mathrm{DBI}} = -\int \frac{16\pi^2}{g^2_{\mathrm{D8}}}\,, \tag{4.14}$$

as expected for an instantonic brane.

4.5 Baryon vertices and t'Hooft monopoles

The original D7-brane baryon vertex configuration is mapped after the duality into a D4-brane wrapped on $S^3_+ \times S^1$ and a D6-brane wrapped on $S^3_+ \times S^1 \times S^2$. The second one however has vanishing tadpole charge in the absence of large gauge transformations, given that

$$\begin{aligned} S^{\mathrm{D6}}_{\mathrm{CS}} &= 2\pi\, T_6 \int (C_5 - B_2 \wedge C_3) \wedge F = -2\pi\, T_6 \int_{S^3_+ \times S^1 \times S^2} \hat{F}_6 \int \mathrm{d}tA_t \\ &= -nN_4 \int \mathrm{d}tA_t\,. \end{aligned} \tag{4.15}$$

For the $D4$ wrapped on $S^3_+ \times S^1$ we find

$$S^{\mathrm{D4}}_{\mathrm{CS}} = -2\pi\, T_4 \int_{S^3_+ \times S^1} \hat{F}_4 \int \mathrm{d}tA_t = -N_4 \int \mathrm{d}tA_t\,. \tag{4.16}$$

As a result, there is one candidate for baryon vertex in the non-Abelian T-dual background, realized as a D4-brane wrapped on $S^3_+ \times S^1$.

Similarly, in the original background we had $D3^{\pm}$-branes wrapped on $S^3_{\pm}$ t'Hooft monopoles whose tadpole charges were given by the ranks of the flavor groups. The $D3^+$ is mapped after the duality into a D4 wrapped on $\{S^3_+, \rho\}$ and a D6 wrapped on $\{S^3_+, \rho, S^2\}$ with tadpole charges

$$S^{\mathrm{D4}}_{\mathrm{CS}} = -2\pi\, T_4 \int_{S^3_+} \int_{\rho_n}^{\rho_{n+1}} \mathrm{d}\rho \hat{F}_4 \int \mathrm{d}tA_t = N_4^f \int \mathrm{d}tA_t\,, \tag{4.17}$$

and

$$S_{\rm CS}^{\rm D6} = -2\pi T_6 \int_{S^3_+\times S^2}\int_{\rho_n}^{\rho_{n+1}} \mathrm{d}\rho \hat{F}_6 \int \mathrm{d}t A_t = -N_2^f \int \mathrm{d}t A_t\,. \tag{4.18}$$

Given that N_4^f is not associated to a BPS D4-brane in the absence of large gauge transformations it is sensible to also not associate to it a 't Hooft monopole configuration. The D6-brane thus remains as the candidate 't Hooft monopole, with tadpole charge given by the charge of the D2 flavor brane.

The $D3^-$ 't Hooft monopole of the original background is in turn mapped into a D0-brane and a D2-brane wrapped on the S^2. We indeed find that these branes have tadpoles with charges

$$S_{\rm CS}^{\rm D0} = -2\pi\, T_0\, m \int \mathrm{d}t A_t = -N_8 \int \mathrm{d}t A_t\,, \tag{4.19}$$

and

$$S_{\rm CS}^{\rm D2} = -2\pi\, T_2 \int_{S^2} \hat{F}_2 \int \mathrm{d}t A_t = -n N_8 \int \mathrm{d}t A_t\,. \tag{4.20}$$

Clearly the second brane does not carry any tadpole charge in the absence of large gauge transformations. Thus, only the D0-brane remains as candidate 't Hooft monopole, with tadpole charge given by the charge of the D8 flavor brane.

Consistently with our previous results we find two 't Hooft monopole configurations in the dual background whose tadpole charges are given by the charges of the two D2 and D8 dual flavor branes.

4.6 Central charge

Finally in this section we compute the central charge of the dual supergravity solution. We show that as in the original theory it is possible to define two R-symmetry currents from which

$$c = 2\frac{k^+k^-}{k^+ + k^-} \tag{4.21}$$

as in [65]. We take the general expressions in [70], to which the reader is referred for more details.

Rewriting the original IIB metric as

$$\mathrm{d}s^2_{str} = \alpha(r)\beta(r)\mathrm{d}r^2 + \alpha(r)\mathrm{d}x^2_{1,1} + g_{ij}\mathrm{d}y^i\mathrm{d}y^j\,, \tag{4.22}$$

we read off

$$\alpha = L^2 r^2, \quad \beta = \frac{1}{r^4}\,. \tag{4.23}$$

Substituting these in the expressions for the internal volume[12] and r-dependent quantity κ we obtain

$$V_{\rm int} = \int \mathrm{d}^7 y\, e^{-2\Phi}\sqrt{\det(g_{ij})} = 4\pi^4 R_+^3 R_-^3 \delta x \tag{4.24}$$
$$\kappa = V_{\rm int}^2\, \alpha(r) = V_{\rm int}^2 L r.$$

[12] We have generalized these as in [54] to account for the y-dependent dilaton in the dual background.

The central charge of the original theory can then be computed as

$$c \sim \beta^{d/2} \kappa^{3d/2} (\kappa')^{-d} \tag{4.25}$$

where $d = 1$ in our case and $\kappa' \equiv d\kappa/dr$, to obtain

$$c = \frac{1}{(2\pi)^2} L R_+^3 R_-^3 \,\delta x = 2L^2 N_1 = 2N_1 \frac{N_5^+ N_5^-}{N_5^+ + N_5^-}\,, \tag{4.26}$$

where we have substituted δx from (3.6), $L^2 = N_5^+ N_5^- /(N_5^+ + N_5^-)$, and have fixed the normalization factor in (4.25) to agree with the central charge computed in [65], with $k^\pm = N_1 N_5^\pm$.

Similarly for the non-Abelian T-dual solution we find

$$\tilde{V}_{\text{int}} = \int \mathrm{d}^7 y\, e^{-2\Phi} \sqrt{\det(\tilde{g}_{ij})} = \frac{1}{3}\pi^6 R_+^3 R_-^3 \,\delta x\ , \tag{4.27}$$

from where, taking the same normalization factor as in (4.26),

$$\tilde{c} = \frac{1}{48} L R_+^3 R_-^3 \,\delta x = \frac{\pi}{3} L^2 N_4 = 2N_4 \frac{N_2^f N_8^f}{3N_2^f + N_8^f}\,. \tag{4.28}$$

Note that it is not possible to bring the dual central charge into the form (4.21) unless we change the normalization factor. Indeed, the change in the internal volume produced by the non-Abelian T-duality transformation translates generically into central charges differing by constant factors (see for instance [45, 49]). Still, up to this normalization factor, the central charge is of the form (4.21), with two levels that depend differently on the products of color and flavor charges. We denote these by $k^+ = 3N_4 N_2^f$, $k^- = N_4 N_8^f$. Note that consistently with the form of the dual geometry, the $+ \leftrightarrow -$ symmetry of the original background has now disappeared. It would be interesting to understand the field theory origin of the values for the two levels that we obtain. The central charge is thus compatible with a large $\mathcal{N} = (0,4)$ superconformal theory dual to our solution.

5 Example in new class of $AdS_3 \times S^2$ geometries in 11D

In this section, following section 2, we manipulate the massive IIA solution of the previous section by performing two Abelian T-dualities, in the process rendering it as a solution to massless IIA supergravity. We will then be in a position to uplift the solution to 11D supergravity. As we detail in section 7, while not entirely obvious, there are indeed *two* manifest global U(1) isometries, namely the overall transverse x-direction and the remaining Hopf-fibre, which becomes a global symmetry after the initial T-duality.

Performing the T-duality on the x-direction, the NS sector is unchanged, while the T-dual RR sector becomes

$$\begin{aligned}
F_1 &= \frac{R_-^2}{4}\mathrm{d}x,\\
F_3 &= \frac{4R_-^2\,\rho^3}{16\rho^2 + R_-^4}\sin\chi \mathrm{d}\chi\wedge\mathrm{d}\xi\wedge\mathrm{d}x - \frac{R_-^3}{4R_+ L}[L^4 \mathrm{Vol}(AdS_3) + R_+^4 \mathrm{Vol}(S_+^3)],
\end{aligned}$$

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$$
\begin{aligned}
F_5 &= [2L^2\mathrm{Vol}(AdS_3) + 2R_+^2\mathrm{Vol}(S_+^3)] \wedge \rho\,\mathrm{d}\rho \wedge \mathrm{d}x \\
&\quad - \frac{4R_-^3\,\rho^3}{L\,R_+(16\rho^2+R_-^4)}[L^4\mathrm{Vol}(AdS_3) + R_+^4\,\mathrm{Vol}(S_+^3)]\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi.
\end{aligned} \tag{5.1}
$$

We can further T-dualise on the Hopf-fibre direction, which we parametrise through the coordinate ψ, to get the massless IIA solution:

$$
\begin{aligned}
\mathrm{d}\hat{s}^2 &= L^2\mathrm{d}s^2(AdS_3) + \frac{R_+^2}{4}(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2) + \mathrm{d}x^2 + \frac{4}{R_+^2}\mathrm{d}\psi^2 \\
&\qquad + \frac{4}{R_-^2}\mathrm{d}\rho^2 + \frac{4R_-^2\,\rho^2}{16\rho^2+R_-^4}(\mathrm{d}\chi^2 + \sin^2\chi\mathrm{d}\xi^2), \\
\hat{B} &= \frac{16\rho^3}{16\rho^2+R_-^4}\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi + \cos\theta\mathrm{d}\phi\wedge\mathrm{d}\psi, \\
e^{-2\hat{\Phi}} &= \frac{R_-^2\,R_+^2}{256}(16\rho^2+R_-^4), \\
F_2 &= -\frac{R_-^2}{4}\mathrm{d}x\wedge\mathrm{d}\psi - \frac{R_-^3\,R_+^3}{32L}\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\phi, \\
F_4 &= -\frac{4R_-^2\,\rho^3}{16\rho^2+R_-^4}\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi\wedge\mathrm{d}x\wedge\mathrm{d}\psi + \frac{R_-^3\,L^3}{4R_+}\mathrm{vol}(AdS_3)\wedge\mathrm{d}\psi, \\
&\quad + \frac{\rho\,R_+^2}{4}\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\wedge\mathrm{d}\rho\wedge\mathrm{d}x - \frac{R_-^3\,R_+^3\,\rho^3}{2L(16\rho^2+R_-^4)}\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\wedge\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi.
\end{aligned}
$$

Uplifting to 11D, we get:

$$
\begin{aligned}
\mathrm{d}s_{11}^2 &= e^{2\lambda}\left[L^2\mathrm{d}s^2(AdS_3) + e^{2A}(\mathrm{d}\chi^2 + \sin^2\chi\mathrm{d}\xi^2) + \mathrm{d}s_6^2\right], \\
G_4 &= -\frac{4R_-^2\,\rho^3}{16\rho^2+R_-^4}\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi\wedge\mathrm{d}x\wedge\mathrm{d}\psi + \frac{R_-^3\,L^3}{4R_+}\mathrm{vol}(AdS_3)\wedge\mathrm{d}\psi, \\
&\quad + \frac{R_+^2}{4}\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\wedge\rho\mathrm{d}\rho\wedge\mathrm{d}x - \frac{R_-^3\,R_+^3\,\rho^3}{2L(16\rho^2+R_-^4)}\sin\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\wedge\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi \\
&\quad + \left[\frac{16\rho^2(16\rho^2+3R_-^4)}{(16\rho^2+R_-^4)^2}\mathrm{d}\rho\wedge\sin\chi\mathrm{d}\chi\wedge\mathrm{d}\xi - \sin\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\wedge\mathrm{d}\psi\right]\wedge\mathrm{D}z,
\end{aligned} \tag{5.2}
$$

where we have defined

$$
\begin{aligned}
e^{2\lambda} &= e^{-\frac{2}{3}\hat{\Phi}}, \qquad e^{2A} = \frac{4R_-^2\,\rho^2}{16\rho^2+R_-^4}, \\
\mathrm{d}s_6^2 &= \frac{R_+^2}{4}(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2) + \mathrm{d}x^2 + \frac{4}{R_+^2}\mathrm{d}\psi^2 + \frac{4}{R_-^2}\mathrm{d}\rho^2 + \frac{256}{R_-^2R_+^2(16\rho^2+R_-^4)}\mathrm{D}z^2, \\
\mathrm{D}z &\equiv \mathrm{d}z + C_1, \\
C_1 &= -\frac{R_-^2}{8}(x\mathrm{d}\psi - \psi\mathrm{d}x) + \frac{R_-^3R_+^3}{32L}\cos\theta\mathrm{d}\phi.
\end{aligned} \tag{5.3}
$$

One can check that the Bianchi identity and the equations of motion are satisfied. As we argue in section 7, this uplifted geometry is expected to be $\frac{1}{4}$-BPS. What is particularly interesting about this uplift is that the internal manifold exhibits SU(2)-structure, yet it is beyond the scope of the ansatz in [2], since $\mathcal{A}$ and $\mathcal{G}$ in (2.1) are clearly non-zero. This opens up the possibility that we can read off the relation between the 6D Killing spinors appearing in the more general classification [22], feed them into supersymmetry conditions and identify a more general class of supersymmetric $AdS_3 \times S^2$ solutions in 11D supergravity with SU(2)-structure manifolds. One can then use the supersymmetry conditions to find further explicit solutions, some of which may be, in contrast to non-Abelian T-duals, compact. We hope to report on this in future work [24].

6 A new IIB $AdS_3 \times S^2 \times S^2$ solution

In this section we dualize once more the $AdS_3 \times S^3 \times S^2$ solution of section 4 with respect to the $\mathrm{SU}(2)_L^+$ acting on the S^3_+. We show that this dualization produces a new AdS_3 solution, this time in Type IIB. As we discuss in section 7, and further in appendix A, the new solution we generate will be $\frac{1}{4}$-BPS and still preserve $\mathcal{N} = (0, 4)$ supersymmetry in 2D.[13]

The new background is given by

$$ds^2_{IIB} = L^2 \mathrm{d}s^2(AdS_3) + \mathrm{d}x^2 + \frac{4}{R_+^2}\left(\mathrm{d}\rho_+^2 + \frac{R_+^6\rho_+^2}{64\Delta_+}(\mathrm{d}\chi_+^2 + \sin^2\chi_+ \mathrm{d}\xi_+^2)\right)$$
$$+ \frac{4}{R_-^2}\left(\mathrm{d}\rho_-^2 + \frac{R_-^6\rho_-^2}{64\Delta_-}(\mathrm{d}\chi_-^2 + \sin^2\chi_- \mathrm{d}\xi_-^2)\right), \tag{6.1}$$

where we have introduced (ρ_-, χ_-, ξ_-) to equal our previous (ρ, χ, ξ) after the first dualization on S^3_-, and (ρ_+, χ_+, ξ_+) to denote the new coordinates arising after the second dualization on S^3_+. $\Delta_\pm$ are given by

$$\Delta_\pm = \frac{R_\pm^6 + 16R_\pm^2\rho_\pm^2}{64}. \tag{6.2}$$

The corresponding dilaton is just

$$e^{-2\Phi} = \Delta_+\Delta_-, \tag{6.3}$$

and the new NS-NS 2-form is given by

$$B_2 = \frac{R_+^2\rho_+^3}{4\Delta_+}\mathrm{Vol}(S_+^2) + \frac{R_-^2\rho_-^3}{4\Delta_-}\mathrm{Vol}(S_-^2) \tag{6.4}$$

where $S^2_\pm$ are the 2-spheres parameterized by $(\chi_\pm, \xi_\pm)$, respectively. The dual RR sector is given by[14]

$$\hat{F}_1 = \frac{R_-^3 R_+^3}{32L}\mathrm{d}x + \frac{1}{4}R_-^2\rho_+\mathrm{d}\rho_+ - \frac{1}{4}R_+^2\rho_-\mathrm{d}\rho_-,$$
$$\hat{F}_3 = \frac{1}{4}R_+^2\rho_-^2\,\mathrm{d}\rho_- \wedge \mathrm{Vol}(S_-^2) - \frac{1}{4}R_-^2\rho_+^2\,\mathrm{d}\rho_+ \wedge \mathrm{Vol}(S_+^2),$$

[13] The only subtlety here would appear to be the correct identification of the global SU(2) with respect to which one T-dualises.

[14] Note that these are the fluxes associated to the Page charges.

$$
\begin{aligned}
\hat{F}_5 &= 2L^2\rho_-\rho_+\,\mathrm{Vol}(AdS_3)\wedge \mathrm{d}\rho_-\wedge \mathrm{d}\rho_+ \\
&\quad -\frac{L^3}{4}\mathrm{Vol}(AdS_3)\wedge \mathrm{d}x\wedge\left(\frac{R_+^3}{R_-}\rho_-\,\mathrm{d}\rho_- + \frac{R_-^3}{R_+}\rho_+\,\mathrm{d}\rho_+\right), \\
\hat{F}_7 &= \frac{L^3}{4}\mathrm{Vol}(AdS_3)\wedge \mathrm{d}x\wedge\left(\frac{R_+^3}{R_-}\rho_-^2\mathrm{d}\rho_-\wedge\mathrm{Vol}(S_-^2) + \frac{R_-^3}{R_+}\rho_+^2\mathrm{d}\rho_+\wedge\mathrm{Vol}(S_+^2)\right), \\
&\quad -2L^2\rho_-\rho_+\mathrm{Vol}(AdS_3)\wedge\mathrm{d}\rho_-\wedge\mathrm{d}\rho_+\wedge\left(\rho_-\mathrm{Vol}(S_-^2)+\rho_+\mathrm{Vol}(S_+^2)\right), \\
\hat{F}_9 &= 2L^2\rho_-^2\rho_+^2\,\mathrm{Vol}(AdS_3)\wedge\mathrm{d}\rho_-\wedge\mathrm{Vol}(S_-^2)\wedge\mathrm{d}\rho_+\wedge\mathrm{Vol}(S_+^2).
\end{aligned}
\tag{6.5}
$$

This solution satisfies the IIB equations of motion and preserves eight supersymmetries. As our previous massive AdS_3 solution, it is perfectly regular, with the range of the new $\mathbb{R}^+$ direction, ρ_+, also to be determined. As we did after the first dualization, we link the running of both non-compact directions $\rho_\pm$ to large gauge transformations in this background. The ranges of these coordinates must then be divided in $[\rho_{\pm(n_\pm)}, \rho_{\pm(n_\pm+1)}]$ intervals in which large gauge transformations with $n_\pm$ parameters on the non-trivial $S_\pm^2$ cycles ensure that B_2 lies in the fundamental region.

The field theory analysis that can be made from this supergravity solution follows very closely the one we made for the previous massive AdS_3 solution, so we will omit the details. As in that case each of the brane configurations that we described in section 2 is mapped to a single brane configuration in the dual for $n_\pm = 0$, and no dual configurations exist otherwise unless $R_- = R_+ = 0$. For $n_\pm = 0$ we find the brane configurations:

- Color branes: D7 on $\{t, x_1, \rho_-, S_-^2, \rho_+, S_+^2\}$
- Flavor branes: D5 on $\{t, x_1, r_-, \rho_+, S_+^2\}$ (at $\rho_- = 0$)
  D5' on $\{t, x_1, r_+, \rho_-, S_-^2\}$ (at $\rho_+ = 0$)

This can be summarized pictorially as

$$
\begin{tikzcd}[column sep=small]
 & \cancel{D5} & & & \cancel{D7} & & & \cancel{D3} \\
D4 \arrow[ur] \arrow[dr] & & & D8 \arrow[ur] \arrow[dr] & & & D2 \arrow[ur] \arrow[dr] & \\
 & D7 & & & D5' & & & D5
\end{tikzcd}
$$

where we have crossed out the branes not occurring as BPS configurations but expected a priori from the analysis of the fluxes. The charges of the surviving BPS D7, D5 and D5' are:

$$N_7 = +\frac{1}{2\kappa_{10}^2 T_7}\int_{S^1}\hat{F}_1 = \frac{R_+^3 R_-^3}{32L}\delta x \tag{6.6}$$

$$N_5^+ = -\frac{1}{2\kappa_{10}^2 T_5}\int_{S_-^2}\int_0^{\rho_{-(1)}}\mathrm{d}\rho_-\,\hat{F}_3 \tag{6.7}$$

$$N_5^- = +\frac{1}{2\kappa_{10}^2 T_5}\int_{S_+^2}\int_0^{\rho_{+(1)}}\mathrm{d}\rho_+\,\hat{F}_3 \tag{6.8}$$

where once again δx is the hand-set length of the x-direction. $\rho_{\pm(1)}$ satisfy $16\rho^3_{\pm(1)}/(R^4_\pm + 16\rho^2_{\pm(1)}) = \pi$. Hence, a candidate brane intersection is:

$$\begin{aligned} N_5^+\; D5: &\quad 013456 \\ N_5^-\; D5': &\quad 012789 \\ N_7\; D7: &\quad 01345789 \end{aligned} \tag{6.9}$$

which realizes the $\mathrm{SU}(2)^+ \times \mathrm{SU}(2)^-$ symmetries of the background. As shown in section 7 (see also the appendix) these correspond to R-symmetries in the dual theory. Thus, the dual field theory is still a large $\mathcal{N} = (0, 4)$ SCFT. The field theory living at the intersection would have gauge group $\mathrm{U}(N_7)$ and a global symmetry $\mathrm{SU}(N_5^+) \times \mathrm{SU}(N_5^-)$.
Consistently with this picture we also have:

- Baryon vertices: D1 on $\{t, S^1\}$ with tadpole charge N_7
- 't Hooft monopoles: D3$^\pm$ on $\{t, \rho_\pm, S^2_\pm\}$ with tadpole charge $N_5^\mp$
- Central charge:

$$c = \frac{2}{3} N_7 \frac{N_5^+ N_5^-}{N_5^+ + N_5^-} \tag{6.10}$$

This form for the central charge agrees with a large $\mathcal{N} = (0, 4)$ dual CFT with affine $\mathrm{SU}(2)^\pm$ current algebras at levels $k^\pm = N_7 N_5^\pm$, even if with a different overall factor compared to [65]. This is consistent with the supersymmetry analysis. Together with the analysis of brane configurations this suggests a dual field theory in which D7 branes substitute the D1-branes of the original field theory dual to the $AdS_3 \times S^3_+ \times S^3_- \times S^1$ solution. In this theory the global $\mathrm{SU}(2)^L_+ \times \mathrm{SU}(2)^L_-$ symmetries have disappeared. It would be interesting to see if one can indeed derive these properties from the brane intersection given by (6.9).

7 Comments on supersymmetry

In this section we comment on the number of supersymmetries the various solutions to 10D Type II supergravity preserve. To make the text self-contained, we start by recalling our supersymmetry conventions [66, 71]. The fermionic supersymmetry variations for Type IIA and Type IIB supergravity are respectively

$$\begin{aligned} \delta\lambda &= \frac{1}{2}\partial\!\!\!/\Phi\eta - \frac{1}{24}H\!\!\!/_3\sigma_3\eta + \frac{1}{8}e^\Phi\left[5m\sigma_1 + \frac{3}{2}F\!\!\!/_2 i\sigma_2 + \frac{1}{24}F\!\!\!/_4\sigma_1\right]\eta, \\ \delta\Psi_\mu &= \nabla_\mu\eta - \frac{1}{8}H_{3\,\mu\nu\rho}\Gamma^{\nu\rho}\sigma_3 + \frac{1}{8}e^\Phi\left[m\sigma_1 + \frac{1}{2}F\!\!\!/_2 i\sigma_2 + \frac{1}{24}F\!\!\!/_4\sigma_1\right]\Gamma_\mu\eta, \end{aligned} \tag{7.1}$$

and

$$\begin{aligned} \delta\lambda &= \frac{1}{2}\partial\!\!\!/\Phi\eta - \frac{1}{24}H\!\!\!/_3\sigma_3\eta + \frac{1}{2}e^\Phi\left[F\!\!\!/_1 i\sigma_2 + \frac{1}{12}F\!\!\!/_3\sigma_1\right]\eta, \\ \delta\Psi_\mu &= \nabla_\mu\eta - \frac{1}{8}H_{3\,\mu\nu\rho}\Gamma^{\nu\rho}\sigma_3 - \frac{1}{8}e^\Phi\left[F\!\!\!/_1 i\sigma_2 + \frac{1}{6}F\!\!\!/_3\sigma_1 + \frac{1}{240}F\!\!\!/_5 i\sigma_2\right]\Gamma_\mu\eta, \end{aligned} \tag{7.2}$$

where λ denotes the dilatinos, Ψ_μ the gravitinos and η is a Majorana-Weyl spinor

$$\eta = \begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix}. \tag{7.3}$$

The supersymmetry preserved by the non-Abelian T-dual of $AdS_3 \times S^3 \times CY_2$ is well-documented [17, 66] and analysis leads to the conclusion that half the supersymmetry is broken in the transformation. Therefore, for the geometries exhibited in section 2, all solutions preserve eight supersymmetries, or $\mathcal{N} = (0, 4)$ supersymmetry in 2D. We have noted that the 11D uplift fits into the classification of [2] and further demonstrated that supersymmetry is not enhanced beyond $\frac{1}{4}$-BPS in 11D, thus providing the first concrete example in the class of [2].

For the geometry $AdS_3 \times S^3 \times S^3 \times S^1$, supersymmetry breaking is not a foregone conclusion. To see why this may be the case, we recall that the geometry $AdS_3 \times S^3 \times S^3 \times S^1$ possesses an $\mathrm{SU}(2) \times \mathrm{SU}(2)$ R-symmetry, yet is manifestly $\mathrm{SO}(4) \times \mathrm{SO}(4)$-invariant. Therefore, it could be expected that a judicious choice of the T-duality SU(2) factor would result in a geometry preserving the same amount of supersymmetry as the original solution. This intuition is based on ref. [37], where T-duality with respect to a global SU(2) isometry generated a surprising new supersymmetric AdS_6 solution to IIB supergravity.

Here we correct statements in the literature[15] and show that picking out a left or right-acting SU(2) isometry from one of the three-spheres leads to broken supersymmetry in an analogous fashion to $AdS_3 \times S^3 \times CY_2$ non-Abelian T-duals. For completeness, we do this in two ways, uncovering a consistent picture.

Firstly, and most easily, we can import the findings of ref. [66]. We recall for spacetimes with SO(4) isometry - with generalisations to SU(2) isometry [53] - that supersymmetry breaking is encoded in a single condition, namely (3.11) of ref. [66],

$$\left[-\frac{1}{2R_-}\Gamma^{\chi\xi}\sigma_3 - \frac{1}{4R_-}\Gamma^{\chi\xi\rho}i\sigma_2 - \frac{1}{4}\left(\frac{1}{L}\Gamma^{012} + \frac{1}{R_+}\Gamma^{678}\right)\sigma_1\right]\tilde{\eta} = 0, \tag{7.4}$$

where, assuming we T-dualise from the IIB form for the geometry, $\tilde{\eta}$ is related by a factor to the Killing spinor of IIA supergravity η,

$$\tilde{\eta} \equiv e^{-X}\eta = \exp\left(\frac{1}{2}\tan^{-1}\left(\frac{R_-^2}{4\rho}\right)\Gamma^{\chi\xi}\sigma_3\right)\eta. \tag{7.5}$$

Using (3.5), we can rewrite these conditions as:

$$\begin{aligned}\left[-\frac{1}{R_-}\Gamma^{\chi\xi} + \frac{1}{L}\Gamma^{012\rho} + \frac{1}{R_+}\Gamma^{678\rho}\right]\tilde{\epsilon}_+ &= 0,\\ \left[\frac{1}{R_-}\Gamma^{\rho\chi\xi} - \frac{1}{L}\Gamma^{012} - \frac{1}{R_+}\Gamma^{678}\right]\tilde{\epsilon}_- &= 0,\end{aligned} \tag{7.6}$$

[15]It was initially reported in ref. [53] that an application of non-Abelian T-duality to an SU(2) factor in one of the SO(4) isometries resulted in a T-dual preserving sixteen supersymmetries. The analysis of ref. [53] failed to take account of an additional condition, which breaks supersymmetry to eight.

and further using (3.14) of ref. [66], which in this case reads,

$$\tilde{\epsilon}_+ = \Gamma^{\rho}\epsilon_+, \quad \tilde{\epsilon}_- = -\epsilon_-, \quad \Gamma^{\rho\chi\xi} = -\Gamma^{345}, \tag{7.7}$$

we recover what turns out to be the original projection condition of the IIB geometry (3.1)

$$\left[\frac{1}{L}\Gamma^{012} + \frac{1}{R_+}\Gamma^{345} + \frac{1}{R_-}\Gamma^{678}\right]\eta = 0. \tag{7.8}$$

We observe that squaring this expression, we recover (3.5). We note also that in the process of redefining the spinors, the chirality of $\tilde{\epsilon}_+$ is flipped so that it now corresponds to a Killing spinor of Type IIB supergravity. On its own, this projection condition would suggest the background is $\frac{1}{2}$-BPS, however we also find that the following identification is also implied

$$\epsilon_+ = \epsilon_-. \tag{7.9}$$

This constitutes an additional condition, which breaks supersymmetry to $\frac{1}{4}$-BPS, or eight supersymmetries.

To develop a better understanding of what has just happened, it is also useful to explicitly work out the Killing spinors for the original solution (3.1). Following a calculation similar to ref. [61], except translated into our conventions, and making use of the projection condition (7.8), which falls out from the analysis, we can determine the precise form of the Killing spinors in their original IIB setting:

$$\begin{aligned}
\epsilon_+ &= \left[r^{\frac{1}{2}} + r^{-\frac{1}{2}}(t{\Gamma_0}^2 + x{\Gamma_1}^2)\right](\alpha_1 + \Omega\beta_1) + r^{-\frac{1}{2}}(\Omega\alpha_2 + \beta_2), \\
\epsilon_- &= \left[r^{\frac{1}{2}} + r^{-\frac{1}{2}}(t{\Gamma_0}^2 + x{\Gamma_1}^2)\right](\alpha_1 - \Omega\beta_1) - r^{-\frac{1}{2}}(\Omega\alpha_2 - \beta_2),
\end{aligned} \tag{7.10}$$

where we have defined the constant spinors, $\Gamma^{01}\alpha_i = \alpha_i$, $\Gamma^{01}\beta_i = -\beta_i$, and the matrix,

$$\Omega = e^{-\frac{1}{2}\psi_1\Gamma^{34}} e^{-\frac{1}{2}\theta_1\Gamma^{53}} e^{-\frac{1}{2}\phi_1\Gamma^{34}} e^{-\frac{1}{2}\psi_2\Gamma^{67}} e^{-\frac{1}{2}\theta_2\Gamma^{86}} e^{-\frac{1}{2}\phi_2\Gamma^{67}}, \tag{7.11}$$

where the angular dependence follows for the explicit form of left-invariant one-forms τ_α, which satisfy $\mathrm{d}\tau_\alpha = \frac{1}{2}\epsilon_{\alpha\beta\gamma}\tau^\beta \wedge \tau^\gamma$. The existence of Poincaré supersymmetries of both chirality with respect to Γ^{01} indicates that supersymmetry is $\mathcal{N} = (4,4)$ in 2D.

We now can appreciate that the identification (7.9) is a by-product of the fact that all Killing spinors with angular dependence get projected out under the non-Abelian T-duality. It is worth noting that a single Hopf-fibre T-duality also results in the same supersymmetry breaking, although one can consider a linear combination of the Hopf-fibres, which preserves additional supersymmetries [72].[16]

This final observation that angular dependence gets projected out presents us with a small puzzle. Namely, how can the loss of angular dependence be reconciled with $\mathcal{N} = (0,4)$ supersymmetry, which requires, at a very least, the geometric realisation of an associated

[16] That the T-dual geometry in this special case must preserve twelve supersymmetries, and not the generic eight, can be most easily seen by resorting to the Kosmann spinorial-Lie-derivative [73]. One can then use the powerful result in ref. [53] that the supersymmetries uncharged under the T-duality direction are preserved.

SU(2) R-symmetry? To answer this question, we need to recall that an SU(2) transformation on a round three-sphere results in a residual S^2 factor in the metric. This then is one candidate SU(2) R-symmetry. As we shall appreciate later, the Killing spinors of the non-Abelian T-dual also have dependence on $\mathrm{SU}(2)_R$ of the remaining three-sphere. This suggests the presence of large $\mathcal{N} = (0, 4)$ supersymmetry where the corresponding isometry group is $D(2, 1|\gamma) \times \mathrm{SL}(2, \mathbb{R}) \times \mathrm{SU}(2)$, which as we explain in the appendix, is analogous to the Abelian T-dual, i.e. the geometry $AdS_3 \times S^3 \times S^2 \times T^2$.

In a bid to make this work self-contained, we now explicitly check that the residual S^2 becomes the SU(2) R-symmetry, that the remaining SO(4) has an $\mathrm{SU}(2)_L$ global symmetry and that supersymmetry is indeed $\mathcal{N} = (0, 4)$, as claimed. To do so, we solve the Killing spinor equations for IIA supergravity in the T-dual geometry.

We begin by introducing a frame for the remaining three-sphere,

$$\mathrm{d}s^2(S^3_+) = \frac{1}{4}[(\mathrm{d}\psi + \cos\theta \mathrm{d}\phi)^2 + \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2]. \tag{7.12}$$

We next introduce the natural dreibein, $e^6 = \frac{R_+}{2}(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi)$, $e^7 = \frac{R_+}{2}\mathrm{d}\theta$, $e^8 = \frac{R_+}{2}\sin\theta\mathrm{d}\phi$ and reverse the overall sign of the RR sector relative to (4.5), so we can import results from ref. [66], where expressions are given in terms of spherical coordinates, which are best suited to the current example. We note that $H_3 = \mathrm{d}B_2$ has no legs along the ψ-direction, so the gravitino variation in this direction simply reads:

$$\begin{aligned} e^{-X}\delta\Psi_6 = {} & \frac{2}{R_+}\partial_\psi\tilde{\eta} + \frac{1}{2\,R_+}\Gamma^{78}\tilde{\eta} + \frac{e^{-2X}}{R_-\sqrt{16\rho^2 + R_-^4}}\Bigg[-\frac{R_-^2}{4}\sigma_1 - \rho\Gamma^{\chi\xi} i\sigma_2 \\ & -\rho\left(\frac{R_-}{L}\Gamma^{012\rho} + \frac{R_-}{R_+}\Gamma^{678\rho}\right)\sigma_1 - \frac{R_-^3}{4}\left(\frac{1}{R_+}\Gamma^{0129} + \frac{1}{L}\Gamma^{6789}\right)\sigma_1\Bigg]\Gamma_6\tilde{\eta}, \end{aligned} \tag{7.13}$$

where we have multiplied by the matrix e^{-X} and redefined the original Killing spinor as in (7.5). The Killing spinor $\tilde{\eta}$ is a IIA spinor satisfying the projection conditions

$$\begin{aligned} \left(\frac{R_-}{L}\Gamma^{012\chi\xi} i\sigma_2 + \frac{R_-}{R_+}\Gamma^{678\chi\xi} i\sigma_2\right)\tilde{\eta} &= \tilde{\eta}, \\ \Gamma^{\rho}\sigma_1\tilde{\eta} &= -\tilde{\eta}, \end{aligned} \tag{7.14}$$

hopefully making it obvious, through the appearance of two projection conditions, that the number of preserved supersymmetries is eight.

Bearing in mind that $\tilde{\eta}$ is comprised of Majorana-Weyl spinors of opposite chirality, we can dualise gamma matrices as follows

$$\Gamma^{012x}\sigma_1\tilde{\eta} = \Gamma^{678\rho\chi\xi} i\sigma_2\tilde{\eta}, \quad \Gamma^{678x}\sigma_1\tilde{\eta} = \Gamma^{012\rho\chi\xi} i\sigma_2\tilde{\eta}. \tag{7.15}$$

Then using the above expressions, one can rewrite (7.13) as

$$e^{-X}\delta\Psi_6 = \frac{2}{R_+}\partial_\psi\tilde{\eta} + \frac{1}{2\,R_+}\Gamma^{78}\tilde{\eta} + \frac{e^{-2X}}{R_+\sqrt{16\rho^2 + R_-^4}}\Gamma^{78}\left[-2\rho + \frac{R_-^2}{2}\Gamma^{\chi\xi}\sigma_3\right]\tilde{\eta}. \tag{7.16}$$

Finally, we insert the expression for e^{-2X},

$$e^{-2X} = \frac{1}{\sqrt{16\rho^2 + R_-^4}}(4\rho + R_-^2 \Gamma^{\chi\xi}\sigma_3), \tag{7.17}$$

to reach the conclusion that $\partial_\psi \eta = \partial_\psi(e^X \tilde{\eta}) = 0$, so after an SU(2) transformation the Killing spinors are independent of the Hopf-fibre, meaning that we can Abelian T-dualise later with respect to this direction. Similar calculations for the θ and ϕ-directions show that the Killing spinors also do not depend on these. We therefore see in an explicit fashion that the second three-sphere is now comprised of a global $\mathrm{SU}(2)_L$ symmetry, yet with the Killing spinors still dependent on $\mathrm{SU}(2)_R$. In analogy with the Abelian case, we have an $D(2|1,\gamma) \times \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ symmetry algebra.

To extract the R-symmetry dependence on the residual S^2, we consider $e^{-X}\delta\Psi_\alpha$, where $\alpha \in \{\chi, \xi\}$. We find

$$\begin{aligned}
e^{-X}\delta\Psi_\chi &= \frac{\sqrt{16\rho^2 + R_-^4}}{2R_-\rho}\partial_\chi\tilde{\eta} + \frac{e^{-2X}}{(16\rho^2 + R_-^4)}\left(-\frac{R_-^5}{4\rho}\Gamma^\chi\sigma_1 + \frac{(16\rho^2 + 3R_-^4)}{2R_-}\Gamma^\xi i\sigma_2\right)\tilde{\eta} \\
&\quad + \frac{1}{R_-\sqrt{16\rho^2 + R_-^4}}\left(-\frac{R_-^2}{2}\Gamma^\chi\sigma_1 + 2\rho\Gamma^\xi i\sigma_2\right)\tilde{\eta}, \\
&= \frac{\sqrt{16\rho^2 + R_-^4}}{2R_-\rho}\left(\partial_\chi\tilde{\eta} + \frac{1}{2}\Gamma^\xi i\sigma_2\right)\tilde{\eta},
\end{aligned} \tag{7.18}$$

where in the second line we have expanded e^{-2X}. A similar calculation for $e^{-X}\delta\Psi_\xi$, after simplifications leads to

$$e^{-X}\delta\Psi_\xi = \frac{\sqrt{16\rho^2 + R_-^4}}{2R_-\rho}\left(\frac{1}{\sin\chi}\partial_\xi\tilde{\eta} + \frac{1}{2}\frac{\cos\chi}{\sin\chi}\Gamma^{\xi\chi} - \frac{1}{2}\Gamma^\chi i\sigma_2\right)\tilde{\eta}. \tag{7.19}$$

Up to the inclusion of the Killing spinors for AdS_3, we can then write the explicit form for the IIA Killing spinor

$$\eta = e^X e^{-\frac{1}{2}\chi\Gamma^\xi i\sigma_2} e^{-\frac{1}{2}\xi\Gamma^{\chi\xi}} \tilde{\eta}_{AdS_3} \tag{7.20}$$

where $\tilde{\eta}_{AdS_3}$ denotes the Killing spinors for AdS_3,

$$\nabla_\mu\tilde{\eta} = \frac{1}{2}\gamma_3\Gamma_\mu\tilde{\eta}, \tag{7.21}$$

where we have defined $\gamma_3 \equiv \Gamma^{012}$. A calculation similar to appendix A, then shows that supersymmetry is indeed $\mathcal{N} = (0, 4)$. Similar calculations to above show that the dilatino variation vanishes. Again these results are all expected and follow from the analysis presented in [66], and more generally [53].

To go from the massive IIA solution of section 4 to the massless solution in section 5, we perform two T-dualities with respect to both the overall transverse direction x and the Hopf-fibre of the remaining three-sphere. As we have argued, both correspond to global U(1) isometries and it is expected that supersymmetry will be preserved. As one further final check that this is indeed the case, we record some of the gravitino variations after these two Abelian T-dualities. The gravitino variations in the x-direction and ψ-direction, notably those featuring in the T-duality, are respectively

$$e^{-X}\delta\Psi_x = \frac{1}{4L}\Gamma^{\theta\phi\chi\xi}\left[-\frac{L}{R_-}\Gamma^{\theta\phi x\psi} + \frac{L}{R_+}\Gamma^{\rho\chi\xi x} - 1\right]\sigma_1\tilde{\eta}, \tag{7.22}$$

and

$$e^{-X}\delta\Psi_\psi = \frac{1}{2R_+}\Gamma^{\theta\phi}\sigma_3\left[\Gamma^{\rho x\psi}i\sigma_2 + 1\right]\tilde{\eta} + \frac{1}{4L}\Gamma^{\psi\theta\phi\chi\xi}\left[-\frac{L}{R_-}\Gamma^{\theta\phi x\psi} + \frac{L}{R_+}\Gamma^{\rho\chi\xi x} - 1\right]\sigma_1\tilde{\eta}, \tag{7.23}$$

leading to good, commuting projection conditions. Furthermore, up to a redefinition in $\tilde{\epsilon}_+$, namely $\tilde{\epsilon}_+ \rightarrow -\Gamma^{x\psi}\tilde{\epsilon}_+$, with ϵ_- unchanged so it maintains its chirality, these projection conditions can be mapped back to (7.14), so we see that they are consistent. Yet again, by analogy with the Abelian T-duals discussed in the appendix, the isometry group for this geometry is expected to be $D(2|1,\gamma) \times \mathrm{SL}(2,\mathbb{R})$.

8 Conclusions

Non-Abelian T-duality is a symmetry of the equations of motion of type II supergravity. This has been shown explicitly for SO(4)-invariant spacetimes via dimensional reduction [66], results of which featured prominently in this current work. For spacetimes with less symmetry, e. g. the class of Bianchi IX spacetimes with SU(2) isometry, partial results exist [53, 74], but given the number of examples explored to date, it is safe to assume that the non-Abelian T-duality procedure with RR fluxes outlined in [17], and generalised to larger non-Abelian groups in [75], will take Type II supergravity solutions into each other.

We have made use of this solution-generating property in this paper to provide sample geometries for a class of $\frac{1}{4}$-BPS $AdS_3 \times S^2$ spacetimes in 11D supergravity, where the internal space is an SU(2)-structure manifold. Despite a number of studies asserting that the class exists [1, 18, 22], most notably the classification in [2], there was no explicit example known. Not only have we demonstrated that the non-Abelian T-dual of the well-known geometry $AdS_3 \times S^3 \times CY_2$ provides an example in this class, we have exhibited non-Abelian T-duals of a related geometry, $AdS_3 \times S^3 \times S^3 \times S^1$, which fall outside this class. This suggests that the general supersymmetry conditions of ref. [22] can be mined further to extract a larger class of supersymmetric solutions based on SU(2)-structure manifolds, thus extending the [2] class. It may be hoped that the non-Abelian T-duals, despite being manifestly non-compact, may serve to identify compact solutions via ansatz when the full class of supersymmetric $AdS_3 \times S^2$ solutions of 11D supergravity are identified.

On a related note, the $\frac{1}{4}$-BPS AdS_3 solutions we generate involve a Romans' mass. Therefore, they will serve as a test of an ongoing program of work classifying the AdS solutions of massive IIA supergravity [38, 76, 77]. Furthermore, it may be interesting to consider non-Abelian T-duals of general $AdS_3 \times S^3 \times S^3 \times \Sigma_2$ solutions to 11D supergravity [78], where Σ_2 is a Riemann surface.

We have discussed some properties of the field theories associated to the $AdS_3 \times S^3 \times S^2$ and $AdS_3 \times S^2 \times S^2$ backgrounds that we construct with an aim at testing the general ideas on the CFT interpretation of non-Abelian T-duals in [54] (see also [45]). We have seen that as in previous examples there seems to be a doubling of charges after the transformation. In our AdS_3 cases however the branes responsible for the extra charges turn out to be supersymmetric only in the absence of large gauge transformations, in which case the extra charges vanish. The absence of large gauge transformations can be explained in turn either by the non-existence of non-trivial 2-cycles in the dual geometry at finite ρ, or, else, by a geometry terminating at a regular point. As in [45] the termination of the geometry at a regular point is intimately related to the depletion of the rank of one of the gauge groups.

An important piece of information about the CFT duals to the new solutions comes from the analysis of their central charges. We have shown that as in the original theory it is possible to define two R-symmetry currents from which the central charges exhibit the expected $c \sim k^+k^-/(k^+ + k^-)$ behaviour for a large $\mathcal{N} = (0,4)$ superconformal algebra, in full agreement with the supersymmetry properties of the solutions.

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A Hopf-fibre T-duality for $AdS_3 \times S^3 \times S^3 \times S^1$

To support claims in the text concerning the isometry supergroup for non-Abelian T-duals, here we present simpler Hopf-fibre T-duals in an analogous fashion. Abelian Hopf-fibre T-duals of the related IIB geometry with small superconformal symmetry, namely $AdS_3 \times S^3 \times CY_2$, were considered in [79]. There it was noted that supersymmetry can be preserved completely. Here we explicitly show that this is not the case when one starts with a geometry with large superconformal symmetry. Moreover, following [79], we could extend our analysis here to geometries supported by both NS and RR fields, where T-duality results not in $S^1 \times S^2$, but in (squashed) Lens spaces, $S^3/\mathbb{Z}_p$, however we focus on the simplest case with just RR fields.

Starting from $AdS_3 \times S^3 \times S^3 \times S^1$ (3.1), it is known that Abelian T-duality on a given Hopf-fibre will produce a $\frac{1}{4}$-BPS $AdS_3 \times S^3 \times S^2 \times T^2$ geometry, where the corresponding supergroup is $D(2|1,\gamma) \times \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ [60, 61]. Here γ is a real parameter equating to the ratio of the radii of the three-sphere and two-sphere.[17] Recalling that the bosonic subgroup of the supergroup $D(2|1,\gamma)$ is $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$, we recognise that the symmetries simply correspond to the isometries of $AdS_3 \times S^3 \times S^2$.

Assuming we begin in Type IIB with the solution (3.1), the geometry resulting from a Hopf-fibre T-duality may be written as

$$\begin{aligned}
\mathrm{d}s^2 &= L^2 \mathrm{d}s^2_{AdS_3} + \frac{4}{R_+^2}\mathrm{d}\psi^2 + \frac{R_+^2}{4}(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) + R_-^2 \mathrm{d}s^2_{S^3_-} + \mathrm{d}x^2, \\
B_2 &= \cos\theta \mathrm{d}\phi \wedge \mathrm{d}\psi, \quad e^{\Phi} = \frac{2}{R_+}, \\
F_2 &= -\frac{R_+^2}{4}\sin\theta \mathrm{d}\theta \wedge \mathrm{d}\phi, \\
F_4 &= \left[2L^2 \mathrm{Vol}(AdS_3) + 2R_-^2 \mathrm{Vol}(S^3_-)\right] \wedge \mathrm{d}\psi.
\end{aligned} \tag{A.1}$$

As with the original geometry, the Bianchis and the equations of motion are trivially satisfied. Plugging this solution into the dilatino variation, one can extract two commuting projection conditions,

$$\left(-\frac{1}{R_+}\Gamma^{\theta\phi} i\sigma_2 + \frac{1}{L}\Gamma^{012\psi}\sigma_1 + \frac{1}{R_-}\Gamma^{678\psi}\sigma_1\right)\eta = (\Gamma^{\psi}\sigma_1 - 1)\eta = 0, \tag{A.2}$$

confirming that we now have eight preserved supersymmetries versus the original sixteen. As a consistency check, we observe that squaring the first projection condition, we recover the constraint on the radii (3.5). Solving for the Killing spinor along the internal directions, we find

$$\eta = e^{-\frac{1}{2}\theta\Gamma^{\phi} i\sigma_2} e^{\frac{1}{2}\phi\Gamma^{\theta\phi}}\tilde{\eta}, \tag{A.3}$$

where $\tilde{\eta}$ denotes the Killing spinor for AdS_3. Employing the left-invariant one-forms for the inert three-sphere, we see that angular dependence drops out, so we have an $\mathrm{SU}(2)_L$ global symmetry, just as we witnessed in the non-Abelian case. As a direct consequence, the Killing spinors are independent of the Hopf-fibre and we can perform a further Abelian T-duality. It is an interesting feature of this geometry that uplifting on the M-theory circle to 11D, we recover the $AdS_3 \times S^3 \times S^3 \times T^2$ geometry in 11D, so that supersymmetry is restored to $\frac{1}{2}$-BPS.[18] This uplift should be contrasted with the more trivial T-duality on the x-direction and uplift, which leads to the same upstairs solution.

It is instructive to perform another Hopf-fibre T-duality, thus mirroring the combination of non-Abelian transformations in section 6. Doing so with respect to the ψ_2-direction, we get

$$\begin{aligned}
\mathrm{d}s^2 = L^2 \mathrm{d}s^2_{AdS_3} + \frac{4}{R_+^2}\mathrm{d}\psi_1^2 + \frac{R_+^2}{4}(\mathrm{d}\theta_1^2 + \sin^2\theta_1 \mathrm{d}\phi_1^2) + \frac{4}{R_-^2}\mathrm{d}\psi_2^2 \\
+ \frac{R_-^2}{4}(\mathrm{d}\theta_2^2 + \sin^2\theta_2 \mathrm{d}\phi_2^2) + \mathrm{d}x^2,
\end{aligned}$$

[17] Prior to T-duality, this is just the ratio of the radii of the three-spheres.

[18] This is the reverse of the dimensional reduction considered in ref. [61].

$$
\begin{aligned}
B_2 &= \cos\theta_1 \mathrm{d}\phi_1 \wedge \mathrm{d}\psi_1 + \cos\theta_2 \mathrm{d}\phi_2 \wedge \mathrm{d}\psi_2, \quad e^{\Phi} = \frac{4}{R_+ R_-}, \\
F_3 &= \frac{R_+^2}{4} \sin\theta_1 \mathrm{d}\theta_1 \wedge \mathrm{d}\phi_1 \wedge \mathrm{d}\psi_2 - \frac{R_-^2}{4} \sin\theta_2 \mathrm{d}\theta_2 \wedge \mathrm{d}\phi_2 \wedge \mathrm{d}\psi_1, \\
F_5 &= (1 + *_{10}) \left[-2L^2 \mathrm{Vol}(AdS_3) \wedge \mathrm{d}\psi_1 \wedge \mathrm{d}\psi_2 \right], \qquad \text{(A.4)}
\end{aligned}
$$

where we have added subscripts to distinguish the angular coordinates. We note that the NS sector is even under an exchange of angular coordinates, whereas the RR sector is odd. We now check the remaining supersymmetry. From the dilatino variation, we get the projection condition:

$$
\Gamma^{\psi_1 \psi_2} i\sigma_2 \eta = -\eta. \qquad \text{(A.5)}
$$

From the gravitino variations along the x, ψ_1 and ψ_2 directions, we get the additional projection

$$
\left[\frac{L}{R_+} \Gamma^{012\theta_1\phi_1\psi_2} - \frac{L}{R_-} \Gamma^{012\theta_2\phi_2\psi_1} \right] \sigma_1 \eta = -\eta. \qquad \text{(A.6)}
$$

One can check that the two projection conditions we have indeed commute, so supersymmetry is not broken further.

We can once again solve for angular dependence, getting

$$
\eta = e^{-\frac{1}{2}\theta_1 \Gamma^{\phi_1\psi_1}\sigma_3} e^{-\frac{1}{2}\phi_1 \Gamma^{\phi_1\theta_1}} e^{-\frac{1}{2}\theta_2 \Gamma^{\phi_2\psi_2}\sigma_3} e^{-\frac{1}{2}\phi_2 \Gamma^{\phi_2\theta_2}} \tilde{\eta}, \qquad \text{(A.7)}
$$

where $\tilde{\eta}$ is expected to be the Killing spinor for AdS_3. Indeed, one can check that the remaining equation is just the Killing spinor equation for AdS_3, $\nabla_\mu \eta = \frac{1}{2}\gamma_3 \Gamma_\mu \eta$, where we have defined $\gamma_3 \equiv \Gamma^{012}$. Solving the AdS_3 Killing spinor equation, we find

$$
\tilde{\eta} = \left(r^{\frac{1}{2}} + r^{-\frac{1}{2}} (t\Gamma_0{}^2 + x_1 \Gamma_1{}^2) \right) \tilde{\eta}_+ + r^{-\frac{1}{2}} \tilde{\eta}_-, \qquad \text{(A.8)}
$$

where $\tilde{\eta}_\pm$ are constant spinors subject to (A.5) and (A.6) satisfying $\Gamma^{01}\tilde{\eta}_\pm = \pm\tilde{\eta}_\pm$. We clearly see that the preserved supersymmetry is $\mathcal{N} = (0,4)$, since the Killing spinors separate into the usual Poincaré and superconformal Killing spinors, each with a different chirality. The same conclusion can be drawn for the non-Abelian T-dual in section 6. However, in contrast to the usual small superconformal symmetry, we appear to have $\mathrm{SU}(2) \times \mathrm{SU}(2)$ R-symmetry, which is suggested from the angular dependence of the Killing spinors.

To further check the R-symmetry, we can also analyse the isometry algebra in our conventions in 10D, following a procedure outlined in ref. [61]. The first step is to identify the corresponding generic 10D Killing vector field, whose existence is always guaranteed for supersymmetric geometries. Using the Killing spinor equations presented in section 7, standard arguments show that

$$
V = \frac{1}{2} \left(\bar{\epsilon}_+ \Gamma^M \epsilon_+ + \bar{\epsilon}_- \Gamma^M \epsilon_- \right) \partial_M \qquad \text{(A.9)}
$$

is always Killing. Note, we define $\bar{\epsilon} \equiv \epsilon^\dagger \Gamma^0$, with $(\Gamma^0)^\dagger = -\Gamma^0$ and $(\Gamma^i)^\dagger = \Gamma^i, i = 1, \ldots, 9$. From (A.5), we have $\epsilon_- = \Gamma^{\psi_1\psi_2}\epsilon_+$, and as a result,

$$
V^M = \frac{1}{2} \left(\bar{\epsilon}_+ \Gamma^M \epsilon_+ - \bar{\epsilon}_+ \Gamma^{\psi_1\psi_2} \Gamma^M \Gamma^{\psi_1\psi_2} \epsilon_+ \right). \qquad \text{(A.10)}
$$

We immediately recognise that $V^{\psi_i}=0$, which is as expected, since these components of the vector field drop out when we reduce on a Hopf-fibre from 11D [61]. Thus, $V^M=\bar{\epsilon}_+\Gamma^M\epsilon_+$, only depends on one of the Majorana-Weyl spinors.

It is then a simple exercise to determine the internal components of V,

$$\begin{aligned}V_{\text{int}} = &\frac{2}{R_+}\bar{\epsilon}_+\Gamma^{\theta_1}\epsilon_-\,\xi_1^+ + \frac{2}{R_+}\bar{\epsilon}_+\Gamma^{\phi_1}\epsilon_-\xi_2^+ + \frac{2}{R_-}\bar{\epsilon}_+\Gamma^{\theta_2}\epsilon_-\xi_1^- + \frac{2}{R_-}\bar{\epsilon}_+\Gamma^{\phi_2}\epsilon_-\xi_2^- \\ &-\frac{2}{R_+}\bar{\epsilon}_+\Gamma^{\psi_1}\epsilon_-\xi_3^+ - \frac{2}{R_-}\bar{\epsilon}_+\Gamma^{\psi_2}\epsilon_-\xi_3^- + (+\longleftrightarrow -),\end{aligned} \tag{A.11}$$

where we have relabeled $\tilde{\epsilon}$ simply ϵ for convenience and have defined the following two-sphere Killing vectors:

$$\xi_1^+ = \cos\phi_1\partial_{\theta_1} - \sin\phi_1\cot\theta_1\partial_{\phi_1},\quad \xi_2^+ = \sin\phi_1\partial_{\theta_1} + \cos\phi_1\cot\theta_1\partial_{\phi_1},\quad \xi_3^+ = \partial_{\phi_1},$$

with ξ_i^- similarly defined in terms of the coordinates (θ_2,ϕ_2). Note the Killing vectors satisfy the expected SU(2) commutation relations, $[\xi_i^{(+)},\xi_j^{(+)}] = -\epsilon_{ijk}\xi_k^{(+)}$, etc.

The subscripts on ϵ refer to chirality with respect to Γ^{01}. It is straightforward to show that other combinations of spinors cannot contribute to these vector bilinears. We note that since we have eight supersymmetries, there is *a priori* no relation between say $\bar{\epsilon}_+\Gamma^{\theta_1}\epsilon_-$ and $\bar{\epsilon}_+\Gamma^{\theta_2}\epsilon_-$ etc., so the SU(2) symmetries should be viewed as being independent. This suggests an $\mathcal{N}=(0,4)$ SCFT with $\mathrm{SU}(2)\times\mathrm{SU}(2)$ R-symmetry.

Evaluating the external AdS_3 components of the Killing vector V, we find

$$V_{\text{ext}} = (\bar{\epsilon}_+\Gamma^2\epsilon_- + \bar{\epsilon}_-\Gamma^2\epsilon_+)\,(M_{01}+D) + \bar{\epsilon}_-\Gamma^0\epsilon_-(P_0-P_1) + \bar{\epsilon}_+\Gamma^0\epsilon_+(K_0+K_1),$$

where now ϵ denotes the constant chiral spinors appearing in the expression for the AdS_3 Killing spinor (A.8), and we have defined the AdS_3 Killing vectors in Poincaré patch as

$$\begin{aligned} P_0 &= \partial_t, & P_1 &= -\partial_x, \\ M_{01} &= x\,\partial_t + t\,\partial_x, & D &= r\,\partial_r + t\,\partial_t + x\,\partial_x, \\ K_0 &= (t^2+x^2+r^2)\partial_t + 2t(r\partial_r + x\partial_x), \\ K_1 &= (t^2+x^2-r^2)\partial_x + 2x(r\partial_r + t\partial_t). \end{aligned} \tag{A.12}$$

These satisfy the usual conformal algebra:

$$\begin{aligned} &[M_{\mu\nu},P_\rho] = -(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu), && [M_{\mu\nu},K_\rho] = -(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu), \\ &[M_{\mu\nu},D] = 0,\quad [D,P_\mu] = -P_\mu, && [D,K_\mu] = K_\mu, \\ &[P_\mu,K_\nu] = 2M_{\mu\nu} - 2\eta_{\mu\nu}D, \end{aligned} \tag{A.13}$$

with $\mu,\nu=0,1$.

Recalling the bosonic subgroup of $D(2|1,\gamma)$, we come to the conclusion that after two Hopf-fibre T-dualities, the isometry supergroup of the $AdS_3\times S^3\times S^3$ geometry, namely $D(2|1,\gamma)\times D(2|1,\gamma)$ becomes simply $D(2|1,\gamma)\times \mathrm{SL}(2,\mathbb{R})$, where γ is the ratio of the two-sphere radii.

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3.2 $\mathcal{N} = 2$ supersymmetric AdS$_4$ solution in M-theory with purely magnetic flux

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A $\mathcal{N} = 2$ supersymmetric AdS$_4$ solution in M-theory with purely magnetic flux

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ABSTRACT: We find a new $\mathcal{N} = 2$ AdS_4 solution in M-theory supported by purely magnetic flux via a sequence of abelian and non-abelian T-dualities. This provides the second known example in this class besides the uplift of the Pernici and Sezgin solution to 7d gauged supergravity constructed in the eighties. We compute the free energy of the solution, and show that it scales as $N^{3/2}$. It is intriguing that even though the natural holographic interpretation is in terms of M5-branes wrapped on a special Lagrangian 3-cycle, this solution does not exhibit the expected N^3 behavior.

KEYWORDS: Supersymmetry and Duality, AdS-CFT Correspondence

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1 Introduction

In recent years non-Abelian T duality (NAT duality) has been very successfully applied as a generator of new supergravity backgrounds that may have interesting applications in the context of the AdS/CFT correspondence [1]–[17]. While some of these backgrounds represent explicit new solutions to existing classifications [1, 4, 5, 12], some of them have been shown to fall outside known classifications [18] or to provide the only explicit solution to some set of PDEs [3].

A very inspiring example is the AdS_6 solution to Type IIB supergravity constructed in [3]. Supersymmetry is known to impose strong constraints on AdS_6 backgrounds [19, 20],[1] even if large classes of fixed point theories are known to exist in 5 dimensions [22–24] with expected AdS_6 duals. The only AdS_6/CFT_5 explicit example identified to date is the duality between the Brandhuber and Oz solution to massive Type IIA [25] (known to be the only possible IIA AdS_6 background [19][2]) and the fixed point theory that arises from the D4/D8/O8 system in [22]. Yet, there are families of 5 and 7-brane webs giving rise to 5d fixed point theories [27–29] whose dual AdS_6 spaces remain to be identified. The solution in [3][3] provides a possible holographic dual to these theories.

The duality between 3d SCFTs arising from M5-branes wrapped on 3d manifolds and AdS_4 spaces is yet another example in which explicit AdS_4 solutions with the required properties are scarce. Remarkable progress has been achieved recently [30] through the construction of explicit $AdS_4 \times \Sigma_3 \times M_4$ solutions to massive IIA which are candidate duals to compactifications of the $(1, 0)$ 6d CFTs living in NS5-D6-D8 systems [31] on a 3-manifold Σ_3, which could eventually lead to a generalization of the 3d-3d correspondence [32–38] to $\mathcal{N} = 1$. The $\mathcal{N} = 2$ case is yet especially interesting, since with this number of supersymmetries the 3d-3d correspondence allows to associate a 3d $\mathcal{N} = 2$ SCFT to the 3d manifold on which the M5-branes are wrapped [32–38]. This field theory arises as a twisted compactification on the 3d Riemann surface of the $(2, 0)$ CFT$_6$ living in the M5-branes.

However, to date only one $\mathcal{N} = 2$ AdS_4 explicit solution to M-theory is known that could provide the holographic dual to these compactifications. This solution is the uplift to eleven dimensions [40, 41] of the Pernici-Sezgin solution [42] to 7d gauged supergravity, that dates back to the 80's. This is of the form $AdS_4 \times M_7$ where M_7 is an S^4-fibration over a hyperbolic manifold H_3, on which the M5-branes are wrapped. The Pernici-Sezgin solution is the only explicit solution of the form $AdS_4 \times \Sigma_3 \times S^4$ in the general class of $\mathcal{N} = 2$ AdS_4 backgrounds obtained from M5-branes wrapping calibrated cycles in [43].

In this paper we construct a new $\mathcal{N} = 2$ AdS_4 solution to M-theory belonging to the general class of $\mathcal{N} = 2$ AdS_4 backgrounds derived in [43]. This class is defined by requiring that the Killing spinors satisfy the same projection conditions as the wrapped branes and that there is no electric flux. Yet the solutions need not describe in general M5-branes wrapped in 3d manifolds in the near horizon limit. Our solution seems to belong to this more general class.

We obtain our solution through non-Abelian T-duality on the $AdS_4 \times CP^3$ background dual to ABJM [44], followed by an Abelian T-duality and an uplift to eleven dimensions. The non-Abelian T-duality transformation is responsible for the breaking of the supersymmetries from $\mathcal{N} = 6$ to $\mathcal{N} = 2$. The detailed properties of the resulting $\mathcal{N} = 2$ AdS_4 solution to Type IIB were studied in [11]. This solution contains two $U(1)'s$, one of which can be further used to (Abelian) T-dualize back to Type IIA without breaking any of the supersymmetries. Finally, the solution is uplifted to eleven dimensions, where it can be shown to fulfill the conditions for 11d $\mathcal{N} = 2$ AdS_4 solutions with purely magnetic flux, derived in [43].[4]

[1]See also [21].
[2]Variations of it such as orbifold solutions have also been constructed in [26].
[3]See [8] for a discussion of the properties of the associated CFT.
[4]A systematic study of the most general class of $\mathcal{N} = 2$ AdS_4 solutions of 11d supergravity, that includes the results in [43], was carried out in [45].

The paper is organized as follows. In section 2 we recall briefly the IIB solution constructed in [11] through non-Abelian T-duality acting on the $AdS_4 \times CP^3$ IIA background. In section 3 we construct its IIA Abelian T-dual, and discuss some properties of the associated dual CFT of relevance for the CFT interpretation of the 11d solution. Section 4 contains the uplift to M-theory. Here we discuss some properties of the CFT associated to the 11d solution, that are implied by the analysis of the supergravity solution as well as its IIA description. We compute the holographic central charge and show that, as expected, it coincides with the central charge of the IIB solution written in terms of the 11d charges. Thus, it scales with $N^{3/2}$, contrary to the expectation for M5-branes. We argue that the field theory analysis that we perform suggests that there should be Kaluza-Klein monopoles sourcing the background, and that M5-branes should only play a role in the presence of large gauge transformations (in a precise way that we define). This is intimately related to the existence of a non-compact direction inherited by the NAT duality transformation, which, as discussed at length in the NAT duality literature (see for instance [8, 11, 17]), represents the most puzzling obstacle towards a precise CFT interpretation of this transformation. Finally, in appendix A we present our conclusions. Here we discuss further our result for the free energy, as well as the view that we have taken to try to give a CFT meaning to the non-compact direction. We have relegated most of the technical details to three appendices. In appendix A we include some details of the derivation of both the NAT and T dual solutions presented in sections 2 and 3. These details are especially relevant for the supersymmetry analysis. In appendix B we review the G-structure conditions for preservation of supersymmetry of $AdS_4 \times M_6$ solutions to Type II supergravities. In appendix C we perform the detailed supersymmetry analysis of the solutions in IIA, IIB and M-theory.

2 The IIB NAT dual AdS_4 solution

This solution was constructed in [11], where some properties of the associated dual CFT were also analyzed. We refer the reader to this paper for more details. In this section we present the background for completeness. More technical properties of the derivation that will be useful for the study of the backgrounds constructed from this one in the following sections are presented in appendix A.

The background arises as the NAT dual of the $AdS_4 \times \mathbb{CP}^3$ background with respect to a freely acting SU(2) in the parameterization of the $\mathbb{CP}^3$ as a foliation in $T^{1,1} = S^2 \times S^3$:

$$\begin{aligned} ds^2(\mathbb{CP}^3) = d\zeta^2 + \frac{1}{4}\big(&\cos^2\zeta(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \sin^2\zeta(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \\ &+ \sin^2\zeta\cos^2\zeta(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2\big) \\ = d\zeta^2 + \frac{1}{4}&\big(\cos^2\zeta(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \sin^2\zeta(\omega_1^2 + \omega_2^2) + \sin^2\zeta\cos^2\zeta(\omega_3 + \cos\theta_1 d\phi_1)^2\big) \end{aligned} \tag{2.1}$$

where $0 \le \zeta < \frac{\pi}{2}$, $0 \le \theta_i < \pi$, $0 \le \phi_i \le 2\pi$, $0 \le \psi \le 4\pi$.

Dualising with respect to the SU(2) acting on the 3-sphere parameterized by (ψ,θ_2,ϕ_2) we obtain

$$d\tilde{s}^2=\frac{L^2}{4}ds^2(AdS_4)+L^2\Big(d\zeta^2+\frac{1}{4}\cos^2\zeta\,(d\theta_1^2+\sin^2\theta_1 d\phi_1^2)\Big)+ds^2(M_3)\,, \tag{2.2}$$

where $ds^2(M_3)$ stands for the 3-dimensional metric:

$$\begin{aligned}ds^2(M_3)\;=\;&\frac{1}{16\,\det M}\Big[L^4\sin^4\zeta\Big(dr^2+r^2d\chi^2-\sin^2\zeta\,(\sin\chi dr+r\cos\chi d\chi)^2+\\&+r^2\cos^2\zeta\sin^2\chi\,(d\xi+\cos\theta_1 d\phi_1)^2\Big)+16r^2dr^2\Big]\,.\end{aligned} \tag{2.3}$$

det$M$ is given by:

$$\det M=\frac{L^2}{64}\sin^2\zeta\Big(16r^2(\sin^2\chi+\cos^2\chi\cos^2\zeta)+L^4\sin^4\zeta\cos^2\zeta\Big)\,. \tag{2.4}$$

Here (χ,ξ) parameterize the new 2-sphere arising through the NAT duality transformation, that we will denote by $\tilde{S}^2$. r is the non-compact coordinate generated by the transformation, which lives in $\mathbb{R}^+$. The presence of this non-compact direction is intimately related to the long-standing open problem of extending NAT duality beyond spherical world sheets. In the context of AdS/CFT applications this poses a problem to the CFT interpretation of AdS backgrounds generated through this transformation. Some ideas to provide a consistent interpretation have been proposed in [8, 11] (see also [17]), which we will partially use in this paper. The reader is referred to these papers for more details.

The dilaton reads in turn

$$e^{\phi}=\frac{L}{k}\frac{1}{\sqrt{\det M}}\,. \tag{2.5}$$

A B_2 field is also generated that reads:

$$\begin{aligned}B_2=\;&\frac{L^2\sin^2\zeta}{64\det M}\Big[-L^4 r\cos^2\zeta\sin^4\zeta\cos\theta_1\sin\chi\,d\phi_1\wedge d\chi\\&-16\,r^2\Big(r(\cos^2\zeta\cos^2\chi+\sin^2\chi)\,\mathrm{Vol}(\tilde{S}^2)+\sin^2\zeta\sin^2\chi\cos\chi\,d\xi\wedge dr\Big)\\&-\cos^2\zeta\cos\theta_1\cos\chi\,(L^4\sin^4\zeta+16r^2)\,dr\wedge d\phi_1\Big]\,.\end{aligned} \tag{2.6}$$

Together with this we find the RR sector:

$$F_1=\frac{k}{2}\Big(r\sin^2\zeta\sin\chi\,d\chi-\sin^2\zeta\cos\chi\,dr-r\sin 2\zeta\cos\chi\,d\zeta\Big) \tag{2.7}$$

$$\begin{aligned}\hat{F}_3=\;&-\frac{3}{128}kL^4\sin^3 2\zeta\,d\zeta\wedge\mathrm{Vol}(S_1^2)+\frac{k}{2}\Big(r\,dr\wedge(\cos^2\zeta\,\mathrm{Vol}(S_1^2)\\&+\sin 2\zeta\cos\theta_1\,d\zeta\wedge d\phi_1+\sin 2\zeta\sin^2\chi\,d\zeta\wedge d\xi)-r^2\sin 2\zeta\cos\chi\,d\zeta\wedge\mathrm{Vol}(\tilde{S}^2)\Big)\end{aligned} \tag{2.8}$$

$$\begin{aligned}\hat{F}_5=\;&\frac{1}{64}kL^6\sin^3\zeta\cos\zeta\,\mathrm{Vol}(AdS_4)\wedge d\zeta-\frac{3}{8}kL^2 r\,\mathrm{Vol}(AdS_4)\wedge dr\\&+\frac{k}{2}r^2\Big(\cos^2\zeta\,\mathrm{Vol}(S_1^2)+\sin 2\zeta\cos\theta_1\,d\zeta\wedge d\phi_1\Big)\wedge dr\wedge\mathrm{Vol}(\tilde{S}^2)\,,\end{aligned} \tag{2.9}$$

where $F_p=d\,C_{p-1}-H_3\wedge C_{p-3}$ and $\hat{F}=F\wedge e^{-B_2}$ are the fluxes associated to the Page charges.

Note that after the dualisation a singularity has appeared at the fixed point $\zeta = 0$, where the squashed S^3 used to dualise shrinks to zero size. This singularity is associated to the component of the metric on the r-direction, and is always compensated with the singularity in the dilaton in computations of physical quantities such as gauge couplings, internal volumes, etc. We will see that it will be inherited by the IIA and M-theory solutions where physical quantities will however be perfectly well defined as well.

3 The IIA NAT-T dual AdS_4 solution

Following the steps in appendix A we get the following solution in Type IIA after dualizing the previous background along the ϕ_1 direction, that we will simply rename as ϕ [5]

$$ds^2 = \frac{L^2}{4} ds^2(AdS_4) + L^2 d\zeta^2 + \frac{L^2}{4}\cos^2\zeta d\theta^2 + \sum_{i=1}^{4} \left(\mathcal{G}^i\right)^2 , \tag{3.1}$$

where

$$\begin{aligned}
\mathcal{G}^1 &= \frac{L}{2\sqrt{\Xi}} y_1 \sin^2\zeta\cos\zeta\sin\theta d\xi, \\
\mathcal{G}^2 &= -\frac{2}{L\sqrt{\Delta}\sqrt{\mathcal{Z}}}\left(\mathcal{Z} dy_1 + y_1 y_2 dy_2\right), \\
\mathcal{G}^3 &= -\frac{L}{2\sqrt{\mathcal{Z}}}\sin\zeta dy_2, \\
\mathcal{G}^4 &= \frac{2}{L\cos\zeta\sqrt{\Delta}\sqrt{\Xi}}\left[\Delta d\phi - \sin^2\zeta\cos^2\zeta\cos\theta\left\{y_1y_2dy_1 + \left(y_2^2 + \frac{L^4}{16}\sin^4\zeta\right)dy_2\right\}\right],
\end{aligned} \tag{3.2}$$

and we have defined

$$\Delta = \sin^2\zeta\left(y_1^2 + \cos^2\zeta y_2^2 + \frac{L^4}{16}\sin^4\zeta\cos^2\zeta\right), \qquad \Xi = \Delta\sin^2\theta + y_1^2\sin^4\zeta\cos^2\theta, \tag{3.3}$$

$$\mathcal{Z} = y_1^2 + \frac{L^4}{16}\sin^4\zeta\cos^2\zeta,$$

and

$$y_1 = r\sin\chi, \qquad y_2 = r\cos\chi\,, \tag{3.4}$$

so that we have

$$\begin{aligned}
64L^2\sum_{i=1}^{4}\left(\mathcal{G}^i\right)^2 = {}& \frac{1}{\Delta\,\Xi}\cos^2\zeta\Big\{16\Delta\sec^2\zeta d\phi + \sin^2\zeta\cos\theta\left[L^4\sin^4\zeta(\cos\chi dr - r\sin\chi d\chi)\right. \\
&\left. +16r^2\cos\chi dr\right]\Big\}^2 + \frac{16L^4}{\Xi} r^2\sin^4\zeta\cos^2\zeta\sin^2\theta\sin^2\chi\, d\xi^2 \\
&+ \frac{16L^4}{\mathcal{Z}}\sin^2\zeta(\cos\chi\, dr - r\sin\chi\, d\chi)^2 \\
&+ \frac{256}{\Delta\,\mathcal{Z}}\left[\frac{L^4}{16}\sin^4\zeta\cos^2\zeta(\sin\chi dr + r\cos\chi d\chi) + r^2\sin\chi dr\right]^2 .
\end{aligned} \tag{3.5}$$

[5] Also $\theta_1 \equiv \theta$ and $S^2 \equiv \tilde{S}^2$.

The NS 2 form is given by

$$B_2 = \frac{r^2}{\Xi}\sin^2\zeta\sin\chi\Big[\sin^2\zeta\sin\chi\big(\cos\theta d\xi\wedge d\phi - \cos\chi d\xi\wedge dr\big) \\ - r\big(\cos^2\zeta\sin^2\theta + \sin^2\zeta\sin^2\chi\big)d\chi\wedge d\xi\Big]\,, \tag{3.6}$$

while the dilaton is

$$e^{\Phi} = \frac{4}{kL\cos\zeta\sqrt{\Xi}}\,. \tag{3.7}$$

Notice that this blows up at $\zeta=0$ indicating that the geometry is singular here, which is confirmed when one studies the curvature invariants.

The RR sector is given by

$$\hat{F}_2 = \frac{k}{16}\Big[3L^4\sin^3\zeta\cos^3\zeta\sin\theta d\zeta\wedge d\theta + 8r\sin 2\zeta\big(\cos\theta d\zeta\wedge dr - \cos\chi d\zeta\wedge d\phi\big) \\ + 8r\cos^2\zeta\sin\theta d\theta\wedge dr - 8\sin^2\zeta\big(\cos\chi dr\wedge d\phi + r\sin\chi d\phi\wedge d\chi\big)\Big], \tag{3.8}$$

$$\hat{F}_4 = \frac{k}{2}r\cos\zeta\sin\chi\Big[2\sin\zeta\Big(\sin\chi d\zeta\wedge d\xi\wedge dr\wedge d\phi + r\cos\theta d\zeta\wedge d\xi\wedge dr\wedge d\chi \\ r\cos\chi d\zeta\wedge d\xi\wedge d\phi\wedge d\chi\Big) + r\cos\zeta\sin\theta d\theta\wedge d\xi\wedge dr\wedge d\chi\Big], \tag{3.9}$$

where the gauge invariant fluxes are expressed in terms of these via $\hat{F} = F\wedge e^{-B_2}$.

3.1 Supersymmetry

It was shown in [11, 13] that the NAT dual of ABJM preserves $\mathcal{N}=2$ supersymmetry in 3d, which means the R-symmetry is U(1) in the dual CFT. The argument relies on a proof from [13]. In order to see this we must package all the dependence of the original geometry on the SU(2) isometry in a canonical frame[6]

$$e^{a+3} = e^{C_a(x)}(\omega_a + A_a(x)) \tag{3.10}$$

where each left-invariant one form ω_a appears only once, and x^μ are coordinates on some 7d base which fibers the squashed 3-sphere containing the SU(2) isometry. Then there is a bijective map between spinors independent of the SU(2) directions in this frame and those preserved by the NAT dual solution. The map acts on the 10 dimensional MW Killing spinors as

$$\hat{\epsilon}_1 = \epsilon_1, \qquad \hat{\epsilon}_2 = \Omega_{\mathrm{SU}(2)}\epsilon_2, \tag{3.11}$$

with the matrix[7]

$$\Omega_{\mathrm{SU}(2)} = \Gamma^{(10)}\frac{-e^{C_1+C_2+C_3}\Gamma^{456} + v_a e^{C_a}\Gamma^{a+3}}{\sqrt{e^{2(C_1+C_2+C_3)} + e^{2C_a}v_a^2}}, \tag{3.12}$$

[6]We write e^{a+3} to match notation elsewhere where the canonical vielbeins are e^4, e^5, e^6.

[7]This expression originally appeared in [5], where it was conjectured to hold by analogy with the Abelian case.

where v_a are dual coordinates in the NAT dual geometry, which we are expressing elsewhere in terms of spherical or cylindrical polar coordinates $v_1 = y_1 \cos\xi,\ v_2 = y_2 \sin\xi,\ v_3 = y_2$.

In appendix C.1 we derive a spinor for ABJM independent of the SU(2) directions. This may be written in terms of 6 dimensional MW spinors on $\mathbb{CP}^3$ as in appendix B

$$\eta^1_+ = e^{i\frac{3\pi}{4}}\eta_+, \qquad \eta^2_+ = e^{-i\frac{3\pi}{4}}\eta_+ \tag{3.13}$$

where $(\eta^{1,2}_+)^* = \eta^{1,2}_-$ with the sign labeling chirality. It is possible to decompose the 6d spinors in terms of two in linearly independent parts $\eta_+ = \pi_+ + \tilde{\pi}_+$ obeying the projections of eq. (C.32). We can then make the coordinate dependence explicit as

$$\pi_+ = e^{\zeta\gamma^{34}}\pi^0_+, \qquad \tilde{\pi}_+ = e^{-\zeta\gamma^{34}}\tilde{\pi}^0_+, \tag{3.14}$$

where we have introduced linearly independent constant spinors obeying the projections

$$\begin{aligned}
\gamma^{1456}\pi^0_+ &= -\pi^0_+, \qquad \gamma^{2345}\pi^0_+ = \pi^0_+,\\
\gamma^{1456}\tilde{\pi}^0_+ &= -\tilde{\pi}^0_+, \qquad \gamma^{2345}\tilde{\pi}^0_+ = \tilde{\pi}^0_+,
\end{aligned} \tag{3.15}$$

in the frame of eq. (A.3). The 10d spinor is constructed as in eq. (B.3), but all dependence on the $\mathbb{CP}^3$ directions is in eq. (3.14), which is clearly independent of the SU(2) directions.

So $\mathcal{N} = 2$ is preserved under the NAT duality transformation. We show in appendices C.2, C.3 that the solution does this by mapping a U(1)'s worth of the SU(3)-structures supported by $\mathbb{CP}^3$ to a U(1)'s worth of dynamical SU(2)-structures defined on the dual internal space $\hat{M}_6$. Of course only two of these dual objects are truly distinct: those that are defined in terms of the two linearly independent Killing spinors on $\hat{M}_6$. These may be summed to give the internal part of the $\mathcal{N} = 2$ Killing spinor in IIB, namely

$$\begin{aligned}
\hat{\eta}^1_+ &= e^{i\frac{3\pi}{4}}\left(e^{\zeta\gamma^{34}}\pi^0_+ + e^{-\zeta\gamma^{34}}\tilde{\pi}^0_+\right),\\
\hat{\eta}^2_+ &= e^{i\frac{3\pi}{4}}\left(\kappa_\parallel\big(e^{\zeta\gamma^{34}}\pi^0_+ + e^{-\zeta\gamma^{34}}\tilde{\pi}^0_+\big) + \kappa_\perp\mathcal{J}\big(e^{-\zeta\gamma^{34}}\pi^0_- + +e^{\zeta\gamma^{34}}\tilde{\pi}^0_-\big)\right),
\end{aligned} \tag{3.16}$$

where $\kappa_\perp$ and $\kappa_\parallel$ satisfy $\kappa_\perp^2 + \kappa_\parallel^2 = 1$ and are given in eq. (C.27). The Matrix $\mathcal{J}$ is defined as

$$\mathcal{J} = i\frac{e^{C_1+C_2+C_3}\cos 2\zeta\gamma^1 + e^{C_1}v_1\gamma^4 + e^{C_2}v_2\gamma^5 + e^{C_3}v_3\gamma^6}{\sqrt{e^{2(C_1+C_2+C_3)}\cos^2 2\zeta + e^{2C_1}v_1^2 + e^{2C_2}v_2^2 + e^{2C_3}v_3^3}}, \tag{3.17}$$

in the frame of eq. (A.3), where e^{C_a} are given in eq. (A.2). However for what follows it is only important that $\mathcal{J}$ is independent of ϕ.

It turns out that the amount of preserved supersymmetry is left invariant when we perform an additional T-duality on ∂_ϕ. As proved in [55] (see also [13]), to see this it is sufficient to show that the Killing spinors are independent of ϕ in the canonical frame of T-duality, where ϕ only appears in the vielbein

$$e^\phi = e^{C(x)}\big(d\phi + A(x)\big). \tag{3.18}$$

We get to such a frame by performing a SO(4) transformation $\mathcal{R}$ in eq. (A.14), on the canonical NAT dual vielbeins in eq. (A.8). The action of this rotation on the 10d Majorana Killing spinor will be $\epsilon \to \mathcal{S}\epsilon$, where

$$\mathcal{S}^{-1}\gamma^a\mathcal{S} = \mathcal{R}^a{}_b\gamma^b. \tag{3.19}$$

The matrix $\mathcal{S}$ will be complicated but will not depend on ϕ because $\mathcal{R}$ does not, which is all that matters. This together with the fact that eq. (3.16) is ϕ independent ensures all supersymmetry must be preserved. Indeed in the next section (see also appendix C.4) we see that upon lifting to M-theory the NAT-T dual solution preserves $\mathcal{N}=2$ supersymmetry in the form of a local SU(2) structure in 7d of the form given in [45].

3.2 Properties of the CFT

In this section we briefly discuss some properties of the CFT associated to the NAT-T dual solution. The discussion follows very closely the analysis in reference [11] for the IIB NAT solution. For this reason we will omit most of the explicit computations. The reader is referred to this reference for more details.

It was shown in [8, 11] that the presence of large gauge transformations in NAT dual backgrounds allows to constrain quite non-trivially their global properties. In our particular background (see [11]) it is easy to see that at the singularity $\zeta = 0$ the NS 2-form given by (3.6) reduces to

$$B_2|_{\zeta\sim 0} = -r\mathrm{Vol}(S^2)\,, \tag{3.20}$$

while the space spanned by (ζ, S^2) becomes conformal to a singular cone with boundary S^2. Therefore large gauge transformations can be defined on this non-trivial 2-cycle, which must render

$$b = \frac{1}{4\pi^2}\left|\int_{S^2} B_2\right| \tag{3.21}$$

in the fundamental region. For this, B_2 must transform into $B_2 \to B_2 + n\pi\mathrm{Vol}(S^2)$ when $r \in [n\pi, (n+1)\pi]$.

In turn, the $\hat{F}_3$ and $\hat{F}_5$ field strengths lying on the ζ, θ, ϕ and ζ, θ, ϕ, S^2 directions of the NAT dual solution in [11] give rise after the T-duality to $\hat{F}_2$ and $\hat{F}_4$ field strengths lying on the ζ, θ and ζ, θ, S^2 directions, with the second one non-vanishing only in the presence of large gauge transformations. Accordingly, a Page charge associated to the ζ, θ components of $\hat{F}_2$ is generated in IIA from the quantization condition

$$\frac{1}{2\kappa_{10}^2}\int \hat{F}_p = T_{8-p}N_{8-p}\,. \tag{3.22}$$

As in [11] this charge is to be interpreted as the rank of the gauge group of the CFT dual to the solution in the $r \in [0, \pi]$ interval. We indeed get $N_6 = N_5$, with N_5 given by (3.18) in [11]. Specifically this fixes L to satisfy

$$kL^4 = 64\pi N_6. \tag{3.23}$$

Note that this means that color branes are now D6-branes spanned on the $\mathbb{R}_{1,2}\times M_1\times S^1_\phi\times S^2$ directions. One can indeed check that these branes are BPS when placed at the $\zeta = 0$

singularity. As in [11] the combination $e^{-\phi}\sqrt{g_{rr}}$ in the DBI action is non-singular, rendering well-defined color branes.

In the presence of large gauge transformations with parameter n there is also a non-vanishing D4-brane charge $N_4 = nN_6$, equal to the N_3 charge in [11]. Indeed one can check that D4-branes are also BPS when placed at $\zeta = 0$. These branes should also play a role as color branes for $n \neq 0$, that is, in the $r \in [n\pi, (n+1)\pi]$ intervals. The physical interpretation is that N_4 charge is created in the worldvolume of a D6 when it crosses n NS5. It is then plausible that the field theory dual to the solution in the $[n\pi, (n+1)\pi]$ intervals arises in a (D4, D6) bound state — NS5 intersection. A similar realization was suggested in [17] for AdS_3 duals.

The charge interpreted as level in ABJM is also doubled under the NAT duality transformation. As a result, after the new Abelian T-duality we get two charges, k_6, k_4, associated to the (r,θ) and (r,θ,S^2) components of $\hat{F}_2$ and $\hat{F}_4$, respectively. They thus correspond to D6 and D4 branes or, equivalently, to D4-branes carrying both monopole and dipole charges. We may express the levels in terms of k and the number of large gauge transformations performed as

$$k_6 = k\frac{(2n+1)\pi}{4}, \qquad k_4 = k\frac{(3n+2)\pi}{12}\,. \tag{3.24}$$

Finally, it is easy to check that particle-like brane configurations can be associated to each of the charges with an interpretation as either rank or level in the IIA background. These branes are in all cases D2 or D4 branes wrapped on different sub-manifolds of the internal space. In particular:

- Di-monopoles $\leftrightarrow$ D2 on $M_1 \times S^1_\phi$, D4 on $M_1 \times S^1_\phi \times S^2$
- ’t Hooft monopoles $\leftrightarrow$ D2 on $\{M_1, \theta\}$, D4 on $\{M_1, \theta, S^2\}$
- Di-baryons $\leftrightarrow$ D2 on $\{\zeta, S^1_\phi\}$, D4 on $\{\zeta, S^1_\phi, S^2\}$
- Baryon vertices $\leftrightarrow$ D2 on $\{\zeta, \theta\}$, D4 on $\{\zeta, \theta, S^2\}$

As for the IIB AdS_4 solution (see [11] for the details) the di-monopoles and ’t Hooft monopoles have to sit at $\zeta = 0$ while the di-baryons sit at $r = 0$.

The previous analysis suggests a dual CFT in the $r \in [0, \pi]$ region defined in terms of a $\mathrm{U}(N_6)_{k_4} \times \mathrm{U}(N_6)_{-k_4}$ quiver gauge theory with $\mathcal{N} = 2$ supersymmetry, sourced by D6-branes spanned on the $\mathbb{R}_{1,2} \times M_1 \times S^1_\phi \times S^2$ directions.[8] In turn, for $r \in [n\pi, (n+1)\pi]$ the gauge theory would arise from (D4, D6) - NS5 intersections. It was argued in [11] that invariance under large gauge transformations would imply that the seemingly different CFTs dual to the solution as the non-compact internal direction increases, could be related by some kind of duality, as in [39], with the essential difference that in this case the flow parameter would not be the energy scale but the non-compact internal direction. Reference [17] proposed an alternative mechanism which, applied to our solution, would imply that new $\mathrm{U}(N_6) \times \mathrm{U}(N_6)$ gauge groups would be created by some kind of un-higgsing mechanism, also not related to

[8] One can see (see [11]) that k_4 is the level associated to the D6 color branes.

an energy scale, every time a NS5-brane is crossed. It would be interesting to understand better these proposals for the dual CFT as r increases.

In any case, keeping in mind that there is no a priori reason to expect that the new geometry makes sense as a string theory background,[9] we can just take these proposals as stringy inspired arguments in favor of the existence of a fundamental region in which the dual CFT would contain the same number of gauge groups as the original one.

Restricting ourselves to the $r \in [0,\pi]$ region, a candidate brane realization of the dual CFT would then be the T-dual of the brane picture proposed in [11] for the NAT dual of ABJM:

$$\begin{array}{rc|ccccccccc} 5^2_2: & \times & \times & \times & \times & \times & \times & - & z_1 & z_2 & - \\ N_6\, D6: & \times & \times & \times & - & \times & - & \times & \times & \times & - \\ (5^2_2, k_4\, \mathrm{D4}): & \times & \times & \times & - & \times & \cos\theta & - & - & - & \sin\theta \end{array} \tag{3.25}$$

where 5^2_2 denotes the IIA exotic brane that arises after a T-duality transformation along a worldvolume direction of the 5^2_2 exotic brane of the IIB configuration [11], and z_1 and z_2 denote the two special Killing directions of this brane [49, 50]. In our notation the $(5^2_2, k_4\, \mathrm{D4})$ bound state would be extended along the 0124 and $x^5\cos\theta + x^9 \sin\theta$ directions, and its relative orientation w.r.t. the 5^2_2-brane in the 59 plane would depend on k_4.

Note that the previous picture implies that in M-theory the corresponding AdS_4 geometry would be sourced in the fundamental region $r \in [0,\pi]$ by Kaluza-Klein monopoles, as we discuss in the next section.

4 The purely magnetic AdS_4 solution in M-theory

In this section we lift the solution of the previous section to M-theory and show that it falls into the general class of solutions with purely magnetic flux considered in [43]. The analysis of quantized charges suggests a dual CFT arising from Kaluza-Klein monopoles and M5-branes wrapped on the Taub-NUT direction of the monopoles. We compute the central charge and show that it scales with $(N_5 + N_6/2)^{3/2}$, where N_5 is the number of wrapped M5-branes and N_6 the number of Kaluza-Klein monopoles. This becomes simply $N_6^{3/2}$ in the fundamental region $r \in [0,\pi]$.

4.1 Fluxes

The RR potentials of the IIA solution are given by

$$C_1 = \frac{k}{16}\Big(\cos^2\zeta\cos\theta(3L^4\sin^3\zeta\cos\zeta d\zeta - 8rdr) - 8r\cos\chi\sin^2\zeta d\phi\Big), \tag{4.1}$$

$$C_3 - B_2\wedge C_1 = \frac{k}{2}r^2\cos\zeta\sin\chi\Big(\sin\zeta\sin\chi d\zeta\wedge d\xi\wedge d\phi - \cos\theta\cos\zeta d\xi\wedge dr\wedge d\chi\Big). \tag{4.2}$$

C_1 gives rise to the $g_{\mu z}/g_{zz}$ component of the 11d metric, where z denotes the eleventh direction. Given that there is a magnetic charge associated to C_1 in IIA, a quantized Taub-NUT charge arises in 11d. The brane that carries Taub-NUT charge is the Kaluza-Klein

[9] An essential difference with respect to its Abelian counterpart is that non-Abelian T-duality has not been proved to be a symmetry of String Theory (see [51]).

monopole, which is connected to the IIA D6-brane upon reduction along the eleventh, Taub-NUT direction. Since the IIA solution was sourced by D6-branes in the fundamental region $r \in [0, \pi]$, Kaluza-Klein monopoles should play the role of color branes in M-theory in this region. As we will discuss, BPS KK-monopoles spanned on the $\mathbb{R}_{1,2} \times M_1 \times S^1_\phi \times S^2$ directions can indeed be constructed in 11d that give rise to the D6 color branes in IIA upon reduction.

$(C_3 - B_2 \wedge C_1)$ gives rise in turn to the 3-form[10]

$$\hat{C}_3 = C_3 - i_k C_3 \wedge \frac{k_1}{k^2} \tag{4.3}$$

in 11d. Note that $\hat{C}_3$ has no components along the eleventh direction. This will be of relevance in our later discussion. The M-theory 4-form flux is then given by

$$G_4 = dC_3 = d\left(\hat{C}_3 + i_k C_3 \wedge \left(\frac{k_1}{k^2} + dz\right)\right) \tag{4.4}$$

which, as we can see, is purely magnetic. Therefore there will be no M2-branes sourcing the 11d solution.

As we have noted, $\hat{C}_3$ is by construction transverse to the eleventh direction. This potential couples in the worldvolume of M2-branes constrained to move in the space transverse to the Killing direction and in the worldvolume of M5-branes wrapped on this direction [46]. Moreover, its magnetic components are associated to wrapped M5-branes. Indeed one can show that these branes are BPS in the 11d background and are to be interpreted as color branes. They give rise upon reduction to the color D4-branes of the IIA background. Other field theory observables that we will be able to describe holographically will be constructed in terms of M2-branes transverse to the Killing direction or M5-branes wrapped on this direction.

4.2 Geometry and local SU(2) structure

In [45] it was shown that the most general $\mathcal{N} = 2$ preserving solution in M-theory with an AdS_4 factor supports an SU(2) structure in 7d. As the M-theory 4-form G_4 is purely magnetic it actually falls into the more constrained class of solutions originally considered in [43]. In this section we show that we can uplift the IIA solution to M-theory and fit it into this class of solutions.

The metric ansatz of [45] is of the form

$$ds^2_{11} = e^{2\tilde{\Delta}}\Big(ds^2(AdS_4) + ds^2(\mathcal{M}_7)\Big) \tag{4.5}$$

where we have

$$e^{2\tilde{\Delta}} = L^2 e^{-\frac{2}{3}\Phi}, \qquad ds^2(\mathcal{M}_7) = \frac{1}{L^2}\Big[ds^2(\mathcal{M}_6) + e^{2\Phi}\big(C_1 + dz\big)^2\Big], \tag{4.6}$$

[10]Our notation is that $i_k C_3$ denotes the interior product of C_3 with the Killing vector $k^\mu = \delta^\mu_z$, that points on the eleventh direction, k_1 is the 1-form $k_1 = i_k g$ and k^2 the scalar $k^2 = i_k i_k g$, where g stands for the eleven dimensional metric.

so that Ricci$(AdS_4) = -12\, g(AdS_4)$ to match the conventions of [45]. The metric on $\mathcal{M}_6$ is mearly the internal part of the IIA metric in eq. (3.1).

It is possible to express the internal 7d metric in the form

$$ds^2(\mathcal{M}_7) = ds^2(\mathrm{SU}(2)) + E_1^2 + E_2^2 + E_3^2 \tag{4.7}$$

where $ds^2(\mathrm{SU}(2))$ is the metric on a 4 manifold supporting a canonical SU(2)-structure with associated real 2-form $J = J_3$ and holomorphic 2-form $\Omega = J_1 + iJ_2$, satisfying

$$J_3 \wedge J_3 = \frac{1}{2}\Omega \wedge \bar{\Omega}, \qquad J_3 \wedge \Omega = 0, \qquad \iota_{E_i} J_3 = \iota_{E_i}\Omega = 0. \tag{4.8}$$

Since G_4 is purely magnetic it is possible to define local coordinates such that [43]

$$E_1 = \frac{1}{4} e^{-3\tilde{\Delta}} \rho\, d\xi, \qquad E_2 = \frac{1}{4} e^{-3\tilde{\Delta}} \frac{d\rho}{\sqrt{1 - e^{-6\tilde{\Delta}}\rho^2}}, \tag{4.9}$$

where ξ parametrizes the U(1) R-symmetry and ρ is defined through the associated Reeb vector $\tilde{\xi}$ as $|\tilde{\xi}|^2 = e^{-6\Delta}\rho^2$. Supersymmetry then requires that the SU(2) forms satisfy

$$\begin{aligned} d\big(e^{3\tilde{\Delta}}\sqrt{1 - |\tilde{\xi}|^2} E_3\big) &= e^{3\tilde{\Delta}}\big(2J_3 - 2|\tilde{\xi}| E_2 \wedge E_3\big), \\ d\big(|\tilde{\xi}|^2 e^{9\tilde{\Delta}} J_2 \wedge E_2\big) &= e^{3\tilde{\Delta}}|\tilde{\xi}| d\big(e^{6\tilde{\Delta}} J_1 \wedge E_3\big), \\ d\big(e^{6\tilde{\Delta}} J_1 \wedge E_2\big) &= -e^{3\tilde{\Delta}}|\tilde{\xi}| d\big(e^{3\tilde{\Delta}} J_2 \wedge E_3\big), \end{aligned} \tag{4.10}$$

and the flux be given by

$$G_4 = \frac{1}{4} d\xi \wedge d\big(e^{3\tilde{\Delta}}\sqrt{1 - |\tilde{\xi}|^2} J_1\big). \tag{4.11}$$

We find that the uplift of the IIA solution fits into the above parametrisation. All the forms are defined in terms of the internal M-theory Killing spinors derived in appendix C.4, one needs only to plug them into the bi-linears in appendix B of [45]. Performing these steps with some liberal application of Mathematica, we find the local coordinate

$$\rho = \frac{k}{8} L^4 y_1 \sin^2 2\zeta \sin\theta, \tag{4.12}$$

and the solutions specific vielbein

$$\begin{aligned} E_3 = -\frac{e^{-3\tilde{\Delta}}}{\sqrt{1 - e^{-6\tilde{\Delta}}\rho^2}} \bigg[&\frac{kL^2}{256} \sin^2 2\zeta \Big(L^4 \sin^2 2\zeta \sin\theta d\theta + 64 y_2 d\phi - 64(y_1 dy_1 + y_2 dy_2)\cos\theta \Big) \\ &+ L^2 \cos 2\zeta \big(dz + C_1\big) \bigg], \end{aligned} \tag{4.13}$$

where C_1 is the potential giving rise to the IIA RR 2-form, which may be found in eq. (4.1). To express the SU(2) forms we introduce the following orthonormal frame

$$
\begin{aligned}
e^1 &= \frac{1}{\sqrt{X_1}\sqrt{X_2}}\Big(2X_1 dy_2 + 32 y_1 y_2 \cos^2\zeta \sin\theta\big(\sin\theta dy_1 + y_1\cos\theta d\theta\big)\Big), \qquad (4.14)\\
e^2 &= \frac{e^{-3\tilde\Delta}kL^2}{8\sqrt{X_1}\sqrt{X_2}}\Big(\cos^2\zeta\cos\theta\big(X_1(16y_2^2+L^4\sin^4\zeta)dy_2 - 256 y_1^3 y_2 \sin^2\theta dy_1\big) - X_2\sin^2\zeta d\phi\Big)\\
&\quad - \frac{32 y_2 e^{3\tilde\Delta}}{kL^6\sin^2\zeta\sqrt{X_1}\sqrt{X_2}}\Big(16 y_2\cos\theta dy_2 - L^4\cos^2\zeta\sin^2\zeta\sin\theta d\theta\Big),\\
e^3 &= \frac{e^{-3\tilde\Delta}kL^4\sin 2\zeta}{32\sqrt{X_1}\sqrt{1-e^{-6\Delta}\rho^2}}\Big(-X_1 d\zeta + 4y_1\sin 2\zeta\cos 2\zeta\sin\theta\big(\sin\theta dy_1 + y_1\cos\theta d\theta\big)\Big),\\
e^4 &= \frac{\cos 2\zeta}{2L^2\sqrt{X_1}}\Big(16 y_2 d\phi + L^4\cos^2\zeta\sin^2\zeta\sin\theta d\theta - 16\cos\theta\big(y_1 dy_1 + y_2 dy_2\big)\Big)\\
&\quad - \frac{kL^6\cos^2\zeta\sin^2\zeta\sqrt{X_1}e^{-3\tilde\Delta}}{8\sqrt{1-e^{-6\Delta}\rho^2}}\Big(dz + C_1\Big),
\end{aligned}
$$

where

$$
\begin{aligned}
X_1 &= 16y_1^2\cos^2\theta\sin^2\zeta + 16y_2^2 sin^2\theta\cos^2\zeta + L^4\sin^2\theta\sin^4\zeta\cos^2\zeta, \qquad (4.15)\\
X_2 &= 16y_1^2\cos^2\theta\big(16y_1^2 + L^4\sin^4\zeta\cos^2\zeta\big)\\
&\quad + L^4\cos^2\zeta\sin^2\zeta\sin^2\theta\big(16y_1^2 + \cos^2\zeta(16y_2^2 + L^4\sin^4\zeta)\big).
\end{aligned}
$$

With respect to this basis we have

$$
J = e^1\wedge e^2 + e^3\wedge e^4, \qquad \Omega = e^{i\alpha}(e^1 + ie^2)\wedge(e^3+ie^4), \tag{4.16}
$$

where the phase α is defined through

$$
\tan\alpha = -\frac{e^{3\tilde\Delta}}{kL^2\cos\theta\, y_1^2}. \tag{4.17}
$$

4.3 Properties of the CFT

As in the previous section, some properties of the CFT dual can be inferred by analyzing the 11d supergravity solution. The picture that arises is simply the 11d realization of the IIA picture described in subsection 3.2, apart from some subtleties that have to do with the existence of the special Taub-NUT direction. Indeed, all brane configurations that play a role in 11d will be either transverse to this direction or wrapped on it.

The non-trivial S^2 of the IIA geometry is also present in the 11d uplift. Therefore one can define large gauge transformations for the uplift of the B_2 field, which is the 11d 3-form potential with a component along the Taub-NUT direction, i_kC_3. Thus, as in the IIA background, we need to divide r in intervals of length π in order to have i_kC_3 lying in the fundamental region. From here the discussion parallels exactly the IIA discussion.

In 11d we find quantized charges N_6 and $N_5 = nN_6$, equal to the N_6 and N_4, respectively, in IIA. N_6 is associated to KK-monopoles and N_5 to M5-branes wrapped on the Taub-NUT direction of the monopole. The interpretation is that M5-brane charge (with the M5-brane wrapped in the Taub-NUT direction of the monopole) is created in the worldvolume of the KK-monopole when it crosses M5-branes transverse to the Taub-NUT direction.[11] Using the worldvolume effective action that describes a KK-monopole in 11d [47, 48] one can easily check that it is BPS when placed at $\zeta = 0$. The calculation parallels the D6-brane calculation in IIA with the only difference that the action is now written in terms of eleven dimensional fields. Similarly an M5-brane wrapped on the Taub-NUT direction is also BPS at this location.

As in IIA, the charge interpreted as level in 11d is also doubled, and we get two values k_6 and k_5 equal to the k_6 and k_4 charges, respectively, in IIA. These are now associated to wrapped M5-branes carrying monopole and dipole charges.[12]

Similarly, we find particle-like brane configurations, which are either M2-branes transverse to the Taub-NUT direction or M5-branes wrapped on this direction. These branes are wrapped on the same sub-manifolds of the internal space as in IIA. Namely,

- Di-monopoles $\leftrightarrow$ M2 on $M_1 \times S^1_\phi$, M5 on $M_1 \times S^1_\phi \times S^2 \times S^1_z$
- 't Hooft monopoles $\leftrightarrow$ M2 on $\{M_1, \theta\}$, M5 on $\{M_1, \theta, S^2, S^1_z\}$
- Di-baryons $\leftrightarrow$ M2 on $\{\zeta, S^1_\phi\}$, M5 on $\{\zeta, S^1_\phi, S^2, S^1_z\}$
- Baryon vertices $\leftrightarrow$ M2 on $\{\zeta, \theta\}$, M5 on $\{\zeta, \theta, S^2, S^1_z\}$

As for the IIB AdS_4 solution (see [11] for the details) the di-monopoles and 't Hooft monopoles have to sit at $\zeta = 0$ while the di-baryons sit at $r = 0$. In these derivations we have used the action that describes M2-branes transverse to the Taub-NUT direction of the monopole. In this action $i_k C_3$ couples in both the DBI and CS parts, so the M2-branes are sensitive to large gauge transformations. The details of this action can be found in [46].

Putting together this information, and in analogy with the IIA discussion, we expect a field theory in the $r \in [0, \pi]$ region described by a $\mathrm{U}(N_6)_{k_5} \times \mathrm{U}(N_6)_{-k_5}$ quiver with $\mathcal{N} = 2$ supersymmetry, sourced by KK-monopoles spanned on the $\mathbb{R}_{1,2} \times M_1 \times S^2 \times S^1_\phi$ directions, and with Taub-NUT direction z. A possible brane realization in the fundamental region $r \in [0, \pi]$ could be

$$
\begin{array}{rc|ccccccccccc}
5^3: & \times & \times & \times & \times & \times & \times & - & z_1 & z_2 & - & & z \\
N_6\, M6: & \times & \times & \times & - & \times & - & \times & \times & \times & - & & z \\
(5^3, k_4\, \mathrm{M5}): & \times & \times & \times & - & \times & \cos\theta & - & - & - & \sin\theta & & \times
\end{array}
\tag{4.18}
$$

where z denotes the eleventh direction, the M6 is the Kaluza-Klein monopole with Taub-NUT direction z and 5^3 is the exotic brane that gives rise to the IIA 5^2_2 brane upon reduction [49, 50].

[11] Recall that in 11d $i_k C_3 \to i_k C_3 + n\pi \mathrm{Vol}(S^2)$, and the M5 is magnetically charged with respect to this field.

[12] As shown in [46], M5-branes wrapped on an isometric direction can carry KK-monopole charge, with the Taub-NUT direction equal to the isometric direction.

4.4 Free energy

We can now calculate the free energy on a 3-sphere in the CFT dual to the solution in M-theory. This is expressed in terms of the effective 4 dimensional Newton constant G_4 as

$$\mathcal{F}_{S^3} = \frac{\pi}{2G_4}. \tag{4.19}$$

One can determine G_4 via a dimensional reduction of supergravity on the internal space $\mathcal{M}_7$, the result is

$$\frac{1}{16\pi G_4} = \frac{\pi}{2(2\pi)^9}\int_{\mathcal{M}_7} e^{9\tilde{\Delta}}\mathrm{Vol}(\mathcal{M}_7), \tag{4.20}$$

where we work in units such that $l_p = 1$. For the case at hand the relevant quantity is

$$e^{9\tilde{\Delta}}\mathrm{Vol}(\mathcal{M}_7) = \frac{k^2L^6}{32} r^2 \sin^3\zeta \cos^3\zeta \sin\theta \sin\chi d\zeta \wedge d\theta \wedge d\phi \wedge dr \wedge d\chi \wedge d\xi \wedge dz. \tag{4.21}$$

Integrating this in the region $r \in [n\pi, (n+1)\pi]$, $z \in [0, 2\pi]$ and using eqs. (3.23), (3.24) we arrive at

$$\mathcal{F}_{S^3} = \frac{\sqrt{2}\pi}{36}\left(12 + \frac{N_6^2}{\left(N_5 + \frac{N_6}{2}\right)^2}\right)\sqrt{\kappa_6}\left(N_5 + \frac{N_6}{2}\right)^{3/2}. \tag{4.22}$$

This reproduces, as expected, the result in IIB, with $N_5, N_3 \to N_6, N_5$ [11]. Essentially we have $\mathcal{F}_{S^3} \sim \left(N_5 + \frac{N_6}{2}\right)^{3/2}$ which reproduces the $N^{3/2}$ behavior of ABJM. In particular, in the fundamental region $r \in [0, \pi]$, where $N_5 = 0$, we find

$$\mathcal{F}_{S^3} = \frac{\sqrt{2}\pi}{3^{3/2}}\sqrt{k_4}N_6^{3/2}. \tag{4.23}$$

This is not a surprising result, given that the dependence of the free energy in type II theories, like the central charge and entanglement entropy of the strip, depends on the internal directions only through the quantity

$$V_{\mathrm{int}} = \int_{\mathcal{M}_6} e^{-2\Phi}\mathrm{Vol}(\mathcal{M}_6), \tag{4.24}$$

and is thus invariant under Abelian T-duality[13] and uplift to 11d.

So, quite surprisingly, we have found an AdS_4 M-theory solution with purely magnetic flux that falls in the general classification of [43], that originates in M5-branes wrapped on calibrated 3-cycles, but whose free energy does not exhibit the expected N^3 behavior. We leave a further discussion on this issue for the conclusions.

[13] It is not invariant though under non-Abelian T-duality, because even if the integrand is invariant, the S^3 on which the dualisation is performed is transformed into an $M_1 \times S^2$ space, where M_1 is the space spanned by the r-direction, and thus the domain of integration changes. This is the reason why the prefactors in (4.23) are not the same as in ABJM.

5 Conclusions

In this work we have presented a new warped AdS_4 solution of M-theory preserving $\mathcal{N}=2$ supersymmetry, giving the only known solution in this class other than the uplift of Pernici-Sezgin. A legitimate question to ask is whether this solution is truly distinct from Pernici-Sezgin, indeed this solution was generated by performing first a NAT then T duality on $AdS_4 \times \mathbb{CP}^3$, and some geometries derived via NAT duality have been shown to fall within the ansatz of previous solutions. This does not seem to be the case with this example: the quickest thing to note is that the free energy of Pernici-Sezgin scales as N^3 while this solution scales as $N^{3/2}$. Additionally the uplift of Pernici-Sezgin is everywhere non singular while the curvature invariants of this solution blow up in certain regions of parameter space. One might still wonder if this solution approximates Pernici-Sezgin at least locally away from the singularity, as was argued in [1] to be the case for the NAT dual of $AdS_5 \times S^5$ and the Gaiotto-Maldacena geometries [53]. This also does not seem to be the case. Sfetsos and Thompson were able to find an additional solution to the Gaiotto-Maldacena Toda equation which gave their solution. The differential equations giving rise to Pernici-Sezgin are more simple and are solved uniquely. So this solution is truly distinct.

In this work, following on [11], we have taken the view that the range of r is restricted to lie in a specific cell of length π after n large gauge transformations of B_2. The reason is to ensure that $0 < |\int_{S^2} B_2| < 4\pi^2$, a restriction motivated by string theory. However this does present an issue for the geometry, we are choosing to end it at a regular point which would usually demand the inclusion of extra localized sources. From a purely geometric view point we might choose to take $0 < r < \infty$, however this would be very undesirable from an AdS/CFT perspective. A continuous r would lead to, among other things, a CFT dual with operators of continuous conformal dimension [8]. An attractive resolution to these issues is that the NAT duality generates a solution which approximates a better defined solution free of these pathologies. At any rate, regardless of these potential criticisms, it seems likely that one could use this work as a stepping stone to further populate the solution space of purely magnetic M-theory solutions.

Supersymmetric probe branes in the 11d uplift of the Pernici-Sezgin solution were considered in [52], with an aim at introducing punctures on the Riemann surface along the lines of [53]. The BPS configurations were shown to preserve two U(1)$'s$, one more than required by the R-symmetry of the 3d $\mathcal{N}=2$ SCFT. This second U(1) corresponds to a global U(1) in the 3d field theory, and seems to play a key role in the 3d-3d correspondence [54]. It was argued in [52] that a large number of supersymmetric M5's would ultimately backreact on the Pernici-Sezgin geometry to produce a new AdS_4 solution with a U(1)^2 isometry. It would be interesting to show whether the AdS_4 solution obtained in this paper, containing a U(1)^2 isometry, could be related to this physical situation.

That the free energy of our purely magnetic AdS_4 solution scales like $\mathcal{F}_{S^3} \sim N^{3/2}$ rather than N^3 is a little puzzling. It was proved in [45] that the presence of M2 branes, whether accompanied by $M5's$ or not, always gives rise to $N^{3/2}$ behavior. However we know that our solution cannot contain M2 branes, indeed it is not possible to accommodate M2 branes in a purely magnetic flux ansatz, so what are we to make of this apparent contradiction.

Firstly it should be noted that, at least as far as the authors are aware, there is no proof $\mathcal{F}_{S^3} \sim N^3$ holds universally for all wrapped M5 brane solutions. However this seems like an inadequate evasion of a confusing result. More likely is that the solution we present does not correspond to wrapped M5 branes. Indeed, the ansatz taken in [43] to derive the purely magnetic solutions is defined by requiring the Killing spinors to satisfy the same projection conditions as the wrapped branes. Yet the solutions need not describe in general M5-branes wrapped in 3d manifolds in the near horizon limit. The metric we have obtained is rather complicated and it seems difficult to identify a 3-cycle in the internal geometry that such branes might wrap. This together with the fact that the free energy does not scale with N^3 is suggesting that this is indeed the case for our solution.

On the other hand, even if the CFT interpretation of the solution is yet very preliminary, we seem to have found that there are quantized charges associated to both KK-monopoles and M5-branes, with the first being the only sources of the geometry in the r-region that we have defined as the fundamental region. This is also suggestive of a geometry not originating from wrapped M5-branes.

Finally, let us comment on something slightly tangential. In the process of discussing the supersymmetry preserved by purely magnetic M-theory solutions we analised the G-structure preserved by the NAT dual of ABJM. We showed in appendix C.3 that this IIB solution preserves a U(1)'s worth of dynamical SU(2)-structures in 6d. We note that, it is possible to take the intersection of two of these and define an identity structure. However, given that a complete systematic study of AdS_4 solutions to type II supergravity preserving $\mathcal{N} = 2$ supersymmetry is currently absent form the literature, we have not pursued this here. Even so we know that, as with the better studied AdS_5, $\mathcal{N} = 1$ cases [61, 62], supersymmetry should be preserved in terms of either a local "SU(2)-structure" or "identity structure" on the internal co-dimensions of the isometry dual to the U(1) R-symmetry. The NAT dual of ABJM will certainly fall into the latter class.

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A Some details on the NAT and T duality transformations

In this appendix we give some details on the derivation of the solution in section 3. The starting point is the $AdS_4 \times \mathbb{CP}^3$ metric written as a Hopf fibration

$$ds^2 = ds^2(M_7) + e^{2C_a}(\omega_a + A_a)^2, \tag{A.1}$$

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where ω_a are SU(2) left-invariant 1-forms satisfying $d\omega_a = \frac{1}{2}\epsilon_{abc}\omega_b\wedge\omega_c$ and

$$\begin{aligned}
ds^2(M_7) &= \frac{L^2}{4}\Big[ds^2(AdS_4)+4d\zeta^2+\cos^2\zeta\big(d\theta^2+\cos^2\theta d\phi^2\big)\Big],\\
e^{2C_1} &= e^{2C_2}=\frac{L^2}{4}\sin^2\zeta, \qquad e^{2C_3}=\frac{L^2}{4}\sin^2\zeta\cos^2\zeta,\\
A_1 &= A_2 = 0, \qquad A_3=\cos\theta d\phi,
\end{aligned}\tag{A.2}$$

where the AdS radius is 1. Specifically we introduce the vielbeins

$$\begin{aligned}
&e^{x^\mu}=\frac{L}{2}\rho\, dx^\mu, \qquad e^\rho=\frac{L}{2\rho}d\rho, \qquad e^1=Ld\zeta, \qquad e^2=\frac{L}{2}\cos\zeta d\theta, \qquad e^3=\frac{L}{2}\cos\zeta\sin\theta d\phi,\\
&e^4=\frac{L}{2}\sin\zeta\,\omega_1, \qquad e^5=\frac{L}{2}\sin\zeta\,\omega_2, \qquad e^6=\frac{L}{2}\sin\zeta\cos\zeta(\omega_3+\cos\theta d\phi).
\end{aligned}\tag{A.3}$$

The dilaton of this solution is constant and set to $e^\Phi=\frac{k}{L}$, while the non trivial fluxes are

$$\begin{aligned}
F_2 &= G_2+J_1^a\wedge\big(\omega_a+A_a\big)+\frac{1}{2}\epsilon_{abc}K_0^a\big(\omega_b+A_b\big)\wedge\big(\omega_c+A_c\big),\\
F_4 &= G_4+K_3^a\wedge\big(\omega_a+A_a\big)+\frac{1}{2}\epsilon_{abc}M_2^a\big(\omega_b+A_b\big)\wedge\big(\omega_c+A_c\big)\\
&\quad + N_1\big(\omega_1+A_1\big)\wedge\big(\omega_2+A_2\big)\wedge\big(\omega_3+A_3\big),
\end{aligned}\tag{A.4}$$

where the only non zero components are

$$\begin{aligned}
G_2 &= -\frac{k}{2}\cos^2\zeta\sin\theta d\theta\wedge d\phi, \qquad J_1^3=-k\sin\zeta\cos\zeta d\zeta, \qquad K_0^3=-\frac{k}{2}\sin^2\zeta,\\
G_4 &= \frac{3kL^2}{8}\mathrm{Vol}(AdS_4).
\end{aligned}\tag{A.5}$$

A.1 The IIB NAT duality

Expressing the solution in this manner allows one to simply read off the result of performing a NAT duality transformation on ω_a using [13]. The dual metric is given by

$$d\hat{s}^2 = ds^2(M_7)+\sum_{a=1}^{3}\hat{e}^{a+3}\tag{A.6}$$

We have introduced cylindrical polars for the dual coordinates

$$v_1=y_1\cos\xi, \qquad v_2=y_1\sin\xi, \qquad v_3=y_2,\tag{A.7}$$

and choose to express the dual canonical vielbeins $\hat{e}$ in a way that makes the residual U(1) isometry given by ∂_ξ explicit

$$\begin{aligned}
\cos\xi\hat{e}^4+\sin\xi\hat{e}^5 = -\frac{1}{8L\Delta}\sin\zeta\Big[&4y_1y_2\big(4dy_2+L^2\sin^2\zeta\cos^2\zeta(d\xi+\cos\theta d\phi)\big)\\
&+dy_1\big(16y_1^2+L^4\sin^4\zeta\cos^2\zeta\big)\Big],
\end{aligned}$$

$$
\begin{aligned}
\cos\xi\hat{e}^5 - \sin\xi\hat{e}^4 &= -\frac{1}{8L\Delta}\sin\zeta\Big[4y_1dy_2 + \cos^2\zeta\big(-4y_2dy_1 + L^2y_1\sin^2\zeta(d\xi+\cos\theta d\phi)\big)\Big],\\
\hat{e}^6 &= -\frac{1}{8L\Delta}\sin\zeta\cos\zeta\Big[16y_1y_2dy_1 + dy_2(16y_2^2 + L^4\sin^4\zeta)\\
&\qquad\qquad -4L^2y_1^2\sin^2\zeta(d\xi+\cos\theta d\phi)\Big]\\
\hat{e}^a &= e^a, \qquad a = x^\mu, \rho, 1, 2, 3,
\end{aligned}
\tag{A.8}
$$

where we define

$$
\Delta = \sin^2\zeta\left(y_1^2 + \cos^2\zeta y_2^2 + \frac{L^4}{16}\sin^4\zeta\cos^2\zeta\right). \tag{A.9}
$$

A NS two form is generated

$$
\begin{aligned}
B_2 &= \frac{1}{\Delta}y_1\sin^2\zeta\Big(y_1dy_2 - y_2\cos^2\zeta dy_1\Big)\wedge d\xi\\
&\quad -\frac{1}{\Delta}\sin^2\zeta\cos^2\zeta\cos\theta\left(y_1y_2\,dy_1 + \left(y_2^2 + \frac{L^4}{16}\sin^4\zeta\right)dy_2\right)\wedge d\phi
\end{aligned}
$$

while the dilaton becomes[14]

$$
e^{-2\hat{\Phi}} = \frac{L^2}{4}\Delta\, e^{-2\Phi}. \tag{A.10}
$$

The solution also has all possible RR forms turned on. These can be found in [11] where this solution was originally derived.

A.2 The IIA NAT-T duality

We would now like to perform a T-duality on the global U(1) corresponding to ∂_ϕ. To do this we can once more make use of the results of [13] (see [55] for the original derivation). In order to do this we need to express the metric and B_2 as

$$
\begin{aligned}
d\hat{s}^2 &= d\hat{s}^2(M_9) + e^{2C}(d\phi + A_1)^2,\\
B_2 &= B + B_1\wedge d\phi.
\end{aligned}
\tag{A.11}
$$

Clearly B_2 is already in this form, while the same can be achieved for the metric with a rotation of the vielbein basis $\hat{e}\to\mathcal{R}\hat{e}$, giving

$$
e^{2C} = \frac{\Xi}{4\Delta}L^2\cos^2\zeta, \qquad A_1 = \frac{y_1^2\sin^4\zeta\cos\theta}{\Xi}\,d\xi, \tag{A.12}
$$

where

$$
\Xi = \Delta\sin^2\theta + y_1^2\sin^4\zeta\cos^2\theta. \tag{A.13}
$$

[14]Notice that, for simplicity in other expressions, we are extracting a factor of $\frac{L^2}{4}$ with respect to the definition of Δ in [13].

A rotation that achieves this is

$$\mathcal{R}=\begin{pmatrix} -\frac{\sqrt{\zeta_1^2+\zeta_2^2}\sin\zeta\cos\theta}{\sqrt{\Xi_0}} & \frac{\sin\theta(\sin\xi-\zeta_3\cos\xi)}{\sqrt{\Xi_0}} & -\frac{\sin\theta(\zeta_3\sin\xi+\cos\xi)}{\sqrt{\Xi_0}} & \frac{\sqrt{\zeta_1^2+\zeta_2^2}\sin\theta}{\sqrt{\Xi_0}} \\ 0 & \frac{\zeta_3\sin\xi+\left(\zeta_1^2+\zeta_2^2+1\right)\cos\xi}{\sqrt{\Delta_0}\sqrt{\zeta_1^2+\zeta_2^2+1}} & \frac{\left(\zeta_1^2+\zeta_2^2+1\right)\sin\xi-\zeta_3\cos\xi}{\sqrt{\Delta_0}\sqrt{\zeta_1^2+\zeta_2^2+1}} & \frac{\sqrt{\zeta_1^2+\zeta_2^2}\zeta_3}{\sqrt{\Delta_0}\sqrt{\zeta_1^2+\zeta_2^2+1}} \\ 0 & -\frac{\sqrt{\zeta_1^2+\zeta_2^2}\sin\xi}{\sqrt{\zeta_1^2+\zeta_2^2+1}} & \frac{\sqrt{\zeta_1^2+\zeta_2^2}\cos(\xi)}{\sqrt{\zeta_1^2+\zeta_2^2+1}} & \frac{1}{\sqrt{\zeta_1^2+\zeta_2^2+1}} \\ -\frac{\sqrt{\Delta_0}\sin\theta}{\sqrt{\Xi_0}} & \frac{\sqrt{\zeta_1^2+\zeta_2^2}\sin\zeta\cos\theta(\zeta_3\cos\xi-\sin\xi)}{\sqrt{\Delta_0}\sqrt{\Xi_0}} & \frac{\sqrt{\zeta_1^2+\zeta_2^2}\sin\zeta\cos\theta(\zeta_3\sin\xi+\cos\xi)}{\sqrt{\Delta_0}\sqrt{\Xi_0}} & -\frac{\left(\zeta_1^2+\zeta_2^2\right)\sin\zeta\cos\theta}{\sqrt{\Delta_0}\sqrt{\Xi_0}} \end{pmatrix} \tag{A.14}$$

which acts on 2456. We have introduced the following expressions

$$\Delta_0=1+\zeta_1^2+\zeta_2^2+\zeta_3^2,\qquad \Xi_0=\sin^2\theta\Delta_0+\sin^2\zeta\cos^2\theta\left(\zeta_1^2+\zeta_2^2\right),\qquad \zeta_a=v_a e^{\sum_{b\neq a}C_a}. \tag{A.15}$$

With the rotated vielbein basis, we may give the RR forms in [11] in terms of them and then use [13, 55] to read off the T-dual solution, getting then the results in section 3.

B Type-II G-structure conditions for AdS_4 solutions

In this appendix we review the G-structure conditions for supersymmetric $AdS_4\times M_6$ solutions, which is a slight modification[15] of what may be found in [56, 57], but with notation more akin to [59, 63]. The metric can be cast in the form

$$ds^2=e^{2A}ds^2(AdS_4)+ds^2(M_6), \tag{B.1}$$

where the AdS radius is 1 and the dilaton has support only in M_6. The fluxes have the same direct product structure, which in terms of the RR polyform we may express as

$$F=F_{\rm int}+e^{4A}{\rm Vol}(AdS_4)\wedge\tilde{F}. \tag{B.2}$$

We use a real representation of the 10d gamma matrices[16] in which the Dirac and ordinary conjugates coincide. A 4+6 split is performed on the 10d MW Killing spinors where $\epsilon=(\epsilon_1,\epsilon_2)^T$ and $\Gamma^{(10)}\epsilon=\sigma_3\epsilon$ so that we can write

$$\begin{aligned}\epsilon_1&=e^{A/2}\big(\zeta_+\otimes\eta^1_++\zeta_-\otimes\eta^1_-\big),\\ \epsilon_2&=e^{A/2}\big(\zeta_+\otimes\eta^2_\mp+\zeta_-\otimes\eta^2_\pm\big),\end{aligned} \tag{B.3}$$

where $\pm$ labels chirality in 4 and 6 dimensions, so that the upper/lower signs should be taken in IIA/IIB and $(\eta_+)^*=\eta_-$ and we take the internal, $\eta^{1,2}$ spinor to have unit norm.

Preservation of supersymmetry may be expressed in terms of differential conditions on two pure spinors

$$\Psi_\pm=8\,\eta^1_+\otimes\eta^{2\dagger}_\pm. \tag{B.4}$$

[15] Specifically with the normalization of the internal spinor.

[16] Specifically for the AdS_4 directions $\Gamma^\mu=\hat{\gamma}^\mu\otimes 1$, while on $\mathbb{CP}^3$ we define $\Gamma^i=\gamma^{(4)}\otimes\gamma^i$, where $\hat{\gamma}^\mu$ and γ^a are representations of the gamma matrices in 3+1 and 6 dimensions respectively. We define $\Gamma^{(10)}=\gamma^{(4)}\otimes\gamma^{(7)}$, where $\gamma^{(4)}=i\hat{\gamma}^{tx^1x^2r}$ and $\gamma^{(7)}=-i\gamma_{123456}$.

These conditions are given by

$$(d-H)\wedge\left(e^{3A-\Phi}\Psi_\pm\right) = -2e^{2A-\Phi}\mathrm{Re}\Psi_\mp,$$
$$(d-H)\wedge\left(e^{4A-\Phi}\Psi_\mp\right) = -3e^{3A-\Phi}\mathrm{Im}\Psi_\pm + e^{4A}\tilde{F}, \tag{B.5}$$

where once more the upper/lower signs should be taken in IIA/IIB and

$$e^{4A}\tilde{F} = \iota_{\mathrm{Vol}(AdS_4)}F. \tag{B.6}$$

The G-structure on M_6 can either be an SU(3), when η_+^1 and η_+^2 are globally parallel, or SU(2) when they are not. Using a Fierz identity and the Clifford map it is possible to express $\Phi_\pm$ as polyforms. In the SU(3)-structure case we can write this in terms of a complex 2-form J and a holomorphic 3-form Ω_{hol} as

$$\Psi_+ = e^{-i\theta_+}e^{-iJ}, \qquad \Psi_- = e^{i\theta_-}\Omega_{\mathrm{hol}}, \tag{B.7}$$

where the forms may be expressed in terms of the internal spinors as

$$J_{ab} = -i\eta_+^\dagger\gamma_{ab}\eta_+, \qquad \left(\Omega_{\mathrm{hol}}\right)_{abc} = -\eta_-^\dagger\gamma_{abc}\eta_+, \tag{B.8}$$

where

$$\eta_+^1 = e^{i\alpha_1}\eta, \qquad \eta_+^2 = e^{i\alpha_2}\eta, \qquad \eta_+^\dagger\eta_+ = 1, \qquad \theta_\pm = \alpha_1 \mp \alpha_2, \tag{B.9}$$

and the forms obey $J\wedge J\wedge J = \frac{3i}{4}\Omega_{\mathrm{hol}}\wedge\bar{\Omega}_{\mathrm{hol}}$, $J\wedge\Omega_{\mathrm{hol}} = 0$.

For the SU(2)-structure case the internal spinor may be expressed as

$$\eta_+^1 = e^{i\alpha_1}\eta_+, \qquad \eta_+^2 = e^{i\alpha_2}\left(\kappa_\parallel\eta_+ + \kappa_\perp\chi_+\right) \tag{B.10}$$

where $\chi_+^\dagger\eta_+ = 0$ and $\kappa_\parallel^2 + \kappa_\perp^2 = 1$. The pure spinors may then be expressed in terms of a holomorphic 1-form z, a real 2-form j and a holomorphic 2-form ω_{hol} as

$$\Psi_+ = i\,e^{i\theta_+}e^{\frac{1}{2}z\wedge\bar{z}}\wedge\left(\kappa_\parallel e^{-ij} - i\kappa_\perp\omega_{\mathrm{hol}}\right),$$
$$\Psi_- = e^{i\theta_-}z\wedge\left(\kappa_\perp e^{-ij} + i\kappa_\parallel\omega_{\mathrm{hol}}\right). \tag{B.11}$$

The various forms may be extracted from the spinor via

$$z_a = -i\eta_-^\dagger\gamma_a\chi_+, \qquad j_{ab} = \frac{1}{2}\left(-i\eta_+^\dagger\gamma_{ab}\eta_+ + i\chi_+^\dagger\gamma_{ab}\chi_+\right), \qquad (\omega_{\mathrm{hol}})_{ab} = i\eta_-^\dagger\gamma_{ab}\chi_-, \tag{B.12}$$

and obey the conditions,

$$j\wedge j = \frac{1}{2}\omega_{\mathrm{hol}}\wedge\bar{\omega}_{\mathrm{hol}}, \qquad j\wedge\omega_{\mathrm{hol}}, \qquad \omega_{\mathrm{hol}}\wedge\omega_{\mathrm{hol}} = 0, \qquad \iota_z\omega_{\mathrm{hol}} = \iota_z j = 0. \tag{B.13}$$

Finally it should be noted that the above conditions are actually the conditions for $\mathcal{N}=1$ in 3d. We will be concerned with $\mathcal{N}=2$ supersymmetry which implies a CFT dual with U(1) R-symmetry. This will manifest itself in the fact that there should be a U(1)'s worth of pure spinors satisfying eq. (B.5), two of which are independent.[17]

[17] In the sense that they can be constructed from two sets of linearly independent internal spinors (η_1, η_2) and $(\tilde{\eta}_1, \tilde{\eta}_2)$.

C Detailed supersymmetry analysis

In this appendix we shall look at how the Killing spinors are transformed under the sequence of dualities we perform to reach the M-theory solution of section 4.2. We shall begin by identifying a set of spinors on $\mathbb{CP}^3$ that are uncharged under the SU(2) on which the NAT-duality is performed.

C.1 A SU(2) T-duality invariant Killing spinor on $AdS_4 \times \mathbb{CP}^3$

We express the metric of ABJM in terms of the vielbein basis of eq. (A.3). Supersymmetry is preserved in type IIA when the variations of the dilatino and gravitino vanish. For ABJM which has a constant dilaton and zero Romans mass these constraints are

$$\begin{aligned}\delta\lambda &= \frac{e^{\phi}}{8}\left(\frac{3}{2}\not{F}_2\Gamma^{ab}(i\sigma_2)+\frac{1}{4!}\not{F}_4(\sigma_1)\right)\epsilon=0, \qquad \text{(C.1)}\\ \delta\Psi_\mu &= D_\mu\epsilon+\frac{e^{\phi}}{8}\left(\frac{1}{2}\not{F}_2\Gamma^{ab}(i\sigma_2)+\frac{1}{4!}\not{F}_4(\sigma_1)\right)\Gamma_\mu\epsilon=0,\end{aligned}$$

where $D_\mu\epsilon=(\partial_\mu+\frac{1}{4}\omega_{\mu,ab}\Gamma^{ab})\epsilon$. Specifically we have

$$\frac{1}{2}\not{F}_2=-\frac{2k}{L^2}(\Gamma^{16}+\Gamma^{23}+\Gamma^{45}),\qquad \frac{1}{4!}\not{F}_4=\frac{6k}{L^2}\Gamma_{AdS_4}, \tag{C.2}$$

and

$$\begin{aligned}&\omega^{x^\mu\rho}=\frac{2}{L}e^{x^\mu},\qquad \omega^{12}=-\omega^{36}=\frac{\tan\zeta}{L}e^2,\qquad \omega^{13}=\omega^{26}=\frac{\tan\zeta}{L}e^3\\ &\omega^{14}=-\omega^{56}=-\frac{\cot\zeta}{L}e^4,\qquad \omega^{15}=\omega^{46}=-\frac{\cot\zeta}{L}e^5,\qquad \omega^{16}=-\frac{2\cot 2\zeta}{L}e^6, \qquad \text{(C.3)}\\ &\omega^{23}=\frac{1}{L}(-2\cot\theta_1\sec\zeta e^3+\tan\zeta e^6),\qquad \omega^{45}=\frac{1}{L}(-2\cot\theta_1\sec\zeta e^3+(\cot\zeta+2\tan\zeta)e^6).\end{aligned}$$

Inserting the fluxes into the variation of the dilatino and manipulating leads to

$$\left(\Gamma^{2345}+\Gamma^{16}(\Gamma^{23}+\Gamma^{45})\right)\epsilon=\epsilon. \tag{C.4}$$

This constraint preserves a maximum of 24 real supercharges, however one finds that such a Killing spinor, which also solves the gravitino variation, must depend on the SU(2) directions [11]. Here we take a different approach and impose the projection

$$\Gamma^{2345}\epsilon=\epsilon, \tag{C.5}$$

which preserves only half the supercharges. Turning attention to the gravitino variation, one finds that the components along the AdS_4 directions give

$$D_\mu\epsilon+\frac{1}{L}\Gamma_{AdS_4}\Gamma_\mu(\sigma_1)\epsilon, \tag{C.6}$$

which is a standard Killing spinor equation for AdS_4 which one can solve without any constraint.

Using the projection or eq. (C.5) it is possible to show that the gravitino variation along the $\mathbb{CP}^3$ directions reduce to a differential equation and an additional projection

$$\partial_\zeta \epsilon + \Gamma^6(i\sigma_2)\epsilon = 0, \tag{C.7}$$

$$\Gamma^{1456}\epsilon = -\Big(\cos 2\zeta + \sin 2\zeta \Gamma^6(i\sigma_2)\Big)\epsilon. \tag{C.8}$$

These are solved by

$$\epsilon = e^{-\Gamma^6(i\sigma_2)}\epsilon_0 \tag{C.9}$$

where ϵ_0 is a spinor which depends only on the AdS_4 coordinates and obeys

$$\Gamma^{2345}\epsilon_0 = -\Gamma^{1456}\epsilon_0 = \epsilon_0. \tag{C.10}$$

Thus we have found a Killing spinor preserving 8 real supercharges which gives $\mathcal{N} = 2$ supersymmetry in 3d. This is the most general spinor which is independent of the SU(2) directions (in the prefered frame) and so [13] informs us that 8 supercharges are preserved under a SU(2) NAT duality transformation.

As the solution is a direct product and we know that there are 4 independent Killing spinors preserved by AdS_4, we must have 2 preserved on $\mathbb{CP}^3$. On the other hand the AdS_4 factor and supersymmety preserved by the spinor imply that we are describing a subsector of ABJM with U(1) R-symmetry. The Killing spinors should be invariant under the action of this U(1). Indeed we can impose an additional projection

$$P_\alpha\epsilon = \Gamma^6(i\sigma_2)\epsilon, \qquad P_\alpha = \Gamma^3(-\cos\alpha\Gamma^4 + \sin\alpha\Gamma^5) \tag{C.11}$$

where α is a constant which parametrizes the U(1). Notice that if one defines two spinors such that $P_{\alpha_1}\chi_{\alpha_1} = \Gamma^6(i\sigma_2)\chi_{\alpha_1}$ and $P_{\alpha_2}\chi_{\alpha_2} = \Gamma^6(i\sigma_2)\chi_{\alpha_2}$ hold, then we have $\chi^\dagger_{\alpha_1}\chi_{\alpha_2} = 0$ when $\alpha_1 - \alpha_2 = \pi$, so we are still describing $\mathcal{N} = 2$ supersymmetry.

C.2 A U(1) of SU(3)-structures on $\mathbb{CP}^3$

We know the 6d Killing spinors of ABJM define an SU(3)-structure [58], so the internal spinors η^1_+ and η^2_+ must match up to a phase. Specifically we define

$$\eta^1_+ = e^{i\frac{\theta_0}{2}}\eta_+, \qquad \eta^2_+ = e^{-i\frac{\theta_0}{2}}\eta_+. \tag{C.12}$$

The projective constraints in 6d become

$$\gamma^{1456}\eta_+ = -\Big(\cos 2\zeta + \hat{P}_\alpha \sin 2\zeta\Big)\eta_+, \qquad \gamma^{2345}\eta_+ = \eta_+, \qquad \hat{P}_\alpha\eta_+ = \gamma^6\eta_-, \tag{C.13}$$

where $\hat{P}_\alpha = \gamma^3(-\cos\alpha\gamma^4 + \sin\alpha\gamma^5)$. These are still a little complicated, to get to a canonical frame we first rotate in γ^4, γ^5, and then γ^3, γ^4 such that $\hat{P}_\alpha = -\tilde{\gamma}^{34}$ and $\gamma^{1456}\eta_+ = -\eta_+$. This leads to new vielbeins which we express in terms of eq. (A.3) as

$$\begin{aligned}
\tilde{e}^a &= e^a, \qquad a = 1, 2, 6,\\
\tilde{e}^3 &= \cos 2\zeta e^3 + \sin 2\zeta(-\cos\alpha e^4 + \sin\alpha e^5),\\
\tilde{e}^4 &= \sin 2\zeta e^3 + \cos 2\zeta(\cos\alpha e^4 - \sin\alpha e^5),\\
\tilde{e}^5 &= \sin\alpha e^4 + \cos\alpha e^5.
\end{aligned} \tag{C.14}$$

With respect to this basis we have

$$\tilde{\gamma}^{16}\eta_+ = \tilde{\gamma}^{32}\eta_+ = \tilde{\gamma}^{45}\eta_+ = +i\eta_+, \qquad \tilde{\gamma}^{346}\eta_+ = -\eta_-, \tag{C.15}$$

and so the SU(3)-structures are given by the forms

$$\begin{aligned} J_\alpha &= \tilde{e}^1 \wedge \tilde{e}^6 + \tilde{e}^3 \wedge \tilde{e}^2 + \tilde{e}^4 \wedge \tilde{e}^5, \\ \Omega_{hol,\alpha} &= -i(\tilde{e}^1 + i\tilde{e}^6) \wedge (\tilde{e}^3 + i\tilde{e}^2) \wedge (\tilde{e}^4 + i\tilde{e}^5). \end{aligned} \tag{C.16}$$

The forms satisfy eq. (B.5) for any constant α provided

$$\theta_+ = \theta_0 = \frac{3\pi}{2}, \qquad \theta_- = 0, \qquad e^{2A} = \frac{L^2}{4}. \tag{C.17}$$

One should note that if we take $(J_0, \Omega_{hol,0})$ we can generate the whole U(1) again by sending $\psi \to \psi - \alpha$, inside the left invariant 1-forms ω_i. This is what we expect since the isometry ∂_ψ gives the geometric realisation of the U(1) subgroup of the R-symmetry of ABJM.

C.3 A U(1) of SU(2)-structures in the non-Abelian T-dual

We would now like to find the G-structure and Killing spinors of the geometry after performing the SU(2) isometry non-Abelian T-duality. Fortunately we can exploit a map for the SU(2) transformation of the pure spinors that was proposed in [6]

$$\hat{\Psi}_\pm = \Psi_\mp \, \Omega^{-1}_{\mathrm{SU}(2)}, \tag{C.18}$$

where in general, in the frame of eq. (A.3)

$$\Omega_{\mathrm{SU}(2)} = \frac{1}{\sqrt{1+\zeta_a^2}} \Gamma^{(10)} \Big(-\Gamma_{456} + \sum_{a=1}^{3} \zeta_a \Gamma^{a+3} \Big), \tag{C.19}$$

for ζ_a defined as in [5], which for our parametrisation of ABJM specifically is

$$\zeta_1 = \frac{4}{L^2 \cos\zeta \sin^2\zeta} y_1 \cos\xi, \qquad \zeta_2 = \frac{4}{L^2 \cos\zeta \sin^2\zeta} y_1 \sin\xi, \qquad \zeta_3 = \frac{4}{L^2 \sin^2\zeta} y_2. \tag{C.20}$$

Although eq. (C.18) will give us the pure spinors in type IIB, it is still instructive to study the MW Killing spinors. The action of the NAT duality transformation on this is given by [13]

$$\hat{\epsilon}_1 = \epsilon_1, \qquad \hat{\epsilon}_2 = \Omega_{\mathrm{SU}(2)} \, \epsilon_2, \tag{C.21}$$

which corresponds to the following 6d spinors

$$\begin{aligned} \hat{\eta}^1_+ &= e^{i\frac{3\pi}{4}} \eta_+, \\ \hat{\eta}^2_+ &= -i\, e^{-i\frac{3\pi}{4}} \left[i \, \frac{\cos 2\zeta \tilde{\gamma}^1 + \tilde{\zeta}_1 \tilde{\gamma}^4 + \tilde{\zeta}_2 \tilde{\gamma}^5 + \zeta_3 \tilde{\gamma}^6}{\sqrt{1+\zeta_a \zeta_a}} \eta_- + \frac{\sin 2\zeta}{\sqrt{1+\zeta_a\zeta_a}} \eta_+ \right], \end{aligned} \tag{C.22}$$

with the spinors on AdS_4 unchanged. Here we use the frame of eq. (C.14), but with dual vielbeins, have made use of the projections and defined

$$\tilde{\zeta}_1 = \frac{4}{L^2 \cos\zeta \sin^2\zeta} y_1 \cos(\xi + \alpha), \qquad \tilde{\zeta}_2 = \frac{4}{L^2 \cos\zeta \sin^2\zeta} y_1 \sin(\xi + \alpha). \tag{C.23}$$

Here we see that α only appears in the combination $\xi+\alpha$, which indicates that ∂_ξ plays the role of the U(1) R-symmetry in the NAT dual solution, indeed this can be confirmed by computing the Kosmann derivative along ∂_ξ.

The spinors in eq. (C.22) actually define a dynamical SU(2) structure, which means $\hat\eta^1_+$ and $\hat\eta^2_+$ are not globally parallel and the angle between them is point dependent. We can simplify the expression for $\hat\eta^2_+$ considerably with further rotations of the vielbein basis. There is an optimum frame, in which all components of the SU(2)-structure are relatively simple. The vielbeins are given in this frame by

$$\begin{aligned}
\hat e^1 &= \frac{1}{4L^3\sin^3\zeta\cos\zeta\sqrt{\Delta_q}}\Big(L^4\sin^2\zeta\sin4\zeta-32(y_1dy_1+y_2dy_2)\Big), \qquad \text{(C.24)}\\
\hat e^2 &= \frac{1}{4L^3\sin^3\zeta\cos\zeta\sqrt{\Delta_p}\sqrt{\Delta_q}}\Bigg[\cos\zeta\Big(\sin2\zeta\big(L^4\sin^2\zeta\Delta_p d\theta-32y_1^2\sin2(\xi+\alpha)\sin\theta d\phi\big)\\
&\quad-128y_1y_2\cos(\xi+\alpha)d\zeta\Big)-64y_1\cos(\xi+\alpha)\sin\zeta\cos2\zeta dy_2\Bigg],\\
\hat e^3 &= \frac{1}{2L^3\sin^2\zeta\cos\zeta\sqrt{\Delta_0}\sqrt{\Delta_p}}\Bigg[\cos^2\zeta\Big(32y_1y_2\cos(\xi+\alpha)d\xi+32y_2\sin(\xi+\alpha)dy_1\\
&\quad+\big(32y_1y_2\cos(\xi+\alpha)\cos\theta+L^4\Delta_0\cos2\zeta\sin^2\zeta\sin\theta\big)d\phi\Big)-32y_1\sin(\xi+\alpha)dy_2\Bigg],\\
\hat e^4 &= \frac{2}{L^5\sin^3\zeta\cos\zeta\sqrt{\Delta_0}\sqrt{\Delta_p}\sqrt{\Delta_q}}\Bigg[\cos\zeta\Big(y_1\sin(\xi+\alpha)\big(64y_1^2\cos^2(\xi+\alpha)+L^4\sin^2\zeta\Delta_p\big)d\xi\\
&\quad+\cos(\xi+\alpha)\big(64y_1^2\sin^2(\xi+\alpha)dy_1+64y_1y_2dy_2-L^4\Delta_p\sin^2\zeta dy_1\big)\\
&\quad+L^4y_1\Delta_q\sin(\xi+\alpha)\sin^2\zeta\cos\theta d\phi\Big)-2L^4y_1\Delta_0\cos(\xi+\alpha)\cos2\zeta\sin\zeta d\zeta\Bigg],\\
\hat e^5 &= -\frac{2}{L\sin^2\zeta\cos\zeta\sqrt{\Delta_p}}\Bigg[2y_1\sin(\xi+\alpha)d\zeta-y_2\sin2\zeta\cos\zeta^2\sin\theta d\phi\\
&\quad+\frac14\sin4\zeta\Big(\sin(\xi+\alpha)dy_1+y_1\cos(\xi+\alpha)\big(d\xi+\cos\theta d\phi\big)\Big)\Bigg]\\
\hat e^6 &= -\frac{2}{L\sin^2\zeta\cos\zeta\sqrt{\Delta_q}}\Bigg[\cos2\zeta\sin\zeta dy_2\\
&\quad+2\cos\zeta\Big(y_2d\zeta+y_1\cos\zeta\sin\zeta\big(\cos(\xi+\alpha)d\theta+\sin(\xi+\alpha)\sin\theta d\phi\big)\Big)\Bigg],
\end{aligned}$$

where

$$\Delta_0=1+\zeta_a^2,\qquad \Delta_q=\cos^22\zeta+\zeta_a^2,\qquad \Delta_p=\Delta_q-\sin^22\tilde\zeta\zeta_1^2. \qquad \text{(C.25)}$$

In this basis the action of NAT duality on the 6d spinors is simply

$$\hat\eta^1=e^{i\frac{\theta_0}{2}}\eta_+,\qquad \hat\eta^2_+=-e^{-i\frac{\theta_0}{2}}\left[\kappa_\parallel\eta_++i\kappa_\perp\hat\gamma^1\eta_-\right] \qquad \text{(C.26)}$$

where

$$\kappa_\| = \frac{\sin 2\zeta}{\sqrt{1+\zeta_a\zeta_a}}, \qquad \kappa_\perp = \sqrt{\frac{\cos 2\zeta + \zeta_a\zeta_a}{1+\zeta_a\zeta_a}} \tag{C.27}$$

and $\kappa_\|^2+\kappa_\perp^2 = 1$. The projections the original spinor obeys are most succinctly expressed as

$$\hat{\gamma}_{2345}\eta_+ = \eta_+, \qquad \hat{\gamma}_{1456}\eta_+ = -(\kappa_\perp - \kappa_\| \hat{\gamma}^{34})\eta_+, \tag{C.28}$$

in the basis where $\gamma^{(7)}\eta_+ = \eta_+$ as before. The U(1)'s worth of SU(2)-structure is given by the following forms

$$\begin{aligned}
z_\alpha &= \hat{e}^1 + i\hat{e}^6,\\
j_\alpha &= (\kappa_\perp \hat{e}^3 - \kappa_\| \hat{e}^4)\wedge \hat{e}^2 + (\kappa_\perp \hat{e}^4 + \kappa_\| \hat{e}^3)\wedge \hat{e}^5,\\
\omega_{hol,\alpha} &= -i\big((\kappa_\perp \hat{e}^3 - \kappa_\| \hat{e}^4) + i\hat{e}^2\big)\wedge\big((\kappa_\perp \hat{e}^4 + \kappa_\| \hat{e}^3) + i\hat{e}^5\big),
\end{aligned} \tag{C.29}$$

which satisfy the supersymmetry conditions of eq. (B.5) for any constant α provided

$$\theta_+ = 0, \qquad \theta_- = \theta_0 = \frac{3\pi}{2}, \qquad e^{2A} = \frac{L^2}{4}. \tag{C.30}$$

We could take the intersection of the two linearly independent SU(2) structures defined for $\alpha = 0$ and $\alpha = \pi$, and define an identity structure. However, the supersymmetry conditions of such an object are absent from the literature at present and deriving them is outside the scope of this work.

C.4 Killing spinors in M-theory

Before we can define the M-theory Killing spinor, we must first derive the MW Killing spinors in type IIA after an additional T-duality is performed. As we want to make contact with [45] we need to work with the two linearly independent spinors in 6d. These are given by

$$\pi^1_+ = e^{i\frac{\theta_0}{2}}\pi_+, \qquad \pi^2_+ = e^{-i\frac{\theta_0}{2}}\pi_+, \qquad \tilde{\pi}^1_+ = e^{i\frac{\theta_0}{2}}\tilde{\pi}_+, \qquad \tilde{\pi}^2_+ = e^{-i\frac{\theta_0}{2}}\tilde{\pi}_+, \qquad \theta_0 = \frac{3\pi}{2} \tag{C.31}$$

and are such that

$$\begin{aligned}
&\gamma^{1456}\pi_+ = -\big(\cos 2\zeta - \sin 2\zeta\gamma^{34}\big)\pi_+, \qquad \gamma^{2345}\pi_+ = \pi_+, \qquad \gamma^{246}\pi_+ = -\pi_-\\
&\gamma^{1456}\tilde{\pi}_+ = -\big(\cos 2\zeta + \sin 2\zeta\gamma^{34}\big)\tilde{\pi}_+, \qquad \gamma^{2345}\tilde{\pi}_+ = \tilde{\pi}_+, \qquad \gamma^{246}\tilde{\pi}_+ = \tilde{\pi}_-,
\end{aligned} \tag{C.32}$$

in the frame of eq. (A.3). The independent 10d MW spinors $\epsilon_{1,2}$ and $\tilde{\epsilon}_{1,2}$ are then constructed in the obvious way from eq. (B.3), with $\eta \to \pi$ and using the same spinors on AdS_4. We must act on these spinors first with $\Omega_{\mathrm{SU(2)}}$, which in this frame is as in eq. (C.19), then with $\Omega_{\mathrm{U(1)}}$, which gives the transformation of the spinor under the Abelian T-duality [55]. In the frame of eqs. (A.3), (A.8) this is most succinctly expressed as

$$\begin{aligned}
\Omega_{\mathrm{U(1)}} = &\frac{1}{\sqrt{\Delta_0}\sqrt{\Xi_0}}\sin\zeta\cos\theta\Gamma^{(10)}\Big[\big(-\zeta_2+\zeta_1\zeta_3\big)\Gamma^4 + \big(\zeta_1+\zeta_2\zeta_3\big)\Gamma^5 - \big(\zeta_1^2+\zeta_2^2\big)\Gamma^6\Big]\\
&-\frac{\sqrt{\Delta_0}}{\sqrt{\Xi_0}}\sin\theta\Gamma^{(10)}\Gamma^3.
\end{aligned}$$

We take the 10d MW Killing spinors in IIA after the NAT-T duality transformation to be

$$\hat{\hat{\epsilon}}_1 = \epsilon_1, \qquad \hat{\hat{\epsilon}}_2 = \Omega_{\mathrm{U}(1)}\Omega_{\mathrm{SU}(2)}\epsilon_2, \tag{C.33}$$

with an equivalent expression with $\epsilon \to \tilde{\epsilon}$, which means that the new 6d Killing spinors are given by

$$\begin{aligned}
\hat{\hat{\pi}}^1_+ &= e^{i\frac{\theta_0}{2}}\pi_+, \qquad \hat{\hat{\pi}}^2_+ = -e^{-i\frac{\theta_0}{2}}\Big(\hat{\kappa}_\parallel \pi_+ + \hat{\kappa}_\perp \mathcal{F}\pi_-\Big),\\
\hat{\hat{\tilde{\pi}}}^1_+ &= e^{i\frac{\theta_0}{2}}\tilde{\pi}_+, \qquad \hat{\hat{\tilde{\pi}}}^2_+ = +e^{-i\frac{\theta_0}{2}}\Big(\hat{\kappa}_\parallel \tilde{\pi}_+ + \hat{\kappa}_\perp \mathcal{F}\tilde{\pi}_-\Big),
\end{aligned} \tag{C.34}$$

where

$$\hat{\kappa}_\parallel = \frac{\sin 2\zeta \sin\theta\zeta_2}{\sqrt{\Xi_0}}, \qquad \hat{\kappa}_\perp = \sqrt{1-\hat{\kappa}_\parallel^2},$$
$$\mathcal{F} = \frac{i}{\sqrt{\Xi_0 - \sin^2 2\zeta \sin^2\theta\,\zeta_2^2}}\Big(\sin\zeta\cos\theta\big(\zeta_2\gamma^2 - \zeta_1\gamma^3\big) - \sin\theta\big(\cos 2\zeta\,\zeta_2\gamma^1 - \gamma^5 - \zeta_3\gamma^4 + \zeta_1\gamma^6\big)\Big). \tag{C.35}$$

Clearly eq. (C.34) supports a U(1) of dynamical SU(2)-structures, as was the case in type-IIB, which we will not explicitly derive.

We are know ready to construct the two independent M-theory Killing spinors. These can be expressed in terms of the spinors in IIA as

$$\eta^1 = e^{-\Phi/6}(\epsilon_1 + \epsilon_2), \qquad \eta^2 = e^{-\Phi/6}(\tilde{\epsilon}_1 + \tilde{\epsilon}_2). \tag{C.36}$$

In the conventions of [45] the M-theory spinors are

$$\eta^i = e^{\tilde{\Delta}/2}\Big(\psi^i_+ \otimes \chi_i + \big(\psi^i_+\big)^c \otimes \chi^c_i\Big), \tag{C.37}$$

where $e^{\frac{\tilde{\Delta}}{2}} = e^{\frac{\tilde{A}}{2} - \frac{\Phi}{6}}$ and $e^{2\tilde{A}}$ is a modified warp factor of AdS_4 in IIA such that $Ricci(AdS_4) = -12g(AdS_4)$. Thus if we identify the AdS_4 spinors of IIA with those of eq. (C.37) we see that

$$\begin{aligned}
\chi_1 &= \frac{1}{\sqrt{2}}\big(\hat{\hat{\pi}}^1_+ + \hat{\hat{\pi}}^2_-\big), \qquad \chi^c_1 = \frac{1}{\sqrt{2}}\big(\hat{\hat{\pi}}^1_- + \hat{\hat{\pi}}^2_+\big),\\
\chi_2 &= \frac{1}{\sqrt{2}}\big(\hat{\hat{\tilde{\pi}}}^1_+ + \hat{\hat{\tilde{\pi}}}^2_-\big), \qquad \chi^c_2 = \frac{1}{\sqrt{2}}\big(\hat{\hat{\tilde{\pi}}}^1_- + \hat{\hat{\tilde{\pi}}}^2_+\big),
\end{aligned} \tag{C.38}$$

which clearly satisfy $\bar{\chi}_1\chi_1 = \bar{\chi}_2\chi_2 = 1$, and from these one can construct spinors of charge ± 2 under the U(1) R-symmetry

$$\chi_\pm = \frac{1}{\sqrt{2}}\big(\chi_1 \pm \chi_2\big). \tag{C.39}$$

It is then simply a matter of plugging the $\chi_\pm$ of this section into the spinor bi-linears in appendix B of [45], and rotating the frame to reproduce the results of section 4.2. Note that the frame used in this section needs to be rotated as in eq. (A.14) to reach the vielbein basis where flat directions 2456 may be identified with $\mathcal{G}^{1,2,3,4}$ of eq. (3.2) and the rest with eq. (A.3).

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3.3 Large superconformal near-horizons from M-theory

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Large superconformal near-horizons from M-theory

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We report on a classification of supersymmetric solutions to 11D supergravity with $SO(2,2)\times SO(3)$ isometry, which are AdS/CFT dual to 2D CFTs with $\mathcal{N}=(0,4)$ supersymmetry. We recover the Maldacena, Strominger, Witten near-horizon with small superconformal symmetry and identify a class of $\mathrm{AdS}_3\times S^2\times S^2\times CY_2$ geometries with emergent large superconformal symmetry. This exhausts known compact geometries. Compactification of M-theory on CY_2 results in a vacuum of 7D supergravity with large superconformal symmetry, providing a candidate near-horizon for an extremal black hole and a potential new setting to address microstates.

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I. INTRODUCTION

To obey the second law of thermodynamics, black holes must possess entropy, which Bekenstein and Hawking showed is proportional to the area of the event horizon [1]. This observation paved the way for the holographic principle and AdS/CFT [2]. One of the earliest AdS/CFT calculations (it predates the conjecture) shows that asymptotic symmetries of gravity in AdS_3 correspond to the Virasoro algebra [3], a feature of 2D CFTs. This observation together with the Cardy formula [4] for the asymptotic growth of states for a CFT with central charge c is enough to provide a microscopic derivation for the Bekenstein-Hawking (BH) entropy [5,6]. For black holes with AdS_3 near-horizons, this methodology has been an incredible success, culminating in recent years in generalizations to extreme Kerr black holes [7], potential astrophysical bodies [8].

However, Einstein's gravity is at best an effective description [9], and the BH entropy is expected to be corrected in a candidate UV complete theory, such as M-theory. More concretely, compactifying the 6D M5-brane theory on a four-cycle in a Calabi-Yau three-manifold, CY_3, gives rise to the Maldacena, Strominger, Witten (MSW) CFT [10], with $\mathcal{N}=(0,4)$ supersymmetry at low energies. The corresponding black hole exhibits the near-horizon $\mathrm{AdS}_3\times S^2\times CY_3$, and subleading corrections to the BH entropy have been shown to perfectly match corrections to the central charge [10,11].

The MSW CFT exhibits *small* superconformal symmetry [12] with an $SU(2)$ R symmetry that is manifest in the two-sphere in the dual geometry. Since other superconformal symmetries exist [13–15], a rich class of AdS/CFT geometries can be expected, e.g. [16]. In this paper, we identify a new class of M-theory vacua $\mathrm{AdS}_3\times S^2\times S^2\times CY_2$, implying the existence of a distinct class of 2D $\mathcal{N}=(0,4)$ CFTs with *large* superconformal symmetry and R symmetry $SU(2)\times SU(2)$. We recall that CFTs with large superconformal symmetry remain largely enigmatic. While constructions based on string theory, such as $\mathrm{AdS}_3\times S^3\times S^3\times S^1$ [17,18], exist, contrary to small superconformal CFTs, interpretation as a symmetric product CFT is problematic [19]. This issue continues to attract exciting new holographic proposals [20,21], against a backdrop where we have witnessed a deeper understanding of the role of integrability [22–24].

More concretely, we report the results of a complete classification of supersymmetric solutions to 11D supergravity, the low-energy description of M-theory, where we assume $SO(2,2)\times SO(3)$ isometry, i.e. warped $\mathrm{AdS}_3\times S^2\times\mathcal{M}_6$ spacetime, with $\mathcal{M}_6$ being an $SU(2)$-structure manifold. Since 2D $\mathcal{N}=(0,4)$ superconformal field theories (SCFTs) are expected to exhibit at least an $SU(2)$ isometry, corresponding to the R symmetry, this is a minimal requirement. One may contemplate a distinct class where the $SU(2)$ R symmetry is realized as a squashed three-sphere, $\tilde{S}^3$, but such an ansatz would preclude the MSW geometry. Indeed, noncompact $\mathrm{AdS}_3\times\tilde{S}^3\times S^2\times T^3$ geometries, generated via non-Abelian T-duality [25,26], were identified recently in Ref. [27]. It is also an immediate corollary of this work that compact $\mathrm{AdS}_3\times\tilde{S}^3\times\tilde{S}^3\times T^2$ geometries with $\mathcal{N}=(0,4)$ supersymmetry may be generated through T-duality-shift-T-duality (TsT) transformations [28].

Our results pertain to general warped $\mathrm{AdS}_3 \times S^2$ spacetimes and are not intended to apply to all M-theory geometries dual to 2D $\mathcal{N} = (0,4)$ SCFTs. Within our assumptions, we prove that $\mathcal{M}_6$ is either CY_3, thus recovering the MSW geometry, or it possesses an additional $SU(2)$ R symmetry that emerges from the supersymmetry analysis. Truncating the emergent $SU(2)$ to $U(1)$, we recover a known class [29,30] of spacetimes with $SU(2) \times U(1)$ isometry [31].

The existence of a class of $\mathrm{AdS}_3 \times S^2 \times S^2 \times CY_2$ solutions to 11D supergravity, with 2D $\mathcal{N} = (0,4)$ SCFT duals, comes somewhat as a surprise. In the case where $CY_2 = T^4$, it was shown long ago that there are geometries related through T-duality to well-known $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ solutions in 10D [17]. When $CY_2 = K_3$, the class appears new. It did not feature in a study of wrapped M5-brane geometries [29]. More recently, M-theory geometries dual to 2D $\mathcal{N} = (0,2)$ SCFTs have been discussed, but where supersymmetry is enhanced to $\mathcal{N} = (0,4)$, the geometry is either MSW [32,33], or no good AdS_3 vacuum exists [34,35]. Moreover, it is expected that M-theory on K_3 is dual to heterotic string theory on T^3 [36], a statement that can be made precise in the supergravity limit [37]. Despite this, in a recent classification of heterotic supergravity [38], the only compact, regular solutions with eight supersymmetries are shown to be $\mathrm{AdS}_3 \times S^3 \times CY_2$ [39].

It can be expected our simply stated results will be of interest to anyone studying the holography of 2D $\mathcal{N} = (0,4)$ CFTs.

II. $SO(2,2) \times SO(3)$-INVARIANT SPACETIMES

We recall that bosonic sector of 11D supergravity consists of a metric, g, and a three-form potential, C, with four-form field strength, $G = \mathrm{d}C$. The equations of motion follow from the action

$$S = \frac{1}{2\kappa^2}\int *R - \frac{1}{2}G \wedge *G - \frac{1}{6}C \wedge G \wedge G. \quad (1)$$

Supersymmetric solutions satisfy the Killing spinor equation (KSE):

$$\nabla_M \eta + \frac{1}{288}[\Gamma_M^{NPQR} - 8\delta_M^N \Gamma^{PQR}]G_{NPQR}\eta = 0, \quad (2)$$

where $M, N = 0, \ldots, 10$, $\nabla_M \eta \equiv \partial_M \eta + \frac{1}{4}\omega_{MNP}\Gamma^{NP}\eta$, with spin connection ω, and η is a Majorana Killing spinor. It is well known that the Einstein equation is implied by the KSE once the Bianchi identity, $\mathrm{d}G = 0$, and equation of motion for C hold [40].

2D $\mathcal{N} = (0,4)$ CFTs enjoy both $SO(2,2)$ conformal symmetry and $SU(2) \simeq SO(3)$ R symmetry, which motivates the general ansatz

$$\mathrm{d}s^2 = e^{2A}\left[\frac{1}{m^2}\mathrm{d}s^2(\mathrm{AdS}_3) + e^{2B}\mathrm{d}s^2(S^2) + \mathrm{d}s^2(\mathcal{M}_6)\right],$$
$$G = \frac{1}{m^3}\mathrm{vol}(\mathrm{AdS}_3) \wedge \mathcal{A} + \mathrm{vol}(S^2) \wedge \mathcal{H} + \mathcal{G}, \quad (3)$$

where m is the inverse AdS_3 radius, A, B denote scalar warp factors and $\mathcal{A}$, $\mathcal{H}$, $\mathcal{G}$ are respectively closed one-, two- and four-forms. The curvatures of symmetric spaces are canonically normalized and fields depend only on the coordinates of the internal 6D Riemannian manifold $\mathcal{M}_6$.

In order to characterize the internal space and the fields, we decompose the 11D gamma matrices [41],

$$\Gamma_\mu = \tau_\mu \otimes \sigma_3 \otimes \gamma_7,$$
$$\Gamma_\alpha = 1_2 \otimes \sigma_\alpha \otimes \gamma_7,$$
$$\Gamma_m = 1_2 \otimes 1_2 \otimes \gamma_m, \quad (4)$$

and 11D Killing spinor,

$$\eta = \psi \otimes e^{A/2}[\chi_+ \otimes \epsilon_+ + \chi_- \otimes \epsilon_-], \quad (5)$$

where $\mu = 0, 1, 2$ label AdS_3 directions, $\alpha = 1, 2$ denote those of S^2, $m = 1, \ldots, 6$ correspond to $\mathcal{M}_6$ and we define $\gamma_7 \equiv i\gamma_{123456}$. ψ is a solution to the AdS_3 KSE, $\nabla_\mu \psi = \frac{1}{2}\tau_\mu \psi$, resulting in Poincaré spinors of definite chirality, while $\chi_\pm$ denote an $SU(2)$-doublet satisfying the KSE on S^2, $\nabla_\alpha \chi_\pm = \pm\frac{i}{2}\sigma_\alpha \chi_\pm$, with $\chi_- = \sigma_3 \chi_+$. It is a common feature of Refs. [30,41,42] that the Majorana condition is not manifest, however conjugate spinors, η^c, may easily be constructed e.g. [43]. Following the decomposition through, one determines the effective 6D KSE equations in terms of $\epsilon_\pm$ [41] and recasts them in terms of conditions on differential forms [44], which we illustrate later.

We stress that there is *a priori* no relation between $\epsilon_\pm$, even if one is to be expected [45]. In related work, the authors of Ref. [42] simplified the problem by omitting a term in the four-form flux, which enabled a simplification of the KSE analysis, before showing that the omitted term could not be reconciled perturbatively. This term was later ruled out in general [46]. In the current setting, this simplification involves fixing $\mathcal{A} = \mathcal{G} = 0$. However, since geometries with nonzero $\mathcal{A}$, $\mathcal{H}$, $\mathcal{G}$ can be generated via T-duality [27], this simplification is difficult to motivate.

III. SUPERSYMMETRY CONDITIONS

We review the salient conditions on bilinears, defined in the Appendix, which we construct from spinors $\epsilon_\pm$ [41], which encapsulate the local supersymmetry conditions we must solve. First, supersymmetry demands that the following bilinears vanish [41],

$$W^- = X^+ = \mathrm{Re}(Y) = \tilde{Z} = 0. \quad (6)$$

Moreover, the remaining bilinears are constrained:

$$2mV^+ + e^{-B}\mathrm{Im}(Y) = \frac{e^{-3A}}{2}\left[\frac{1}{2!}\mathrm{Im}(L^3)_{mn}(*_6\mathcal{G})^{mn} + K^+_m\mathcal{A}^m\right], \quad (7)$$

$$\tilde{Y} = -\frac{i}{2me^B}W^+, \qquad Z = -\frac{i}{2me^B}X^-. \quad (8)$$

Thus, there are only three real scalars, $V^\pm$, W^+, and one complex scalar, X^-, which can be independent.

From the vector spinor bilinears, one can identify four *real* Killing vectors on $\mathcal{M}_6$ [41], three of which, $\mathrm{Im}(\tilde{K}^3)$, $\mathrm{Re}(K^4)$ and $\mathrm{Im}(K^4)$ extend to symmetries of the overall solution (3). In contrast, the S^2 warp factor (also $\mathcal{H}$) depends on $\tilde{K}^+$, thus hinting at spacetimes with larger superisometry groups [47]. Thankfully, $\tilde{K}^+$ may be truncated out consistently provided $V^- = 0$, i.e. for 6D spinors $\epsilon_\pm$ with equal norm. Henceforth, we consider $\epsilon_+^\dagger\epsilon_+ = \epsilon_-^\dagger\epsilon_-$, so that $V^- = \tilde{K}^+ = 0$.

The scalars satisfy differential constraints [41],

$$\mathrm{d}V^+ = 0, \quad (9)$$

$$\mathrm{d}[e^{-B}\mathrm{Im}(Y)] = 0, \quad (10)$$

$$e^{-3A}\mathrm{d}[e^{3A}X^-] = -2m\tilde{K}^4, \quad (11)$$

$$e^{-3A}\mathrm{d}[e^{3A}W^+] = 2m\mathrm{Re}(K^3), \quad (12)$$

while the vectors must satisfy

$$\mathrm{d}[e^{3A+B}K^-] = -e^{-B}\mathrm{Im}(Y)\mathcal{H} + e^{3A}\tilde{L}^1, \quad (13)$$

$$\mathrm{d}[e^{6A+B}\mathrm{Re}(\tilde{K}^3)] = -e^{3A+B}\mathrm{Im}(Y) *_6 \mathcal{G} - e^{3A+B}\mathcal{A}\wedge K^- - e^{6A}\mathrm{Im}(L^3), \quad (14)$$

$$\mathrm{d}[e^{6A+2B}\mathrm{Im}(\tilde{K}^3)] = -e^{3A}W^+\mathcal{H} + 2me^{6A+2B}L^1 + 2e^{6A+B}\mathrm{Re}(L^3), \quad (15)$$

$$\mathrm{d}[e^{6A+2B}K^4] = ie^{3A}X^-\mathcal{H} + 2me^{6A+2B}L^6 - 2ie^{6A+B}\tilde{L}^4. \quad (16)$$

We have removed all trivial bilinears and conditions that play no role in our analysis [48]. With $\tilde{Y}$ pure imaginary from (8), and $\tilde{K}^+$ zero, $\mathcal{A}$ and $\mathcal{G}$ are fully determined in terms of bilinears:

$$\mathcal{A} = \frac{2me^{3A}}{V^+}K^+, \qquad \mathcal{G} = \frac{2me^{3A}}{V^+} *_6 \mathrm{Im}(L^3). \quad (17)$$

This ends our review of the supersymmetry conditions of Ref. [41]. We will now solve the conditions by evoking G-structures to characterize the internal manifold $\mathcal{M}_6$.

We introduce two unit-norm, chiral spinors, ξ_i, which are orthogonal, $\xi_i^\dagger\xi_j = \delta_{ij}$. Each chiral spinor individually defines an $SU(3)$-structure. To see this, we introduce projection conditions,

$$\gamma_{12}\xi_1 = \gamma_{34}\xi_1 = \gamma_{56}\xi_1 = i\xi_1 \qquad \Rightarrow \gamma_7\xi_1 = \xi_1, \qquad -\gamma_{135}\xi_1 = \xi_1^*, \quad (18)$$

permitting us to specify the $SU(3)$-structure through a two-form $J_i^{(3)} = -\frac{i}{2}\xi_i^\dagger\gamma_{mn}\xi_i e^m\wedge e^n$ and (3,0)-form $\Omega_i^{(3)} = -\frac{1}{3!}\xi_i^T\gamma_{mnp}\xi_i e^m\wedge e^n\wedge e^p$. With the second spinor, $\gamma_5\xi_2^* = \xi_1$, whose projection conditions follow from (18), we can define two canonical $SU(3)$-structures, with forms $(J_1^{(3)}, \Omega_1^{(3)})$ and $(J_2^{(3)}, \Omega_2^{(3)})$, or equivalently, a canonical $SU(2)$-structure, which is specified by three two-forms and two one-forms:

$$J^\alpha = -\frac{i}{4}(\sigma^\alpha)^{ij}\xi_i^\dagger\gamma_{mn}\xi_j e^m\wedge e^n, \quad (19)$$

$$K^1 + iK^2 = -\frac{1}{2}\epsilon^{ij}\xi_i^T\gamma_m\xi_j e^m, \quad (20)$$

where σ^α, $\alpha = 1, 2, 3$ denote Pauli matrices. In general, we expand $\epsilon_\pm$ in terms of the chiral spinors and their conjugates

$$\epsilon_+ = \alpha_1\xi_1 + \alpha_2\xi_1^* + \alpha_3\xi_2 + \alpha_4\xi_2^*, \qquad \epsilon_- = \beta_1\xi_1 + \beta_2\xi_1^* + \beta_3\xi_2 + \beta_4\xi_2^*, \quad (21)$$

where $\alpha_i, \beta_i \in \mathbb{C}$. Modulo phases of ξ_i, these are the most general spinors consistent with $SU(2)$-structure.

IV. $SU(2)$-STRUCTURE MANIFOLDS

As a warm-up, we consider $SU(3)$-structure manifolds by simply eliminating, α_3, α_4, β_3, β_4, so that only ξ_1 remains in (21). We recall that $SU(3)$-structure manifolds are classified according to five torsion classes W_i [49]. We will now demonstrate that all torsion classes vanish, so Calabi-Yau is the only $\mathcal{M}_6$ with $SU(3)$-structure.

One can use the constraints from vanishing scalars (6) to infer, $\epsilon_- = \pm i\epsilon_+$, where ϵ_+ need not be chiral. If it is chiral, the argument reverts to Ref. [41]; if nonchiral, since $K^+ = \mathrm{Im}(L^3) = 0$, $\mathcal{A}$ and $\mathcal{G}$ also are zero. Next, from (7), we deduce $2me^B = \mp 1$, and from (13), $\mathcal{H} = \pm e^{3A}/(2m)J^{(3)}(V^+ = 1)$. Further differentiating (13)–(16), we can directly confirm that $\mathrm{d}J^{(3)} = \mathrm{d}\Omega^{(3)} = 0$.

We now turn to the generic case. Evaluating the vector bilinears in terms of α_i, β_i using (21), we find that K^- and $\tilde{K}^-$ are orthogonal allowing us without loss of generality to align them with the e^5, e^6 axes of the internal space.

Moreover, imposing (6), and $V^- = \tilde{K}^+ = 0$, we determine the following relations:

$$\beta_1 = -\frac{[\alpha_3(\alpha_4^2+\beta_4^2)+\alpha_1(\beta_2\beta_4+\alpha_2\alpha_4)]}{(\beta_2\alpha_4-\alpha_2\beta_4)},$$
$$\beta_3 = \frac{[\alpha_1(\alpha_2^2+\beta_2^2)+\alpha_3(\beta_2\beta_4+\alpha_2\alpha_4)]}{(\beta_2\alpha_4-\alpha_2\beta_4)},$$
$$if_1 = \beta_2\alpha_2^* + \beta_4\alpha_4^*, \qquad f_2 = \alpha_1^*\alpha_4 + \alpha_2^*\alpha_3, \tag{22}$$

where $f_i \in \mathbb{R}$ are yet to be determined. With $SU(2)$-structure, it follows from $\tilde{K}^+ = 0$ that $K^+ = 0$ and, as a result of (17), $\mathcal{A} = 0$, i.e. no electric flux. As another consequence of these relations, we discover $\mathrm{Re}(\tilde{K}^3) = 0$, which through (14) and (17) leads to the constraint

$$Y = -\frac{i}{2me^B}V^+. \tag{23}$$

Since V^+ is a constant, so too is e^B through (10).

We can now combine this with (8) to find that

$$\left[\beta_4 + \frac{i}{2me^B}\alpha_4\right](f_2 - \alpha_2^*\alpha_3 - \alpha_3^*\alpha_2) = 0. \tag{24}$$

If we impose the vanishing of the first bracket, through the constraints it follows that $\mathcal{G} = 0$ and $\beta_i = -i\alpha_i$, i.e. $\epsilon_- = -i\epsilon_+$, so that we recover Calabi-Yau. To find something new, we impose the second condition, which implies $K^- = \tilde{K}^- = 0$. We recall that these are the original vectors that we aligned with the axes, so now we have the freedom to choose K^3 and $\tilde{K}^3$, which are orthogonal, and rotate them to align with the axes. Doing so, we find it is possible to solve for all the spinor coefficients so that our constraints are satisfied:

$$\alpha_1 = \sqrt{V^+}\cos\frac{\zeta}{2}\cos\frac{\theta}{2}e^{i\varphi_1}, \qquad \alpha_2 = \sqrt{V^+}\sin\frac{\zeta}{2}\cos\frac{\theta}{2}e^{i\varphi_2},$$
$$\alpha_3 = \sqrt{V^+}\cos\frac{\zeta}{2}\sin\frac{\theta}{2}e^{i\varphi_3}, \qquad \alpha_4 = \sqrt{V^+}\sin\frac{\zeta}{2}\sin\frac{\theta}{2}e^{i\varphi_4},$$
$$\beta_1 = \frac{1}{2}\left(\frac{L}{R_2}\cot\frac{\zeta}{2}\alpha_4 - i\frac{L}{R_1}\alpha_1\right), \qquad \beta_4 = \frac{\beta_2}{\beta_1^*}\beta_3^*,$$
$$\beta_3 = -\frac{1}{2}\left(\frac{L}{R_2}\cot\frac{\zeta}{2}\alpha_2 + i\frac{L}{R_1}\alpha_3\right), \qquad \beta_2 = -\frac{\alpha_2}{\alpha_1^*}\beta_1^*, \tag{25}$$

where $\varphi_1+\varphi_2 = \varphi_3+\varphi_4$ and we have redefined $m = L^{-1}$, $R_1 = e^B$, $R_2 = e^B/\sqrt{4m^2e^{2B}-1}$. With these expressions, we determine $W^+ = V^+\cos\zeta$, $X^- = V^+\sin\zeta e^{i\varphi_1+i\varphi_2}$ and solve (11) and (12) to show the warp factor e^A is a constant and

$$e^5 = -R_2\mathrm{d}\zeta, \qquad e^6 = -R_2\sin\zeta\mathrm{d}\chi, \tag{26}$$

where we have defined $\mathrm{d}\chi = \mathrm{d}(\varphi_1+\varphi_2)$. This allows us to identify the one-forms dual to the Killing vectors,

$$\mathrm{Im}(\tilde{K}^3) = -\frac{LV^+}{2}\sin^2\zeta\mathrm{d}\chi,$$
$$K^4 = -\frac{LV^+}{2}e^{i\chi}(\mathrm{d}\zeta + i\cos\zeta\mathrm{d}\chi), \tag{27}$$

which correspond to an emergent $SU(2)$. We can ensure the Killing vectors are canonically normalized through the choice $V^+ = 2R_2^2/L$. Solving the remaining supersymmetry conditions, one arrives at the conclusion that χ aside, the other angular parameters are constant, with $\mathcal{M}_6$ being a direct product of S^2 and CY_2, more concretely T^4 or K_3. The final expression for the four-form flux reads

$$G = \frac{2e^{3A}}{LV^+}[-R_1^2\tilde{L}^1\wedge\mathrm{vol}(S^2) + *_6\mathrm{Im}(L^3)]. \tag{28}$$

It is easy to check that the equations of motion are satisfied, in line with expectations [40]. We also see that both ξ_1, ξ_2 and conjugates need to appear in the spinor. This may be contrasted with the spinor considered in Ref. [50], which is not the most general, and would appear to preclude this outcome. For this reason, setting $\beta_4 = \alpha_4 = 0$ in (24), one recovers the results of existing classifications [29,30]. Setting $A = 0$, since the overall warp factor is constant, we can confirm the radii satisfy

$$\frac{4}{L^2} = \frac{1}{R_1^2} + \frac{1}{R_2^2}. \tag{29}$$

The ratio between S^2 radii, α, corresponds to the supergroup $D(2,1;\alpha)$, with bosonic subgroup $SL(2,\mathbb{R})\times SU(2)\times SU(2)$.

To establish the connection to minimal ungauged supergravity in 7D [51], we exploit the following consistent Kaluza-Klein reduction ansatz:

$$\mathrm{d}s_{11}^2 = e^{-\frac{8}{5}B}\mathrm{d}s_7^2 + e^{2B}\mathrm{d}s^2(CY_2),$$
$$G = F + \sum_{a=1}^{3}F^a\wedge J^a, \tag{30}$$

where J^a denote the three self-dual harmonic two-forms of CY_2, B is a scalar and F and F^a are respectively field strengths corresponding to a three-form and one-form potentials, $F = \mathrm{d}C$, $F^a = \mathrm{d}A^a$. The resulting action in Einstein frame in 7D is

$$\mathcal{L}_7 = R\mathrm{vol}_7 - \frac{36}{5}\mathrm{d}B\wedge *_7\mathrm{d}B - \frac{1}{2}e^{\frac{24}{5}B}F\wedge *_7F$$
$$- e^{-\frac{12}{5}B}F^a\wedge *_7F^a - F\wedge F^a\wedge A^a. \tag{31}$$

To cast the action in the original notation of Ref. [51], one should employ the following redefinitions:

$$B = \frac{\sqrt{5}}{6}\phi, \qquad F_{\mathrm{us}} = \sqrt{2}F_{\mathrm{them}}, \qquad F^a_{\mathrm{us}} = \sqrt{2}F^a_{\mathrm{them}}. \tag{32}$$

V. DISCUSSION

We have initiated a classification of all solutions to 11D supergravity with $SO(2,2)\times SO(3)$ isometry. This is the simplest geometric signature of a supergravity solution dual to a 2D CFT with $\mathcal{N}=(0,4)$ supersymmetry, including the MSW CFT. In the process, we have identified a novel class of near-horizon geometries in M-theory with large superconformal symmetry. Compactifying M-theory on CY_2, we identify a resulting $\mathrm{AdS}_3\times S^2\times S^2$ vacuum to 7D supergravity, thus providing a candidate near-horizon for an extremal black hole and a potential new controlled setting to count black hole microstates.

The M-theory geometry provides a unifying description of well-known $\mathrm{AdS}_3\times S^3\times S^3\times S^1$ geometries of type II string theory through T-duality [27] and heterotic vacua via M-theory/heterotic duality [36]. A careful treatment of the central charge reveals the expected form of a large superconformal algebra [52]

$$c \sim \frac{k^+k^-}{k^+ + k^-}, \tag{33}$$

with affine $SU(2)^\pm$ current algebras at levels $k^\pm$ related to the quantized charges, yet where $c\sim N^2$, for large charge N, and not the more usual $c\sim N^3$ of geometries corresponding to M5-branes.

Our work has two interesting implications. First, it is striking that the $\mathrm{AdS}_3\times S^2\times S^2\times CY_2$ geometries are not identifiable as AdS_3 limits of wrapped M5-branes [29]. This suggests the M5-brane picture is novel and motivates further study to understand anomaly inflow [11]. Second, as we have shown, since 11D supergravity compactifies on CY_2 to 7D minimal supergravity, the $\mathrm{AdS}_3\times S^2\times S^2$ solution hints at being the near-horizon of an extremal black hole. While such solutions have in principle been classified [53], we are not aware of a near-horizon uniqueness theorem in 7D, cf. [54]. Assuming a black hole exists, strong parallels to the MSW case, with M-theory compactified on Calabi-Yau, are expected to facilitate a microscopic derivation of the entropy. Since the small superconformal algebra is recovered from the large one through a decompactification of a two-sphere, it is tempting to speculate that contact with the MSW results may be made in the same limit.

Last, we remark that we have assumed $SU(2)$-structure, and more general solutions with identity structure are known to exist [27]. We hope to extend the classification to consider more general internal manifolds in future work [52].

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APPENDIX: SPINOR BILINEARS

In our conventions, the 6D gamma matrices are Hermitian $\gamma_m^\dagger = \gamma_m$ and antisymmetric $\gamma_m^T = -\gamma_m$. Consistent with the symmetries of the gamma matrices, given $\epsilon_\pm$, we can define an exhaustive set of scalar

$$\begin{aligned}
V^\pm &= \frac{1}{2}(\epsilon_+^\dagger\epsilon_+ \pm \epsilon_-^\dagger\epsilon_-),\\
W^\pm &= \frac{1}{2}(\epsilon_+^\dagger\gamma_7\epsilon_+ \pm \epsilon_-^\dagger\gamma_7\epsilon_-),\\
X^\pm &= \frac{1}{2}(\epsilon_+^T\epsilon_+ \pm \epsilon_-^T\epsilon_-),\\
Y &= \epsilon_+^\dagger\epsilon_-, \qquad \tilde{Y} = \epsilon_+^\dagger\gamma_7\epsilon_-,\\
Z &= \epsilon_+^T\epsilon_-, \qquad \tilde{Z} = \epsilon_+^T\gamma_7\epsilon_-,
\end{aligned} \tag{A1}$$

and vector spinor bilinears:

$$\begin{aligned}
K_m^\pm &= \frac{1}{2}(\epsilon_+^\dagger\gamma_m\epsilon_+ \pm \epsilon_-^\dagger\gamma_m\epsilon_-),\\
\tilde{K}_m^\pm &= \frac{i}{2}(\epsilon_+^\dagger\gamma_m\gamma_7\epsilon_+ \pm \epsilon_-^\dagger\gamma_m\gamma_7\epsilon_-),\\
K_m^3 &= \epsilon_+^\dagger\gamma_m\epsilon_-, \qquad \tilde{K}_m^3 = \epsilon_+^\dagger\gamma_m\gamma_7\epsilon_-,\\
K_m^4 &= \epsilon_+^T\gamma_m\epsilon_-, \qquad \tilde{K}_m^4 = \epsilon_+^T\gamma_m\gamma_7\epsilon_-,
\end{aligned} \tag{A2}$$

where factors of i ensure vectors are real. We define the following two-forms:

$$\begin{aligned}
L_{mn}^1 &= \frac{i}{2}(\epsilon_+^\dagger\gamma_{mn}\epsilon_+ + \epsilon_-^\dagger\gamma_{mn}\epsilon_-),\\
\tilde{L}_{mn}^1 &= \frac{i}{2}(\epsilon_+^\dagger\gamma_{mn}\gamma_7\epsilon_+ + \epsilon_-^\dagger\gamma_{mn}\gamma_7\epsilon_-),\\
L_{mn}^3 &= \epsilon_+^\dagger\gamma_{mn}\epsilon_-, \qquad \tilde{L}_{mn}^4 = \epsilon_+^T\gamma_{mn}\gamma_7\epsilon_-,\\
L_{mn}^6 &= \frac{1}{2}(\epsilon_+^T\gamma_{mn}\gamma_7\epsilon_+ - \epsilon_-^T\gamma_{mn}\gamma_7\epsilon_-),
\end{aligned} \tag{A3}$$

where notation follows Ref. [41].

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3.4 Three-dimensional $\mathcal{N} = 4$ linear quivers and non-Abelian T-duals

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Three-dimensional $\mathcal{N} = 4$ linear quivers and non-Abelian T-duals

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Abstract: In this paper we construct a new Type IIB background with an AdS_4 factor that preserves $\mathcal{N} = 4$ Supersymmetry. This solution is obtained using a non-Abelian T-duality transformation on the Type IIA reduction of the $AdS_4 \times S^7$ background. We interpret our configuration as a patch of a more general background with localised sources, dual to the renormalisation fixed point of a $T^{\hat{\rho}}_{\rho}(\mathrm{SU}(N))$ quiver field theory. This relates explicitly the AdS_4 geometry to a D3-D5-NS5 brane intersection, illuminating what seems to be a more general phenomenon, relating AdS_{p+1} backgrounds generated by non-Abelian T-duality to Dp-$D(p+2)$-NS5 branes intersections.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence

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Contents

1 Introduction

The idea of duality is very old, perhaps dating back to the (self) duality of the Maxwell equations in the absence of charges and currents. The transformation of the fields describing a given dynamics into a *different* set of fields where particular phenomena become more apparent, is a recurrent idea in Theoretical Physics. Indeed, dualities like those proposed by Montonen and Olive [1], Seiberg and Witten [2, 3], Seiberg [4], or the U-duality web in String Theory (see for example [5, 6]) are examples of this. While these dualities are very hard to prove (hence initially conjectured), they have very far reaching consequences in Physics: the phenomena that in one description are highly fluctuating and hence eminently quantum mechanical, become semiclassical and characteristically weakly coupled in the dual set of variables. The AdS/CFT duality [7–9] relating gauge theories and String theories is a paradigmatic example of this.

Other dualities, like the Kramers-Wannier self duality of the two-dimensional Ising model [10], bosonisation in two dimensions [11, 12] or T-duality in the String Theory sigma model [13–15], are within the class of dualities that can be formally proven.

In 1993, Quevedo and de la Ossa [16], following ideas in [15], proposed a non-Abelian generalisation of T-duality, applicable to the Neveu-Schwarz sector of the string sigma model. This was later complemented by Sfetsos and Thompson, who showed how to transform the fields in the Ramond sector [17]. This important work opened the way for further study involving new backgrounds and illuminating some geometrical and dual field theoretic aspects of the non-Abelian T-duality [18–23]–[44]. These works have in turn motivated the search for new classes of supersymetric AdS solutions that were overlooked until recently [45]–[49].

Whilst the sigma-model procedure to calculate the non-Abelian T-dual of a given background is apparently straightforward, many interesting subtleties related to global aspects and invertibility of the duality arise. These subtle aspects were studied in the mid-nineties but not completely resolved, in spite of many serious attempts [50–52]–[55]. Some of such concrete problems are the (im)possibility of extending the non-Abelian duality procedure to all orders in string perturbation theory and α', and the determination of the range of the coordinates and topology of the dual manifold. These issues cast doubts about the 'duality-character' of the non-Abelian T-duality transformation.

One goal of this paper — elaborating on ideas introduced in [44] — is to get information on some of the global problems mentioned above. The example we will consider here involves a Type IIB background with an AdS_4 factor, preserving $\mathcal{N} = 4$ Supersymmetry.

A second goal of this paper — of interest in a broader context — will be to produce a new analytic solution to the Type IIB Supergravity equations of motion with an AdS_4 factor, that can be interpreted as an intersection of D3-D5-NS5 branes. Our example illuminates what is surely a more general phenomenon, relating AdS_{p+1} geometries generated by non-Abelian T-duality with Dp-$D(p+2)$-NS5 branes intersections — see for example [45, 56, 57] for other recent studies of such configurations.

Furthermore, our case-study provides an interesting arena where the CFT interpretation of non-Abelian T-duality put forward in [44] can be tested. Indeed, using the results in [58, 59] (see also [60]), which elaborate on certain limits of Type IIB Supergravity solutions discussed in [61, 62], it is possible to associate a concrete CFT dual to our AdS_4 solution. This will be a $\mathcal{N} = 4$, d=3 conformal field theory, arising as the Renormalisation Group fixed point of a $T^{\hat{\rho}}_{\rho}(\mathrm{SU}(N))$ quantum field theory that belongs to the general class introduced by Gaiotto and Witten in [63]. These conformal field theories can be described in terms of a linear quiver with bi-fundamental and fundamental matter or, equivalently, in terms of Hanany-Witten set ups [64] containing D3, NS5 and D5 branes.

This work extends the ideas in [44] to the AdS_4/CFT_3 case. The paper [44] deals with the *singular* background obtained by the application of non-Abelian T-duality on $AdS_5 \times S^5$ and its interpretation as a Gaiotto-Maldacena type of geometry [65]. Using the formal developments of [66, 67], a *completion* to the geometry generated by non-Abelian duality was proposed, with the following relevant properties:

- It is a smooth background, except at isolated points where brane sources are located.
- The dual CFT is known explicitly.

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- The coordinates of the completed geometry have a definite range, determined by imposing the matching between observables calculated with the CFT and with the geometrical description.
- The original non-Abelian T-dual background (that is, the geometry before *completion*) can be seen as a *zoom-in* on a patch of the completed manifold.

In this paper, we will use a combination of insights from three-dimensional $\mathcal{N} = 4$ CFTs and their dual geometries to obtain a similar understanding of an AdS_4 Type IIB background, obtained by the action of non-Abelian T-duality on the Type IIA reduction of $AdS_4 \times S^7$. An outline of this works goes as follows.

In section 2, we present our (new) background, analyse the amount of SUSY preserved and study the structure of its singularities. The calculation of the associated charges leads us to a proposal for the Hanany-Witten set-up [64]. In section 3 we discuss aspects of $\mathcal{N} = 4$ SCFTs in three dimensions. The associated backgrounds containing an AdS_4 sub-manifold are also discussed. In sections 4 and 5, we embed our non-abelian T-dual geometry into the formalism of [58] (ABEG hereafter). This leads us to a precise proposal for the CFT dual to our background. We interpret our singular solution as embedded in a more generic background (with the characteristics itemized above). Section 6 discusses the subtle calculation of the free energy for the CFT defined by the non-abelian T-dual geometry. Conclusions and some further directions to explore are collected in section 7. Appendix A summarises the main properties of the Abelian T-dual limit of the non-Abelian solution, of relevance for the interpretation of the free energy. Finally, appendix B contains an interesting general relation between Abelian and non-Abelian T-duals.

2 The Type IIB $\mathcal{N} = 4$ AdS_4 solution

In this section we present the new type IIB $\mathcal{N} = 4$ AdS_4 background where our ideas will be tested. It is generated from the maximally supersymmetric $AdS_4 \times S^7$ solution in M-theory (once reduced to Type IIA), through a non-Abelian T-duality transformation.

To begin we parametrise the M-theory solution such that we manifestly have two three-spheres S_1^3 and S_2^3, as

$$
\begin{aligned}
ds^2_{11d} &= ds^2(AdS_4) + 4L^2\left(\frac{1}{4}d\mu^2 + \sin^2\left(\frac{\mu}{2}\right) ds^2\left(S_1^3\right) + \cos^2\left(\frac{\mu}{2}\right) ds^2\left(S_2^3\right)\right),\\
G_4 &= \frac{3\rho^2}{L^3} dt\wedge dx_1\wedge dx_2\wedge d\rho = \frac{3}{L}\,\mathrm{Vol}(AdS_4), \qquad ds^2(AdS_4) = \frac{\rho^2}{L^2}dx_{1,2}^2 + L^2\frac{d\rho^2}{\rho^2},
\end{aligned}
\tag{2.1}
$$

where as usual for AdS_4 Freund-Rubin solutions the AdS and internal radii obey the relation $R_{S^7} = 2R_{AdS_4}$. We take the three-spheres to have unit radius, which means $\mu \in [0, \pi)$. With the above parametrisation there is enough symmetry to reduce to IIA within one of the three spheres and then perform a T-duality transformation on the other. Here we will focus on performing an SU(2) non-Abelian T-duality on the residual SU(2). We also give details of the Hopf fibre T-dual in appendix A.

We want to reduce to Type IIA on the Hopf direction of S_2^3 by parametrising it as

$$4ds(S_2^3) = d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 + (d\psi_2 + \cos\theta_2 d\phi_2)^2, \tag{2.2}$$

with $\psi_2 \in [0, 4\pi]$. Since some supersymmetry will be broken in the process, as the isometry parametrised by ∂_{ψ_2} defines a U(1) subgroup of the full SO(8) R-symmetry, we briefly study the Killing spinor equations. To this end we introduce the manifestly $\mathrm{U}(1)_{\psi_2}$ invariant vielbein,

$$\begin{aligned}
&e^{xi} = \frac{R}{L}dx_i \;\; (i = t, x_1, x_2), \quad e^R = \frac{L}{R}dR,\\
&e^\mu = Ld\mu, \quad e^1 = L\sin\left(\frac{\mu}{2}\right)\omega_1, \quad e^2 = L\sin\left(\frac{\mu}{2}\right)\omega_2, \quad e^3 = L\sin\left(\frac{\mu}{2}\right)\omega_3.\\
&e^{\theta_2} = L\cos\left(\frac{\mu}{2}\right)d\theta_2, \quad e^{\phi_2} = L\cos\left(\frac{\mu}{2}\right)\sin\theta_2 d\phi_2, \quad e^{\psi_2} = L\cos\left(\frac{\mu}{2}\right)(d\psi_2 + \cos\theta_2 d\phi_2).\\
&F_4 = \frac{3}{L}e^{tx_1x_2\rho},
\end{aligned} \tag{2.3}$$

where

$$\omega_1 + i\omega_2 = e^{i\psi_1}\left(id\theta_1 + \sin\theta_1 d\phi_1\right), \qquad \omega_3 = d\psi_1 + \cos\theta_1 d\phi_1, \tag{2.4}$$

which makes manifest an additional SU(2) isometry parametrised by S_1^3. The gravitino variation on S^7 is given in flat indices by[1]

$$\nabla_a\eta + \frac{1}{4L}\Gamma_a\hat{\gamma}\eta = 0, \tag{2.5}$$

where $\hat{\gamma} = \Gamma_t\Gamma_{x_1}\Gamma_{x_2}\Gamma_\rho$. The number of preserved supercharges is determined by what fraction of the initial 32 SUSYs are consistent with setting $\partial_{\psi_2}\eta = 0$ in the frame of eq. (2.3). One can show by imposing that η is independent of ψ_2 that one is lead to a single projection that the Killing spinor must obey,

$$\Gamma_{\mu\theta_2\phi_2\psi_2}\eta = -\left(\cos\left(\frac{\mu}{2}\right) + \sin\left(\frac{\mu}{2}\right)\hat{\gamma}\Gamma_\mu\right)\eta, \tag{2.6}$$

which breaks supersymmetry by half, leaving 16 real supercharges preserved by the reduction to Type IIA. In fact the projection also makes the Killing spinor independent of $(\theta_1, \phi_1, \psi_1)$ in the frame of eq. (2.3) and independent of ψ_1 in any frame in which the Hopf isometry of S_1^3 is manifest. These are precisely the conditions for supersymmetry to be unbroken under SU(2) and U(1) T-duality transformations respectively [35, 68]. So 16 supercharges will remain in Type IIB after either of these duality transformations, enough for this background to be dual to a three-dimensional $\mathcal{N} = 4$ SCFT.

2.1 Reduction of $\mathbb{Z}_k$ orbifold to IIA

Let us now proceed with the reduction on ψ_2 with a slight generalisation. Let us reduce the $\mathbb{Z}_k$ orbifold of S_2^3. This has the effect of generating a stack of k D6 branes in Type IIA while leaving the supersymmetry arguments unchanged.

[1]That the AdS_4 directions solve is a standard exercise that we omit for brevity.

Taking the $\mathbb{Z}_k$ orbifold, amounts to sending $S_2^3 \to S_2^3/\mathbb{Z}_k$ in eq. (2.1) with,

$$4ds(S_2^3/\mathbb{Z}_k) = d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 + \frac{4}{k^2}\left(d\psi_2 + \frac{k}{2}\cos\theta_2 d\phi_2\right)^2, \tag{2.7}$$

where ψ_2 now has period 2π. Setting $l_p = \alpha' = g_s = 1$ leads to the type IIA solution,

$$\begin{aligned} ds^2_{IIA} &= e^{\frac{2}{3}\phi_0}\cos\left(\frac{\mu}{2}\right)\left[ds^2(AdS_4) + 4L^2\left(\frac{1}{4}d\mu^2 + \sin^2\left(\frac{\mu}{2}\right)ds^2(S_1^3) + \frac{1}{4}\cos^2\left(\frac{\mu}{2}\right)ds^2(S_2^2)\right)\right],\\ F_4 &= \frac{3}{L}\mathrm{Vol}(AdS_4), \qquad F_2 = -\frac{k}{2}\mathrm{Vol}(S_2^2), \qquad e^{2\Phi_0} = e^{2\phi_0}\cos^3\left(\frac{\mu}{2}\right), \qquad e^{\frac{2}{3}\phi_0} = \frac{2L}{k}. \end{aligned} \tag{2.8}$$

The reduction has generated a singularity at $\mu = \pi$, but this has a physical interpretation, it is due to the k D6 branes mentioned earlier. Indeed close to $\mu = \pi$ the metric has the form,

$$ds^2 \sim \frac{e^{2\phi_0/3}}{2}\left[\sqrt{\nu}\left(ds^2(AdS_4) + 4L^2 ds^2(S_1^3)\right) + \frac{L^2}{4\sqrt{\nu}}\left(d\nu^2 + \nu^2 ds^2(S_2^2)\right)\right], \quad e^{\Phi} \sim \frac{e^{\phi_0}\nu^{3/4}}{2\sqrt{2}}, \tag{2.9}$$

for $\nu = (\pi - \mu)^2$. We see that the reduction has generated D6 branes that extend along AdS_4, wrap S_1^3 and are localised at $\mu = \pi$. Of course this was to be expected as D6 brane singularities are always generated anywhere the M-theory circle shrinks to zero size.

Before moving on, let us quote the D-brane charges,

$$N_{\mathrm{D2}} = \frac{1}{2\kappa_{10}^2 T_2}\int \star F_4 = \frac{2L^6}{k\pi^2}, \qquad Q_{\mathrm{D6}} = \frac{1}{2\kappa_{10}^2 T_4}\int_{S_2^2} F_2 = k. \tag{2.10}$$

In our conventions $2\kappa_{10}^2 T_{Dp} = (2\pi)^{7-p}$. We thus set

$$L^6 = \frac{k\pi^2 N_{\mathrm{D2}}}{2}, \tag{2.11}$$

to have integer D2 brane charge. We find the expected number of D6 branes.

2.2 The non-Abelian T-dual solution

We now present the solution that will be the main focus of this work, which is the result of performing a non-Abelian T-dual transformation on the S_1^3 of eq. (2.8). Using the rules in [35], and parametrising the T-dual coordinates in terms of spherical coordinates (r, S_1^2), we generate the NS sector,

$$\begin{aligned} ds^2_{\mathrm{IIB}} &= e^{\frac{2}{3}\phi_0}\cos\left(\frac{\mu}{2}\right)\left[ds^2(AdS_4) + L^2\left(d\mu^2 + \frac{k^2}{L^6\sin^2(\mu)}dr^2 + \cos^2\left(\frac{\mu}{2}\right)ds^2(S_2^2)\right)\right]\\ &\quad + \frac{L^6}{k^2\Delta}r^2\sin^2\left(\frac{\mu}{2}\right)\sin^2(\mu)\,ds^2(S_1^2),\\ B_2 &= \frac{L^3}{k\Delta}r^3\sin\left(\frac{\mu}{2}\right)\sin(\mu)\,\mathrm{Vol}(S_1^2), \qquad e^{2\Phi} = \frac{1}{\Delta}e^{2\phi_0}\cos^3\left(\frac{\mu}{2}\right), \end{aligned} \tag{2.12}$$

where we have introduced

$$\Delta = \frac{L^3}{k^3}\sin\left(\frac{\mu}{2}\right)\sin(\mu)\left(k^2r^2 + L^6\sin^2\left(\frac{\mu}{2}\right)\sin^2(\mu)\right). \tag{2.13}$$

The solution is completed with the RR fluxes,

$$\begin{aligned}
F_3 &= \frac{1}{4}\,\mathrm{Vol}(S_2^2)\wedge d\left(kr^2 - \frac{L^6}{k}\left(\cos^2(\mu)-3\right)\cos(\mu)\right),\\
F_5 &= \mathrm{Vol}(AdS_4)\wedge d\left(\frac{L^5}{4k^2}\left(\cos(2\mu)-4\cos(\mu)\right) - \frac{3}{2L}r^2\right)\\
&\quad - \frac{L^9}{4k^2\Delta}r^2\sin^3(\mu)\sin\left(\frac{\mu}{2}\right)\mathrm{Vol}(S_1^2)\wedge\mathrm{Vol}(S_2^2)\wedge\left(3r\sin(\mu)d\mu + 2\sin^2\left(\frac{\mu}{2}\right)dr\right).
\end{aligned} \tag{2.14}$$

We have explicitly checked that the background in eqs. (2.12)–(2.14) solves the Type IIB Supergravity equations of motion, which is also implied by the result of the paper [24].

As is common to all backgrounds generated through an SU(2) non-Abelian T-duality transformation, this solution incorporates a non-compact r-direction. Moreover, this solution has two singularities. The first lies at $\mu = \pi$ and is inherited from the stack of D6 branes in IIA. Indeed, close to $\mu = \pi$ one finds

$$ds^2 \sim \frac{e^{2\phi_0/3}}{2}\left[\sqrt{\nu}\left(ds^2(AdS_4) + L^2 ds^2(S_1^2)\right) + \frac{L^2}{4\sqrt{\nu}}\left(d\nu^2 + d\tilde{r}^2 + \nu^2 ds^2(S_2^2)\right)\right],\quad e^{\Phi} \sim \frac{2\sqrt{\nu}}{L^3\tilde{r}} \tag{2.15}$$

where we have defined $\tilde{r} = 2k/L^2 r$ and $\nu = (\pi-\mu)^2$. This is almost the behaviour of the smeared D5 stack one would generate under Hopf fibre T-duality along ψ_1. The r-dependence of the dilaton however modifies this. Recalling that the dilaton is determined by a one loop effect in T-duality, which essentially amounts to imposing that $e^{-2\Phi}\,\mathrm{Vol}(\mathcal{M}_I)$ (where $\mathcal{M}_I$ is the submanifold where the duality is performed) is duality invariant, the r factor has its origin in the different volumes of the original and non-Abelian T-dual submanifolds, which are respectively S^3 and $\mathbb{R}^3$. This is manifest when we parametrise the volume of $\mathbb{R}^3$ in spherical coordinates (r, S_1^2), where r is the radial direction. The second singularity at $\mu = 0$ is also unsurprising, since we have dualised on a sphere whose radius vanishes at this point. We indeed obtain the non-Abelian T-dual analogue of smeared NS5 branes, since close to $\mu \sim 0$ we have,

$$ds^2 \sim e^{2\phi_0/3}\left[ds^2(AdS_4) + L^2 ds^2(S_2^2) + \frac{L^2}{4\nu}\left(d\nu^2 + d\tilde{r}^2 + \nu^2 ds^2(S_1^2)\right)\right],\qquad e^{\Phi} \sim \frac{8}{L^3\sqrt{\nu}\tilde{r}} \tag{2.16}$$

where now we have defined $\nu = \mu^2$ and once more it is the dependence of the dilaton on r that makes this deviate from the conventional $(\sqrt{\nu})^{-1}$ behaviour.

As previously discussed in other non-Abelian T-dual examples — see [27, 34, 36, 43], the behaviour of the solution close to the location of the NS5-branes brings in interesting information. Close to $\mu = 0$ we have $B_2 = r\,\mathrm{Vol}(S_1^2)$, with the metric spanned by (μ, S_1^2)

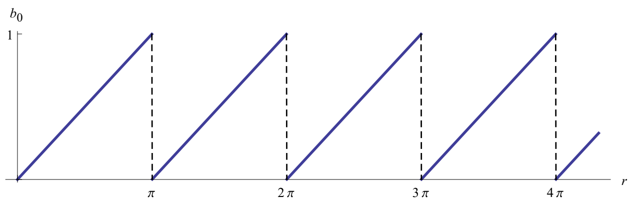

Figure 1. b_0 as a function of r.

becoming a singular cone, which defines a 2-cycle. This means that we must ensure that on this cycle S_1^2, the quantity

$$b_0 = \frac{1}{4\pi^2}\int_{S_1^2} B_2 \tag{2.17}$$

satisfies $b_0 \in [0,1]$ over the infinite range of r. This is achieved by performing a large gauge transformation $B_2 \to B_2 - n\pi\,\mathrm{Vol}(S_1^2)$ every time we cross $r = n\pi$ for $n = 0, 1, 2, \ldots$ so that b_0 is a piecewise linear periodic function as illustrated in figure 1. In this way r is naturally partitioned into intervals of length π, with different brane content in each one of them, as the study of the Page charges reveals. Indeed, there are two charges defined on compact sub manifolds,

$$N_{\mathrm{D5}} = \frac{1}{2\kappa_{10}^2 T_5}\int_{\Sigma_1} F_3 = \frac{L^6}{k\pi}, \qquad N_{\mathrm{D3}} = \frac{1}{2\kappa_{10}^2 T_3}\int_{\Sigma_2}(F_5 - B_2\wedge F_3) = nN_{\mathrm{D5}}, \tag{2.18}$$

where $\Sigma_1 = (\mu, S_2^2)$, $\Sigma_2 = (\mu, S_2^1, S_2^2)$. Thus, we need to tune

$$L^6 = kN_{\mathrm{D5}}\pi. \tag{2.19}$$

Notice that N_{D3} is not globally defined. Instead its value depends on which interval we consider. In addition to this there are three charges that are defined on the non compact sub-manifolds,

$$\tilde{\Sigma}_1 = (r, S_1^2), \qquad \tilde{\Sigma}_2 = (r, S_2^2), \qquad \tilde{\Sigma}_3 = (r, S_1^2, S_2^2). \tag{2.20}$$

We take the non compact r to be indicative of an infinite linear quiver, as shown for a related AdS_5 example in [44]. We calculate the charges in the interval $r \in [n\pi, (n+1)\pi]$ and find,

$$\begin{aligned} N_{\mathrm{NS5}} &= \frac{1}{2\kappa_{10}^2 T_{\mathrm{NS5}}}\int_{S_1^2}\int_{n\pi}^{(n+1)\pi} dr\, H_3 = 1, \\ k_{\mathrm{D5}} &= -\frac{1}{2\kappa_{10}^2 T_5}\int_{S_2^2}\int_{n\pi}^{(n+1)\pi} dr\, F_3 = (1+2n)\frac{k\pi}{4} \equiv (2n+1)k_0. \end{aligned} \tag{2.21}$$

Notice that the parameter k, originally quantised in the Type IIA solution needs to be re-quantised according to $k\pi = 4k_0$, after the non-Abelian T-duality. The same happens to the size of the space L as shown in eq. (2.19).

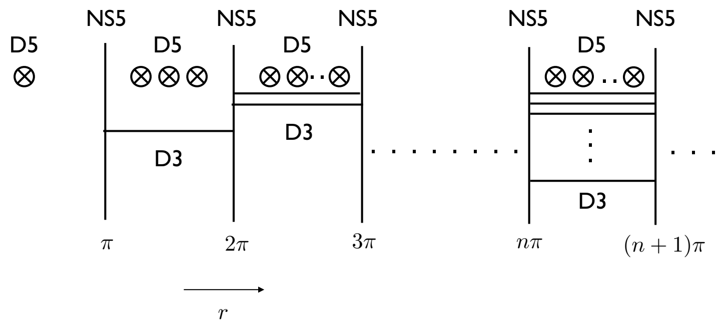

Figure 2. (NS5, D3, D5) brane set-up. The number of D3-branes is given in $N_{\rm D5}$ units and that of D5-branes in k_0 units.

We can also compute

$$k_{\rm D3} = -\frac{1}{2\kappa_{10}^2 T_3}\int_{S_1^2\times S_2^2}\int_{n\pi}^{(n+1)\pi} dr\ (F_5 - B_2\wedge F_3) = (3n+2)\frac{k_0}{3},$$

but this last one will not be relevant in our analysis below. Notice that all these charges are integer provided $\frac{k\pi}{12} = \frac{1}{3}k_0$ is an integer.

The previous analysis suggests a (NS5, D3, D5) brane set-up in which NS5-branes wrapped on $AdS_4\times S_2^2$ are located at $\mu = 0$, $r = \pi, 2\pi, \ldots, n\pi$, with n running to infinity, and there are $nN_{\rm D5}$ D3-branes, extended on $(\mathbb{R}^{1,2}, r)$ stretched among the n'th and $(n+1)$'th NS5's. On top of this, $(2n+1)k_0$ D5-branes, wrapped on $AdS_4\times S_1^2$ and located at $\mu = \pi$, lie between the n'th and $(n+1)$'th NS5-branes. This brane set-up is depicted in figure 2. After we recall some basic properties of 3d $\mathcal{N} = 4$ CFTs and their holographic duals, following [58, 63], we will make a concrete proposal for the field theory living on this brane configuration.

3 Aspects of 3d $\mathcal{N} = 4$ CFTs and their holographic duals

In this section we recall the basic aspects of the three dimensional $\mathcal{N} = 4$ field theories studied in [63] and of their holographic duals, derived in [58, 60]. We start with the field theory description.

3.1 3d $\mathcal{N} = 4$ CFTs

The study of the moduli space of $\mathcal{N} = 4$ SYM in four dimensions defined on an interval with SUSY preserving boundary conditions, lead Gaiotto and Witten [63] to introduce a family of 3d quantum field theories — named $T_\rho^{\hat\rho}(\mathrm{SU}(N))$, characterised by an integer N and two partitions of it, denoted ρ and $\hat\rho$. From $(N,\ \rho,\ \hat\rho)$ it is possible to read the data defining the UV of these theories, namely, the gauge group $G = \mathrm{U}(N_1)\times\ldots.\times\mathrm{U}(N_k)$, the bi-fundamental fields transforming in the $(N_i, \bar N_{i+1})$ representations, and the fundamental matter, transforming under $\mathrm{U}(M_i)$ for each gauge group.

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D5	x	x	x		x	x	x			
NS55	x	x	x					x	x	x

Table 1. Hanany-Witten brane set-up corresponding to the $\mathcal{N}=4$ 3d theory.

Given a list of positive numbers $[l_1 \geq l_2 \geq \ldots. \geq l_p]$, one can define a partition ρ of N by $N = \sum_{r=1}^{p} M_r l_r$. The numbers M_r, which indicate how many times the different integers l_r appear in the partition, give the ranks of the fundamental matter groups in the field theory. Similarly, one can define a second partition $\hat{\rho}$, consisting of the numbers $[\hat{l}_1 \geq \hat{l}_2 \geq \ldots. \geq \hat{l}_{\hat{p}}]$, with multiplicities $\hat{M}_r$, such that $N = \sum_{r=1}^{\hat{p}} \hat{M}_r \hat{l}_r$. From these partitions the ranks of the different $\mathrm{U}(N_i)$ gauge groups are computed from the expressions,

$$N_i = \sum_{s=1}^{i} (m_s - \hat{l}_s)\,, \tag{3.1}$$

where m_s denotes the number of terms that are equal or bigger than a given integer s in the decomposition $N = \sum_{r=1}^{p} M_r l_r$.

Gaiotto and Witten [63] conjectured that the condition for these three-dimensional field theories to flow to a conformal fixed point is (schematically) $\rho^T \geq \hat{\rho}$. More specifically, this condition means that

$$\sum_{s=1}^{i} m_s \geq \sum_{s=1}^{i} \hat{l}_s \qquad \forall i = 1, \ldots \hat{p}\,. \tag{3.2}$$

Associating a Young tableau with rows of lengths $[l_1, \ldots, l_p]$ to the partition ρ and one with columns of lengths $[\hat{l}_1, \ldots, \hat{l}_{\hat{p}}]$ to the partition $\hat{\rho}$, this condition means that the number of boxes in the first i-rows of the Young tableau associated to ρ^T must be larger or equal than the corresponding number in the tableau associated to $\hat{\rho}$. In those cases in which the equality holds, that is,

$$\sum_{s=1}^{i} m_s = \sum_{s=1}^{i} \hat{l}_s \qquad \text{for some} \quad i\,, \tag{3.3}$$

some gauge groups have zero rank, and the quiver becomes disconnected.

The quantum theory defined by $T_{\rho}^{\hat{\rho}}(\mathrm{SU}(N))$ has Coulomb and Higgs branches of vacua, while the theory defined by $T_{\hat{\rho}}^{\rho}(\mathrm{SU}(N))$ has the same moduli space, but with the Coulomb and Higgs vacua interchanged. Both theories are conjectured to flow to the same IR fixed point, which is a reflection of mirror symmetry. The three-dimensional CFT that appears at low energies is invariant under $\mathrm{SO}(2,3)$-reflecting the conformality in 3d, and $\mathrm{SO}(4) \sim \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$-reflecting the R-symmetry of $\mathcal{N}=4$ SUSY in 3d. This field theory can be nicely realised through a Hanany-Witten [64] set-up consisting of p D5 branes and $\hat{p}$ NS5 branes with D3 branes stretched between them. This brane set-up is shown in table 1.

The x_3-direction on which D3 branes stretch is of finite size, thus giving rise at long distances to a three-dimensional QFT on $[0,1,2]$. The $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ R-symmetry is associated with rotations in the $[4, 5, 6]$ and $[7, 8, 9]$ directions. In turn, the $l_1 \geq l_2 \geq \ldots . \geq l_p$ and $\hat{l}_1 \geq \hat{l}_2 \geq \ldots . \geq \hat{l}_{\hat{p}}$ numbers that define the partitions $(\rho, \hat{\rho})$ are respectively, the linking numbers associated to the p D5 and $\hat{p}$ NS5 branes. These are defined by

$$
\begin{aligned}
l_{D5,a} &= l_a = -n_a + R_a^{\mathrm{NS5}}, \qquad & a = 0, \ldots, p,\\
\hat{l}_{NS5,b} &= \hat{l}_b = n_b + L_b^{\mathrm{D5}} \qquad & b = 1, \ldots, \hat{p},
\end{aligned}
$$

where n_a is the net number of D3 branes ending on the given five brane (number of D3 on the right — number of D3 on the left). In turn, R_a^{NS5} is the number of NS5 branes to the *right* of a given D5 brane, while L_b^{D5} is the number of D5 branes to the *left* of a given NS5 brane. The multiplicities of each linking number, M_r, $\hat{M}_r$, are thus the number of branes in the corresponding stack of D5 or NS5 branes.

3.2 The ABEG dual geometries

Following the formulation initiated in [61, 62], the authors of [58, 60] proposed that the supergravity solutions associated to the three dimensional $\mathcal{N} = 4$ CFTs that we just described are fibrations of $AdS_4 \times S^2 \times S^2$ over a Riemann surface Σ_2. We will refer to these geometries as ABEG geometries for brevity. These solutions have manifest $\mathrm{SO}(2,3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry, and can be completely determined from two harmonic functions $h_1(z, \bar{z}), h_2(z, \bar{z})$, defined on the Riemann surface Σ_2. From the functions h_1, h_2, the background and fluxes are given by,

$$
\begin{aligned}
&ds^2 = \lambda^2 ds^2(AdS_4) + \lambda_1^2 ds^2(S_1^2) + \lambda_2^2 ds^2(S_2^2) + ds^2(\Sigma_2),\\
&H_3 = d(b_1) \wedge \mathrm{Vol}(S_1^2), \qquad F_3 = d(b_2) \wedge \mathrm{Vol}(S_2^2), \qquad ds^2(\Sigma_2) = 4\tilde{\rho}^2 |dz|^2\\
&F_5 = 4(1 + \star) f \wedge \mathrm{Vol}(S_1^2) \wedge \mathrm{Vol}(S_2^2),
\end{aligned}
\tag{3.4}
$$

where $\tilde{\rho}^2, \lambda, \lambda_1, \lambda_2, b_1, b_2$ and the dilaton e^{Φ} are real functions and f denotes a 1-form on Σ_2, explicitly written below. These functions can be written in a compact form from h_1, h_2 using,

$$
\begin{aligned}
&W = \partial_z h_1 \partial_{\bar{z}} h_2 + \partial_{\bar{z}} h_1 \partial_z h_2, \qquad X = i(\partial_z h_1 \partial_{\bar{z}} h_2 - \partial_{\bar{z}} h_1 \partial_z h_2)\\
&N_1 = 2h_1 h_2 |\partial_z h_1|^2 - h_1^2 W, \qquad N_2 = 2h_1 h_2 |\partial_z h_2|^2 - h_2^2 W,
\end{aligned}
\tag{3.5}
$$

as,

$$
\begin{aligned}
&\tilde{\rho}^2 = \frac{\sqrt{N_2|W|}}{h_1 h_2}, \qquad \lambda^2 = 2\sqrt{\frac{N_2}{|W|}}, \qquad \lambda_1^2 = 2e^{\Phi} h_1^2 \sqrt{\frac{|W|}{N_1}}, \qquad \lambda_2^2 = 2h_2^2 \sqrt{\frac{|W|}{N_2}},\\
&b_1 = 2h_2^D + 2h_1^2 h_2 \frac{X}{N_1}, \qquad b_2 = -2h_1^D + 2h_1 h_2^2 \frac{X}{N_2}, \qquad e^{2\Phi} = \frac{N_2}{N_1}.
\end{aligned}
\tag{3.6}
$$

Here h_1^D, h_2^D are the harmonic duals of h_1, h_2, defined such that $h_1^D + ih_1$ and $h_2 - ih_2^D$ are holomorphic functions. Notice that we are working in string frame, hence some factors of the dilaton differ from [60, 62], which use Einstein frame. Finally, the 1-form f is given by,

$$
f = 2\,\mathrm{Im}\left(\left[3i\,(h_1\partial_z h_2 - h_2 \partial_z h_1) + \partial_z\left(h_1 h_2 \frac{X}{W}\right)\right] \frac{\lambda_1^2 \lambda_2^2}{\lambda^4} dz\right). \tag{3.7}
$$

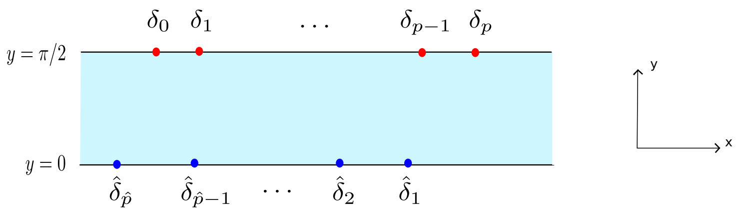

Figure 3. 5-branes distribution along the strip, parameterised by $z = x + iy$.

It was shown in [58] that the two harmonic functions h_1, h_2 that encode the supergravity solution as shown above, can be determined from the (D5, NS5, D3) brane set-up associated to the $T^{\hat{\rho}}_{\rho}(\mathrm{SU}(N))$ theory. Defining the sets of numbers $[N_5^a, \delta_a]$ and $[\hat{N}_5^b, \hat{\delta}_b]$, denoting respectively the number of branes at each stack and the position of this stack, for D5 and NS5 branes, and taking Σ_2 as the strip defined by $-\infty < \mathrm{Re}[z] < \infty$ and $0 \leq \mathrm{Im}[z] \leq \frac{\pi}{2}$,[2] the h_1, h_2 functions are given by,

$$h_1 = -\frac{1}{4}\sum_{a=1}^{p} N_5^a \log\tanh\left(\frac{i\frac{\pi}{2} + \delta_a - z}{2}\right) + cc, \qquad h_2 = -\frac{1}{4}\sum_{b=1}^{\hat{p}} \hat{N}_5^b \log\tanh\left(\frac{z - \hat{\delta}_b}{2}\right) + cc. \tag{3.8}$$

These expressions exhibit logarithmic singularities at the locations of the stacks of D5-branes, at $z = \delta_a + i\pi/2$, for h_1, and at the locations of the NS5-branes $z = \hat{\delta}_b$, for h_2. The brane distribution is depicted in figure 3. The Laplace problem that these functions solve must be complemented by conditions on the boundaries of the Riemann surface [61, 62],

$$h_1\big|_{\mathrm{Im}[z]=0} = \partial_\perp h_2\big|_{\mathrm{Im}[z]=0} = 0, \qquad h_2\big|_{\mathrm{Im}[z]=\frac{\pi}{2}} = \partial_\perp h_1\big|_{\mathrm{Im}[z]=\frac{\pi}{2}} = 0, \tag{3.9}$$

where $\partial_\perp = \partial_z - \partial_{\bar{z}}$, which the h_1 and h_2 in eq. (3.8) satisfy.

From the expressions for h_1, h_2 in eq. (3.8), the fluxes in eq. (3.4) can be calculated using eqs. (3.5) and (3.6). The associated charges are defined as,

$$N_5^a = \frac{1}{2\kappa_{10}^2 T_{\mathrm{D5}}}\int_{I\times S_2^2} F_3, \qquad \hat{N}_5^b = \frac{1}{2\kappa_{10}^2 T_{\mathrm{NS5}}}\int_{\hat{I}\times S_1^2} H_3, \tag{3.10}$$

where the 3-cycles, defined in [58], consist of a shrinking sphere times an interval I or $\hat{I}$, that semi-circles the position of the singularity at δ_a or $\hat{\delta}_b$. As we discussed, $(N_5^a, \hat{N}_5^b)$ should be identified with the multiplicities $(M_r, \hat{M}_r)$ in the two partitions $\rho, \hat{\rho}$.

Similarly, it is possible to define two Page charges associated to D3 branes, one being the S-dual of the other:

$$N_3^a = \frac{1}{2\kappa_{10}^2 T_{\mathrm{D3}}}\int_{I\times S_1^2\times S_2^2} [F_5 - B_2\wedge F_3], \qquad \hat{N}_3^b = \frac{1}{2\kappa_{10}^2 T_{\mathrm{D3}}}\int_{\hat{I}\times S_1^2\times S_2^2} [F_5 + C_2\wedge H_3]. \tag{3.11}$$

These charges are well defined whenever the potential B_2 or C_2 entering in their expression is well-defined, that is, away from the positions where the NS5 or D5 branes are located.

[2] This choice of strip is consistent for linear quivers (see [58, 59]).

From these and the previous charges, the linking numbers associated to the D5 and NS5 branes can be determined as [58],

$$l_a = -\frac{N_3^a}{N_5^a} = \frac{2}{\pi}\sum_{b=1}^{\hat{p}} \hat{N}_5^b \arctan(e^{\hat{\delta}_b-\delta_a}), \qquad \hat{l}_b = \frac{\hat{N}_3^b}{\hat{N}_5^b} = \frac{2}{\pi}\sum_{a=1}^{p} N_5^a \arctan(e^{\hat{\delta}_b-\delta_a}). \quad (3.12)$$

As expected, they satisfy

$$N = \sum_{b=1}^{\hat{p}} \hat{N}_5^b \, \hat{l}_b = \sum_{a=1}^{p} N_5^a \, l_a. \quad (3.13)$$

Finally, in [69] a special limit of the general expressions for h_1 and h_2 given in eq. (3.8) was considered. In this limit, the NS5-branes and D5-branes located at the two boundaries of the strip, $\mathrm{Im}\, z = 0$, $\mathrm{Im}\, z = \pi/2$, are positioned at infinite values of $\mathrm{Re}\, z$. This limit will be useful when we discuss the realisation of the non-Abelian T-dual solution as an ABEG geometry. Specifically, it was shown in [69] that if $\delta_a \to \infty$ and $\hat{\delta}_b \to -\infty$ one can approximate eq. (3.8) by,

$$\begin{aligned} h_1 &= \sin y \sum_{a=1}^{p} N_5^a e^{x-\delta_a} + \ldots && \text{if} \quad x < \delta_1, \\ &= \sin y \sum_{a=i}^{p} N_5^a e^{x-\delta_a} + \ldots && \text{if} \quad \delta_{i-1} < x < \delta_i, \\ h_2 &= \cos y \sum_{\hat{b}=1}^{\hat{p}} \hat{N}_5^b e^{\hat{\delta}_b - x} + \ldots && \text{if} \quad x > \hat{\delta}_1, \\ &= \cos y \sum_{\hat{b}=i}^{\hat{p}} \hat{N}_5^b e^{\hat{\delta}_b - x} + \ldots && \text{if} \quad \hat{\delta}_{i-1} > x > \hat{\delta}_i, \end{aligned} \quad (3.14)$$

where the strip is parameterised by $z = x + iy$. Notice that these expressions still satisfy the boundary conditions in eq. (3.9).

4 The Type IIB $\mathcal{N} = 4$ AdS_4 solution and CFT

After we have discussed the basic ingredients of 3d $\mathcal{N} = 4$ CFTs and their duals, we can go back to our brane configuration, discussed at the end of section 2, and make a concrete proposal for the CFT associated to the brane set-up depicted in figure 2.

Restricting the r direction to lie between zero and $r = (n + 1)\pi$, we have a total number of $n + 1$ NS5-branes (see figure 4). In order to have a field theory that flows to a non-trivial infrared fixed point (see below) we need to add $(n+1)N_{\mathrm{D5}}$ D3-branes ending on the $(n + 1)$'th NS5-brane from the right. This is achieved inserting a stack of $(n + 1)N_{\mathrm{D5}}$ D5-branes to the right of the $(n + 1)$'th NS5-brane, each one connected to this NS5-brane by a D3-brane. In turn, this is equivalent up to a Hanany-Witten move [64] to just taking the n'th stack of D5-branes with $(2n + 1)k_0 + (n + 1)N_{\mathrm{D5}}$ branes. This field theoretical

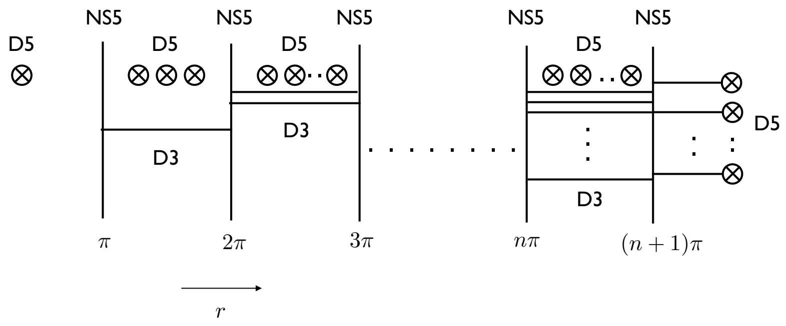

Figure 4. Completed (NS5, D3, D5) brane set-up.

completion of the quiver has the geometric counterpart of making finite the range of the r-coordinate.

Thus, in the notation of ABEG, we have $p = n+1$ and the multiplicity of D5 branes is,

$$N_5^a = (2a+1)k_0\,, \qquad a = 0, \dots, n-1 \qquad N_5^n = (2n+1)k_0 + (n+1)N_{\mathrm{D5}}\,. \tag{4.1}$$

We can now compute the linking numbers associated to the five branes in the Hanany-Witten set-up. These provide an invariant way of encoding the brane configuration, since they do not change under Hanany-Witten moves. The linking numbers associated to the D5-branes are given by ,

$$l_a = -n_a + R_a^{\mathrm{NS5}}, \tag{4.2}$$

where n_a denotes the net number of D3-branes ending on the a'th stack of D5-branes and R_a^{NS5} the number of NS5-branes located at its right. For our brane set-up we find,

$$l_a = n + 1 - a \qquad \text{for} \quad a = 0, 1, \dots, n. \tag{4.3}$$

From here the total number of D3-branes N, reads

$$\begin{aligned} N = \sum_{a=0}^{n} l_a N_5^a &= \sum_{a=0}^{n}(n+1-a)(2a+1)k_0 + (n+1)N_{\mathrm{D5}} \\ &= \frac{k_0}{6}(n+1)(n+2)(2n+3) + N_{\mathrm{D5}}(n+1). \end{aligned} \tag{4.4}$$

Alternatively, we can compute N using the NS5-branes stacks. In this case the linking numbers are computed from,

$$\hat{l}_b = n_b + L_b^{\mathrm{D5}}, \tag{4.5}$$

where n_b denotes once more the net number of D3-branes ending on the b'th NS5-brane, and L_b^{D5} denotes the number of D5-branes to the left of the b'th NS5-brane. We find that,

$$\hat{l}_b = N_{\mathrm{D5}} + k_0 b^2, \qquad b = 1, \dots, n+1. \tag{4.6}$$

Once can easily check that

$$N = \sum_{b=1}^{n+1} \hat{l}_b N_5^b = \sum_{b=1}^{n+1} \hat{l}_b = \frac{k_0}{6}(n+1)(n+2)(2n+3) + N_{\mathrm{D5}}(n+1)\,, \tag{4.7}$$

as in eq. (4.4). Thus, the $\rho, \hat{\rho}$ partitions associated to the brane configuration in figure 4 read

$$\rho: \qquad N = \underbrace{1+\ldots+1}_{(2n+1)k_0+(n+1)N_{\rm D5}} + \underbrace{2+\ldots+2}_{(2n-1)k_0} + \ldots + \underbrace{(n+1)+\ldots+(n+1)}_{k_0}, \tag{4.8}$$

and

$$\hat{\rho}: \qquad N = \underbrace{N_{\rm D5}+(n+1)^2k_0}_{1} + \underbrace{N_{\rm D5}+n^2k_0}_{1} + \ldots + \underbrace{N_{\rm D5}+k_0}_{1}. \tag{4.9}$$

These two partitions define the $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ field theory associated to our brane set-up. Following now the work of ABEG [58] we can read from eq. (4.8) the number of terms, m_l, that are equal or bigger than a given integer l,

$$m_1 = (n+1)N_{\rm D5} + (n+1)^2k_0\,, \qquad m_2 = n^2k_0\,, \qquad \ldots, \qquad m_n = 4k_0, \qquad m_{n+1} = k_0. \tag{4.10}$$

From these, the condition to have a field theory that flows to a non-trivial infrared fixed point, as was conjectured in [63], is,

$$\rho^T \geq \hat{\rho} \Longleftrightarrow \sum_{s=1}^{i} m_s \geq \sum_{s=1}^{i} \hat{l}_s \qquad \forall i = 1, \ldots, n+1, \tag{4.11}$$

where for this to hold the $\hat{l}_i$ must be ordered such that $\hat{l}_1 \geq \hat{l}_2 \cdots \geq \hat{l}_i$. We will use this notation in the rest of this section. In the present case we have,

$$\sum_{s=1}^{i} m_s = (n+1)N_{\rm D5} + \sum_{q=n-i+2}^{n+1} q^2 k_0\,, \tag{4.12}$$

which is strictly larger than

$$\sum_{s=1}^{i} \hat{l}_s = iN_{\rm D5} + \sum_{q=n-i+2}^{n+1} q^2 k_0, \tag{4.13}$$

for $i = 1, \ldots, n$, while

$$\sum_{s=1}^{n+1} m_s = \sum_{s=1}^{n+1} \hat{l}_s. \tag{4.14}$$

The last condition is consistent with the fact that there are k_0 D5-branes in the $[0, \pi]$ interval that are disconnected from the rest of the branes, thus leading to a quiver that breaks into two disconnected components. We can also check that, consistently with our brane set-up in figure 4, the ranks of the gauge groups are given by

$$N_i = \sum_{s=1}^{i} (m_s - \hat{l}_s) = (n+1-i)N_{\rm D5}\,, \tag{4.15}$$

and the last gauge group is empty, in agreement with the fact that there are k_0 free hypermultiplets associated to the decoupled k_0 D5-branes. Each of these gauge groups has associated M_j hypermultiplets in the fundamental, with M_j given by,

$$\rho: \qquad N = \underbrace{1+\ldots+1}_{M_1} + \underbrace{2+\ldots+2}_{M_2} + \ldots + \underbrace{(n+1)+\ldots+(n+1)}_{M_{n+1}}. \tag{4.16}$$

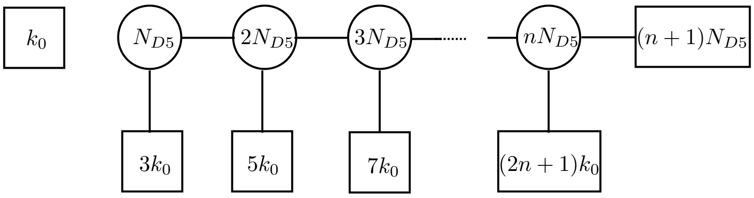

Figure 5. Quiver associated to the (NS5, D3, D5) brane set-up in figure 4.

These can be read from eq. (4.8) in our case. The resulting quiver is represented in figure 5, and we can see that it is fully consistent with the brane configuration in figure 4. We can also check explicitly that

$$M_i + N_{i-1} + N_{i+1} > 2N_i, \tag{4.17}$$

a condition for the quiver to flow towards a superconformal field theory in the infrared [63].

In summary, we have seen that *ending* the Hanany-Witten set-up in figure 2 and *completing* it with flavour branes as in figure 4, lead us to a concrete proposal for a quiver describing a $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ theory. The charges of the non-Abelian T-dual solution calculated in eq. (2.21) were instrumental in identifying the quiver and its completion. We will now show that the metric and other fields in the non-Abelian T-dual background are also consistent with those associated to the quiver of figure 4. The non-Abelian T-dual geometry will arise as a zooming-in on a particular region of the ABEG [58] solution associated to the $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ field theory.

5 The Type IIB $\mathcal{N} = 4$ AdS_4 solution as a ABEG geometry

Since the solution that we generated in section 2 preserves $\mathcal{N} = 4$ SUSY, contains an AdS_4 factor and has SO(4) isometry, it is natural to expect that it should fit within the formalism described in section 3.2. Below, we will prove this. We start by redefining,

$$\sigma = -\cos\mu, \qquad \beta^2 = \frac{k^2}{L^6}. \tag{5.1}$$

For the non-Abelian T-dual solution in eqs. (2.12)–(2.14), we can calculate,

$$\begin{aligned}
&\lambda_1^2 = \frac{r^2}{2\beta^2\Delta}(1-\sigma)(1+\sigma)^2, \quad \lambda_2^2 = \frac{(1-\sigma)^{3/2}}{\sqrt{2}\beta}, \quad \lambda^2 = \frac{\sqrt{2}}{\beta}\sqrt{1-\sigma}, \quad \frac{1}{\tilde{\rho}^2} = 2\sqrt{2}\beta\sqrt{1-\sigma}(1+\sigma),\\
&b_1 = \frac{r^3}{\sqrt{2}\beta\Delta}\sqrt{1-\sigma}(1+\sigma)-n\pi, \quad b_2 = c_0+\frac{k}{4}\left(r^2+\frac{\sigma(\sigma^2-3)}{\beta^2}\right), \quad z = \sigma+i\beta r,\\
&e^{2\Phi} = \frac{2\sqrt{2}}{k^2\beta\Delta}(1-\sigma)^{3/2}, \quad \Delta = \frac{1}{\sqrt{2}\beta^3}\sqrt{1-\sigma}(1+\sigma)\left(\beta^2 r^2+\frac{(1-\sigma)(1+\sigma)^2}{2}\right),
\end{aligned} \tag{5.2}$$

where $d(c_0) = 0$ and the $n\pi$ comes from the contribution to B_2 of n large gauge transformations. From this we find that the functions in eqs. (3.5)–(3.6) read,

$$N_1 = \frac{rk^3\Delta}{64\sqrt{2-2\sigma}\beta^3}, \quad N_2 = \frac{kr(1-\sigma)}{32\beta^4}, \quad W = -\frac{kr}{16\beta^2}, \quad X = -\frac{k}{16\beta^3}(1+\sigma), \tag{5.3}$$
$$h_1 = \frac{kr(1+\sigma)}{4\beta}, \quad h_2 = \frac{1-\sigma}{2\beta}, \quad h_1^D = -\frac{k(1-(2+\sigma)\sigma)+(4c_0+kr^2)\beta^2}{8\beta^2}, \quad h_2^D = \frac{1}{2}(r-n\pi).$$

Notice that the functions h_1, h_2 are harmonic. As established in [62], this implies that the equations of motion of Type IIB Supergravity are satisfied. Also, note that the definition of h_2^D in each $n\pi < r < (n+1)\pi$ cell implies it is a piecewise periodic function such that $0 < h_2^D < \pi/2$.

Thus, we have shown that the solution generated by non-Abelian T-duality — eqs. (2.12)–(2.14), fits within the class of solutions discussed in eqs. (3.4)–(3.7). It is worth stressing nevertheless that it does not satisfy the boundary conditions in eq. (3.9) — nor does it show any of the *isolated* singularities than can be associated to the positions $(\delta, \hat{\delta})$, of the D5 and NS5 branes. This suggests that the solution generated by non-Abelian T-duality could be thought of as a *limit* of the generic solutions in eqs. (3.4)–(3.7), along the lines of eqs. (3.14). We next study this in detail.

Let us start by computing the positions of the D5 and NS5 brane stacks associated to our brane configuration in figure 4. As explained in [58] and summarised in section 3.2, these positions can be computed from,

$$l_a = \frac{2}{\pi}\sum_{b=1}^{\hat{p}} \hat{N}_5^b \arctan(e^{\hat{\delta}_b - \delta_a}) \qquad \hat{l}_b = \frac{2}{\pi}\sum_{a=1}^{p} N_5^a \arctan(e^{\hat{\delta}_b - \delta_a}).$$

These equations are simply solved by

$$e^{\hat{\delta}_b - \delta_a} = \tan\left(\frac{\pi}{2}\frac{l_a \hat{l}_b}{N}\right). \tag{5.4}$$

Using eqs. (4.3) and (4.6) this gives for our brane set-up

$$\hat{\delta}_b - \delta_a = \log\left[\tan\left(\frac{\pi}{2N}(n+1-a)(N_{\text{D5}} + k_0 b^2)\right)\right], \tag{5.5}$$

with N given by eq. (4.4).

Recalling that we read the charges of our brane configuration from the supergravity solution, we expect to find a sensible solution to eq. (5.5) in the supergravity limit $N_{\text{D5}} \to \infty$. Taking this limit we find,

$$\hat{\delta}_b - \delta_a = \log\left[\tan\left(\frac{\pi}{2}\left(1 - \frac{a}{n+1}\right)\right)\right] \tag{5.6}$$

which shows that in this limit all stacks of NS5-branes can be approximately taken at the same position $\hat{\delta}$. Equivalently, we can write eq. (5.6) as

$$\delta_a - \hat{\delta} = \log\left[\tan\left(\frac{\pi a}{2(n+1)}\right)\right]. \tag{5.7}$$

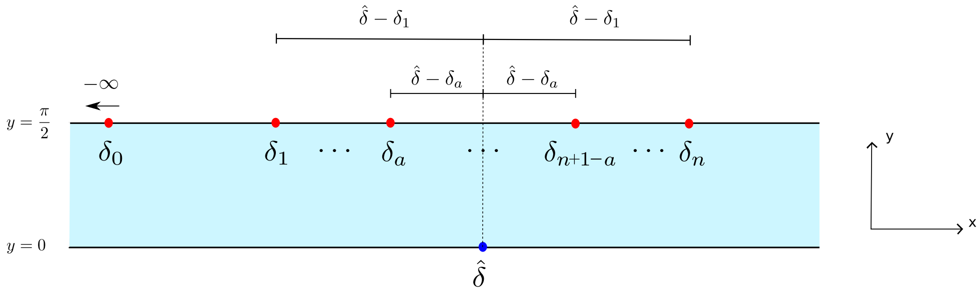

Figure 6. Positions of D5 and NS5 branes in the supergravity limit. In this limit the set-up becomes symmetric around $\hat{\delta} = \delta_{(n+1)/2}$, with the exception of the detached stack of D5-branes at $\delta_0 = -\infty$.

From here we see that the first stack of (detached) k_0 D5-branes lies strictly at $\delta_0 - \hat{\delta} = -\infty$, while the rest of stacks lie symmetrically at both sides of the NS5-branes, given that

$$\log\left[\tan\left(\frac{\pi c}{2(n+1)}\right)\right] = -\log\left[\tan\left(\frac{\pi a}{2(n+1)}\right)\right] \qquad \text{for} \quad c = n+1-a. \tag{5.8}$$

Thus,

$$\delta_1 - \hat{\delta} = \hat{\delta} - \delta_n, \qquad \delta_2 - \hat{\delta} = \hat{\delta} - \delta_{n-1}, \qquad \ldots, \qquad \hat{\delta} = \delta_{(n+1)/2}. \tag{5.9}$$

This brane distribution is depicted in figure 6. Let us now obtain the h_1, h_2 functions associated to this configuration, following [58, 59].

In the supergravity limit the main contribution to h_1 in eq. (3.8) comes from the n'th stack, given that the number of branes in this stack goes with N_{D5} as shown by eq. (4.1). We can then approximate,

$$h_1 \sim -\frac{1}{4} N_5^n \log\tanh\left(\frac{i\frac{\pi}{2} + \delta_n - z}{2}\right) + cc. \tag{5.10}$$

For h_2 we have in turn

$$h_2 \sim -\frac{1}{4}(n+1)\log\tanh\left(\frac{z - \hat{\delta}}{2}\right) + cc. \tag{5.11}$$

Choosing

$$\delta_n = -\hat{\delta} = -\frac{1}{2}\log\left[\tan\left(\frac{\pi}{2(n+1)}\right)\right], \tag{5.12}$$

the n'th stack of D5-branes lies approximately at plus infinity for large n while the stack of NS5-branes lies approximately at minus infinity. For finite x we can then use the approximate expressions for h_1, h_2 in eq. (3.14) to produce,

$$\begin{aligned} h_1 &\sim \sin y\, N_5^n\, e^{x-\delta_n} \sim \sin y\, N_{\mathrm{D5}} \sqrt{\frac{\pi(n+1)}{2}} e^{x}, \\ h_2 &\sim \cos y\, (n+1)\, e^{\hat{\delta}-x} \sim \cos y \sqrt{\frac{\pi(n+1)}{2}}\, e^{-x}, \end{aligned} \tag{5.13}$$

where we have approximated $\delta_n = -\hat{\delta} \sim -\frac{1}{2}\log\left[\left(\frac{\pi}{2(n+1)}\right)\right]$ for large n. Close to $y = 0$, $x = 0$ we have,

$$\begin{aligned} h_1 &\sim y\, N_{\rm D5}\sqrt{\frac{\pi(n+1)}{2}}\,(1+x), \\ h_2 &\sim \sqrt{\frac{\pi(n+1)}{2}}\,(1-x). \end{aligned} \tag{5.14}$$

Let us now compare these expressions with those of our non-Abelian T-dual solution. Taking the functions h_1, h_2 for this solution from eq. (5.3),

$$\begin{aligned} h_1 &= \frac{kr(1+\sigma)}{4\beta}, \\ h_2 &= \frac{1-\sigma}{2\beta}, \end{aligned} \tag{5.15}$$

we find that they agree with eq. (5.14) if we identify $x = \sigma$ and

$$\frac{1}{2\beta} \sim \sqrt{\frac{\pi(n+1)}{2}}\,, \qquad r \sim \frac{2}{k} N_{\rm D5}\, y. \tag{5.16}$$

Taking into account that $\beta = k/L^3$, and $N_{\rm D5} = L^6/(\pi k)$, these are equivalent to

$$N_{\rm D5} \sim 2k(n+1)\,, \qquad r \sim 4(n+1)y. \tag{5.17}$$

The output of this analysis is that the non-Abelian T-dual solution comes out when zooming into the region $x = y = 0$ of the ABEG solution associated to the brane set-up in section 4. Further, we have shown that in order to match these solutions n must go to infinity as $N_{\rm D5}/k$, which is consistent with the fact that n is unbounded in the non-Abelian T-dual solution. Note however that this limit should be taken directly in equation (5.5) for consistency of the previous analysis. We have checked numerically that the matching between the non-Abelian T-dual solution and the ABEG geometry still holds in this limit in the region $x \sim 0$, $y \sim 0$. In this matching we must have $\sigma \sim x$, $r \sim 4(n+1)y$. Therefore, σ, which in the non-Abelian T-dual solution ranges in $[-1, 1]$, must be small. The coordinate r in turn, may cover a finite region depending on how the $y \to 0$ limit is taken in the expression above, which is unspecified in our analysis.

The previous agreement suggests that we may see the ABEG solution as a *completion* of the non-Abelian T-dual geometry, that: i) Extends it to $-\infty < \sigma < \infty$, such that the singularities in $\sigma = \pm 1$ are moved to $\pm\infty$, and thus resolved, and ii) Delimits r to a bounded region. This is shown pictorially in figure 7. It is interesting that this completion makes explicit the ideas in [44], where a completion of the non-Abelian T-dual of $AdS_5 \times S^5$ as a superposition of Maldacena-Nunez geometries was outlined.

6 Free energy

The authors of reference [69] computed the free energy of some specific examples of $T^{\hat{\rho}}_{\rho}(\mathrm{SU}(N))$ field theories, both directly in the field theory as well as using holography.

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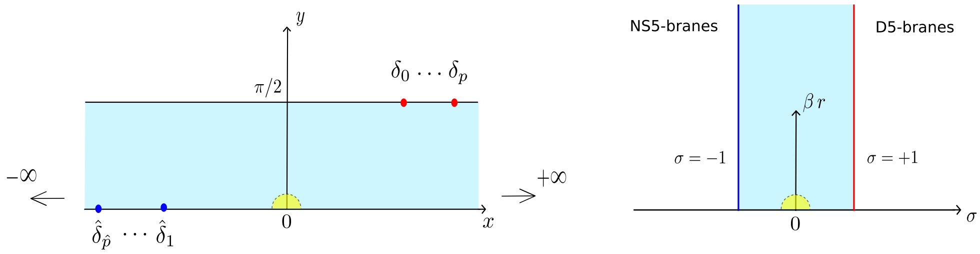

Figure 7. The ABEG set-up in the limit in which the NS5-branes are placed at $-\infty$ and the D5-branes at $+\infty$ (left). The non-Abelian T-dual set-up, with $\sigma \in [-1, 1]$, $r \in [0, +\infty)$, with smeared NS5-branes at $\sigma = -1$ and smeared D5-branes at $\sigma = +1$ (right). The matching of the solutions occurs locally around $x, y \sim 0$. The *completion* of the non-Abelian T-dual geometry is achieved extending σ to $\pm\infty$ and bounding r to an interval. In this completion the NS5-branes are localised at $-\infty$ and the D5-branes at $+\infty$.

This free energy was shown to exhibit a $\frac{1}{2}N^2 \log N$ behaviour at leading order. Further, it was argued that this value should provide an upper bound to the free energy of any, more general, $T^{\hat{\rho}}_{\rho}(\mathrm{SU}(N))$ field theory.

In this section we compute the free energy associated to the non-Abelian T-dual solution and compare it to that of the completed ABEG geometry. We show that, as expected, the free energies do not agree, consistently with the fact that the non-Abelian T-dual geometry approximates the ABEG geometry only in a small patch. On the contrary, this calculation shows explicitly how the completion of the non-Abelian T-dual geometry leads to a sensible value for the free energy of the dual $T^{\hat{\rho}}_{\rho}(\mathrm{SU}(N))$ field theory that is in consonance with previous results in the literature and satisfies the bound found in [69].

We will use the conventions in [69]. In this reference the free energy is computed from

$$S_{\text{eff}} = \frac{1}{2^4 \pi^5} \mathrm{Vol}_6, \tag{6.1}$$

where Vol_6 is the volume of the six dimensional internal space, which can be calculated from the functions h_1, h_2, defined in the 2d manifold Σ_2 as,

$$\mathrm{Vol}_6 = 32(4\pi)^2 \int_{\Sigma} d^2x(-W)h_1 h_2. \tag{6.2}$$

We first use this expression to compute the free energy associated to the non-Abelian T-dual solution. In this case we find,

$$h_1 h_2 = \frac{k}{8\beta^2} r(1 - \sigma^2)\,, \qquad W = -\frac{kr}{16\beta^2}, \tag{6.3}$$

and

$$\mathrm{Vol}_6 = 32(4\pi)^2 \frac{k^2}{2^7 \beta^4} \int_{\Sigma} r^2(1 - \sigma^2)\beta dr d\sigma. \tag{6.4}$$

Here we have used that the differential area of the strip is $d\Sigma = \beta dr d\sigma$. Integrating $r \in [0, (n+1)\pi]$ and $\sigma \in [-1, 1]$ we find

$$S_{\text{eff}} = \frac{\pi^{3/2}}{9} \sqrt{k} N_{\mathrm{D5}}^{3/2} (n+1)^3. \tag{6.5}$$

As in previous non-Abelian T-duals of AdS backgrounds, this free energy exhibits the same behaviour, in this case as $\sqrt{k}N^{3/2}$, of the original AdS background, multiplied by a power of $(n+1)$ coming from the NS5-branes.

Let us now analyse the "Abelian T-dual limit" of this expression. This limit was first discussed in [44] at the level of the central charges. It was shown that the central charge (and the free energy) of the SU(2) non-Abelian T-dual of $AdS_5 \times S^5$ and that of its Abelian T-dual counterpart[3] exactly match if r is taken in a $r \in [n\pi, (n+1)\pi]$ interval and n is sent to infinity. In this limit both metrics do in fact fully agree. We have presented a detailed analysis of this limit in appendix A for the present AdS_4 non-Abelian T-dual solution and its Abelian T-dual counterpart (see also appendix B). Borrowing the result for the free energy of the Abelian T-dual solution we can show that it fully agrees with the free energy of the non-Abelian solution for $r \in [n\pi, (n+1)\pi]$ and $n \to \infty$.

Indeed, integrating $r \in [n\pi, (n+1)\pi]$, $\sigma \in [-1, 1]$ in eq. (6.4) and taking the $n \to \infty$ limit, we find

$$S_{\text{eff}} = \frac{\pi^{3/2}}{3}\sqrt{k}N_{\text{D5}}^{3/2}n^2. \tag{6.6}$$

Using now that $N_{\text{D3}} = nN_{\text{D5}}$ for the non-Abelian solution and that in the large n limit $k_{\text{D5}} = nk\pi/2$, as implied by the second expression in eq. (2.21), eq. (6.6) can be rewritten as

$$S_{\text{eff}} = \frac{\sqrt{2}\pi}{3}\sqrt{k_{\text{D5}}}N_{\text{D3}}^{3/2}. \tag{6.7}$$

One can see that this result matches exactly the free energy of the Abelian T-dual background, given by eq. (A.12). As stressed in [44], this calculation shows that non-Abelian T-duality in an interval of length π corrects the Abelian T-duality calculation by $1/n$ terms. In our present set-up this provides a non-trivial check of the validity of expression (6.4).

Let us now compare the free energy of the full non-Abelian solution, given by eq. (6.5), to the free energy computed from the completed ABEG geometry. As shown in [69] the approximated expressions given by (3.14) are enough to capture the leading order behaviour. Using then these approximated expressions for h_1, h_2 for our particular ABEG geometry, given by eqs. (5.13), we can write $h_1 h_2 \sim \frac{1}{2}\sin 2y N_{\text{D5}}(n+1)^2 e^{-2\delta_n}$ and

$$W = \frac{1}{4}\frac{\partial^2}{\partial y^2}(h_1 h_2) = -h_1 h_2. \tag{6.8}$$

The internal volume then reads,

$$\text{Vol}_6 = 16\,(4\pi)^2 N_{\text{D5}}^2 (n+1)^4 e^{-4\delta_n} \int_0^{\frac{\pi}{2}} dy \sin^2 2y \int_{\hat{\delta}}^{\delta_n} dx, \tag{6.9}$$

which gives to leading order,

$$\text{Vol}_6 = 16\pi^5 N_{\text{D5}}^2 (n+1)^2 \log{(n+1)}, \tag{6.10}$$

[3]Namely, the result of T-dualising the original $AdS_5 \times S^5$ background along the Hopf fibre of the S^3 in the internal space.

and finally,

$$S_{\text{eff}} = N_{\text{D5}}^2 (n+1)^2 \log{(n+1)}. \tag{6.11}$$

We thus see that the free energy of the $T_\rho^{\hat{\rho}}(N)$ theory associated to our configuration exhibits a similar logarithmic behaviour to that of the examples discussed in [69]. As in those examples, the logarithm comes holographically from the size of the configuration. In our case it depends however on the number of NS5-branes, rather than on the number of D3-branes. Interestingly, taking into account the relation between N and n, given by eq. (4.4), the free energy given by eq. (6.11) satisfies the $\frac{1}{2}N^2 \log N$ bound suggested in [69] for the free energy of general $T_\rho^{\hat{\rho}}(N)$ field theories. This is to our knowledge the first check in the literature of the conjecture in [69].

We would like to note that for our particular $T_\rho^{\hat{\rho}}(N)$ theory, there is no field theoretical computation in the literature, along the lines of [70, 71], with which we could compare our holographic result. Indeed, the scaling limit taken in the field theory computation in [69], given by

$$N_5^a = N^{1-\kappa_a}\gamma_a\,; \qquad l_a = N^{\kappa_a}\lambda_a, \tag{6.12}$$

with $0 \leq \kappa_a < 1$ and $\sum_{a=1}^p \gamma_a\lambda_a = 1$, is not fulfilled by our configuration, for which only $\kappa_n = 0$ is well-defined. The reason we avoid this scaling is that there is a further N dependence in the number p that appears in $N = \sum_{a=1}^p N_5^a l_a$, as compared to the situation considered in [69]. It would be interesting to extend the field theory calculation in [69] to cover the present, more general, set-up, and check if the result matches the holographic computation.

As we have previously mentioned, we can see quite explicitly from the calculation of the free energy how the non-Abelian T-dual solution is completed by the ABEG geometry. Indeed, taking into account the different parametrisation of the strip in the non-Abelian T-dual solution, $d\Sigma = \beta dr d\sigma$, and in the ABEG solution, $d\Sigma = dxdy$, and doing the *completions*

$$\int_{-1}^{1} (1-\sigma^2) d\sigma \to \int_{\hat{\delta}}^{\delta_n} e^x e^{-x} dx, \tag{6.13}$$

and

$$\beta \int_0^{(n+1)\pi} r^2 dr \to 2(n+1)^2 \int_0^{\pi/2} \sin^2 2y dy, \tag{6.14}$$

which extend in a particular way the relations $\sigma \sim x$, $r \sim 4(n+1)y$, valid in the $x, y \sim 0$ region, we can recover exactly the free energy associated to the ABEG solution, given by eq. (6.11), from that of the non-Abelian T-dual solution. In this completion the logarithm is associated to the infinite extension of the configuration in the x direction, which is what allows us to send the singularities in $\sigma = \pm 1$ to $\pm\infty$. Note that the completion changes as well, and quite dramatically, the $\sqrt{k} N_{\text{D5}}^{3/2}(n+1)^3$ scaling of the free energy of the non-Abelian T-dual solution into the $N_{\text{D5}}^2(n+1)^2$ scaling associated to the ABEG geometry. Interpreting the behaviour of the free energy of AdS backgrounds generated through non-Abelian T-duality has remained an interesting open problem in the non-Abelian duality literature. Indeed, in all examples analysed so far the free energy of the non-Abelian T-dual was simply that of the original background corrected by a factor of $(n+1)$ to some power,

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associated to the NS5-branes. One was thus led to interpret that non-Abelian T-duality was not changing too much the field theory. Instead, the detailed calculation done in the present example shows that the completion needed to correctly define the dual CFT can change this behaviour quite significantly.

To summarise, we have shown that expressions (6.13), (6.14) inform us about the precise way in which the non-Abelian T-dual solution must be *completed* in order to describe holographically a $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ theory:

- Expression (6.13) shows that the interval $\sigma \in [-1, 1]$ must be extended to $\sigma \in (-\infty, \infty)$. The two singularities at $\sigma = \mp 1$ are then moved to infinity such that a perfectly smooth background remains.
- Expression (6.14) informs us about how precisely the non-compact direction of the non-Abelian T-dual solution must be bounded.

Our AdS_4 example thus provides a new AdS background in which the CFT dual can be used to define the geometry, in complete analogy with the AdS_5 case discussed in [44]. It also shows that the completion can significantly change the scaling of the free energy, and thus the CFT. This may shed some light on the possible interpretation of the behaviour of the free energy under non-Abelian T-duality.

7 Summary and conclusions

Let us start by summarising the contents of this paper. Then we will present some ideas for future work and comment on open problems that our results suggest.

We started by constructing a new solution to the Type IIB equations of motion. This new background consists of an AdS_4 factor and two spheres S_1^2, S_2^2, fibered on a Riemann surface $\Sigma(z, \bar{z})$. A dilaton, NS three form and Ramond three and five forms complete it. The system preserves sixteen supercharges and is obtained acting with non-Abelian T-duality on the dimensional reduction of $AdS_4 \times S^7$ to Type IIA. Both the original type IIA and its type IIB counterpart are singular. An important achievement of this paper is to understand the way of completing the geometry so that the only remaining *isolated* singularities are associated with brane sources. Global aspects of the geometry have also been understood thanks to this completion.

The procedure that we used to achieve these results can be summarised as follows. The study of the Page charges in section 2, suggested the brane distribution and Hanany-Witten set-up. The isometries of the background indicated the global symmetries of the dual field theory and the same goes for the amount of preserved SUSY. These data constrained our system in an important way, and suggested the way in which the Hanany-Witten set up, that in principle is *unbounded*, can be *completed* (hence *closed*) by the addition of flavour branes. This completion, shown explicitly comparing figure 4 with figure 2, is needed in order to define the partitions from which the $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ dual theory can be read. The position (in theory space) where this completion takes place is arbitrary and determines the parameters of the dual field theory. From here, the knowledge of the associated field theory, that in this case flows to a conformal fixed point, is constraining enough to allow us to write a precise completed Type IIB background in terms of a couple of holomorphic functions defined on a Riemann surface. This background describes an intersection of D3-D5-NS5 branes, and is smooth, except at the isolated positions of the five brane sources. Then, we discussed how a particular zoom-in on a region of the completed background gives place to the original Type IIB solution obtained by non-Abelian T-duality. Finally, our calculation of the free energy showed explicitly that this completion produces a sensible result for the free energy of the associated $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ field theory, satisfying the upper bound $\frac{1}{2}N^2 \log N$ found in [69]. This result suggests that there could be a scaling in the field theory side that reproduces our gravitational result.

A couple of points are crucial in the previous summary. On the one hand we have assumed that the fluxes capture faithfully the brane distribution (except, of course, for the completion with flavour branes). This has allowed us to suggest a Hanany-Witten set-up and to calculate (after completing it) the linking numbers that select the particular CFT. On the other hand, the fact that we are using field theory knowledge to smooth out a supergravity background is quite original and key to our procedure.

Interestingly, our approach has also allowed us to find out about global properties, in particular about the range of the r-coordinate (which is one of the long-standing problems of the whole non-Abelian T-duality formalism). It also gives a clean way of resolving or interpreting singularities in terms of sources. This is particularly nice since the presence of these sources is a consequence of the flavour symmetry on the field theory side, that also reflects in the completed quiver. A circle of ideas closes nicely.

What remains to be done (for this particular system and more generally)?

The proposed picture of intersecting Dp-$D(p+2)$-NS5 branes associated with an AdS_{p+1} background should be tested in detail. For this, a more complete case-by-case study is needed. Examples with different dimensionality might reveal new subtleties, that in the present study or in that of [44] do not show. In particular, it is clear that backgrounds with AdS_6 and AdS_3 factors should be studied following the ideas presented here. Progress should be possible in cases with less SUSY and smaller isometry groups.

In relation to the present AdS_4/CFT$_3$ case, it would be interesting to investigate Wilson loops, vortex operators [72] and other subtle CFT aspects — see for example [73–75], to understand, in particular, how our solutions capture these fine-points. The study of the spectrum of glueballs and mesons using our backgrounds (both the one obtained via non-Abelian T-duality and the completed one) is also of potential interest to learn about the nature of the duality. It would also be interesting to understand the geometric realisation of the decoupled flavour group in the quiver associated to our completed geometry

More generally, it would be very interesting to find out a precise answer for what is the effect of a non-Abelian T-duality transformation at the CFT level. In our example we started with a background dual to a CFT with one node and adjoint matter, to which we associated (after a non-Abelian T-duality transformation) a quiver containing a large number of colour and flavour groups. But, how precisely did we go from one quiver to the other? Is an 'unhiggsing' at work, or is the non-Abelian T-duality a genuine non-field theoretical operation? For a quite particular case of non-Abelian T-duality transformation, some progress was recently reported in [76].

Finally, it would be very nice if the ideas developed in this work could be used to answer deep questions about the nature of non-Abelian T-duality in String theory. For example, its invertibility, or the character of the genus and α'-expansions. We have given some evidence that the AdS/CFT correspondence can be very useful also in this regard.

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A The Abelian T-dual limit

In this appendix we summarise the key properties of the $\mathcal{N} = 4$ AdS_4 type IIB solution that is generated from the IIA solution in equation (2.8) by T-duality along the Hopf direction of S^3_1. This solution, dual to a circular quiver, was discussed at length in [59]. Here we recall its more relevant properties in the notation used in this paper. We show that it emerges as the $r \to \infty$ limit of the non-Abelian T-dual solution in section 2 (see appendix B). In this limit the free energies of both solutions also agree, as shown in section 6.

A.1 The solution

The Abelian T-dual of the IIA solution in equation (2.8) along the Hopf direction of S^3_1 reads:

$$\begin{aligned} ds^2_{\rm IIB} &= e^{\frac{2}{3}\phi_0}\cos\left(\frac{\mu}{2}\right)\left[ds^2(AdS_4) + L^2\left(d\mu^2 + \frac{k^2}{L^6\sin^2(\mu)}dr^2 + \cos^2\left(\frac{\mu}{2}\right)ds^2(S^2_2)\right)\right] \\ &\quad + \frac{L^3}{k}\sin\left(\frac{\mu}{2}\right)\sin(\mu)ds^2(S^2_1), \qquad B_2 = \cos\theta_1 d\phi_1\wedge dr, \qquad e^{2\Phi} = \frac{e^{4\phi_0/3}}{L^2\tan^2\left(\frac{\mu}{2}\right)}. \end{aligned} \tag{A.1}$$

Since we dualise on the Hopf fibre $0 < \psi_1 < 4\pi$, we have $0 < \tilde{\psi}_1 < \pi$.[4] To ease notation we choose to label $\tilde{\psi}_1 = r$, as we have a similar coordinate in the non-Abelian T-dual case

[4]Recall that the periodicity of the Abelian T-dual coordinate is fixed by the condition $\int d\psi_1\wedge d\tilde{\psi}_1 = (2\pi)^2$.

of section 2, here though we stress that it is compact. Additionally this background is supported by the gauge invariant RR fluxes

$$F_3 = \frac{k}{2} dr \wedge \text{Vol}(S_2^2), \qquad F_5 = -\frac{3}{L} dr \wedge \text{Vol}(AdS_4) + \frac{3L^6}{4k} \sin^3(\mu) d\mu \wedge \text{Vol}(S_2^1) \wedge \text{Vol}(S_2^2). \tag{A.2}$$

As observed in [43, 44] for other AdS backgrounds (see appendix B for a general analysis), this solution arises in the $r \to \infty$ limit of the non-Abelian T-dual solution derived in section 2. This is straightforward for the metric and the NS-NS 2-form,[5] while the dilatons differ by an r^2 factor that accounts for the different integration measures in the partition functions of the Abelian and non-Abelian T-dual σ-models, as explained in [44]. The RR sector, even if the fields are different (see appendix B), yields to the same quantised charges, as we show below. Finally, in order to match both solutions globally, r must live in an interval of length π, $r \in [n\pi, (n+1)\pi]$, with $n \to \infty$. It is indeed in this limit in which there is precise agreement between the corresponding free energies.

The Abelian T-dual solution is also $\mathcal{N} = 4$ supersymmetric, as discussed in section 2, and has two singularities. The first singularity at $\mu = \pi$ is inherited from the stack of D6 branes in IIA. Indeed, one finds that for $\mu \sim \pi$

$$ds^2 \sim \frac{e^{2\phi_0/3}}{2}\left[\sqrt{\nu}\big(ds^2(AdS_4)+L^2 ds^2(S_1^2)\big)+\frac{L^2}{4\sqrt{\nu}}\big(d\tilde{r}^2+d\nu^2+\nu^2 ds^2(S_2^2)\big)\right], \; e^{\Phi} \sim \frac{e^{2\phi_0/3}}{2L}\sqrt{\nu} \tag{A.3}$$

which is the metric close to flat space smeared D5's, where we have defined $\tilde{r} = 4e^{-2\phi_0/3}/L^2 r$ and $\nu = (\pi - \mu)^2$. The second singularity at $\mu = 0$ is caused by NS5 branes localised there, wrapping S_2^2 and smeared along r. This is a generic result of T-dualising on the Hopf fibre of a 3-sphere with vanishing radius. Close to $\mu = 0$ one finds the metric (now $\nu = \mu^2$)

$$ds^2 \sim e^{2\phi_0/3}\left[ds^2(AdS_4) + L^2 ds^2(S_2^2) + \frac{L^2}{4\nu}\left(d\tilde{r}^2 + d\nu^2 + \nu^2 ds^2(S_1^2)\right)\right], \qquad e^{\Phi} \sim \frac{2e^{2\phi_0/3}}{L\sqrt{\nu}}, \tag{A.4}$$

as expected.

The Page charges of this solution are given by

$$\begin{aligned} N_{\text{D3}} &= \frac{1}{2\kappa_{10}^2 T_3}\int_{\Xi_2}(F_5 - B_2 \wedge F_3) = \frac{N_{\text{D2}}}{2}, \\ k_{\text{D5}} &= \frac{1}{2\kappa_{10}^2 T_5}\int F_3 = \frac{k}{2}, \\ N_{\text{NS5}} &= \frac{1}{2\kappa_{10}^2 T_{\text{NS5}}}\int_{(r,S_1^2)} H_3 = 1, \end{aligned} \tag{A.5}$$

where $\Xi_1 = (r, S_2^1, S_2^2)$, $\Xi_2 = (\mu, S_2^1, S_2^2)$ and we keep L defined as it was for IIA in eq (2.11). The factors of 2 in k_{D5}, N_{D3} originate from the different periodicities of the original and T-dual variables. They are usually absorbed through a redefinition of Newton's

[5] Note that B_2 arises in the gauge $B_2 = r\text{Vol}(S_1^2)$ in the Abelian T-dual.

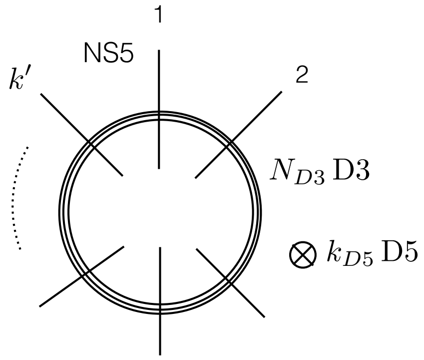

Figure 8. Brane set-up for the Abelian T-dual of the IIA reduction of $AdS_4 \times S^7/(\mathbb{Z}_k \times \mathbb{Z}_{k'})$. At each interval there are $N_{\rm D3}$ D3-branes stretched between the NS5-branes and $k_{\rm D5} = k/2$ transverse D5-branes.

constant. Comparing these charges with those of the non-Abelian T-dual solution, given in expressions (2.18), (2.21), we find that

$$N_{\rm D3}^{\rm NATD} = n\pi N_{\rm D3}^{\rm ATD}\,, \qquad k_{\rm D5}^{\rm NATD} = n\pi k_{\rm D5}^{\rm ATD}\,. \tag{A.6}$$

This same rescaling was found in [44] in the matching between the Abelian and non-Abelian T-dual AdS_5 spaces studied in that paper. As discussed there, the $n\pi$ factor can again be safely absorbed through a redefinition of Newton's constant. We give further details of the general relationship between T-dual and non-Abelian T-dual solutions in appendix B.

A simple generalisation is to allow $0 < r < k'\pi$, which is equivalent to taking the T-dual of the IIA reduction of the $AdS_4 \times S^7/(\mathbb{Z}_k \times \mathbb{Z}_{k'})$ orbifold. In that case $N_{\rm NS5} = k'$. The solution described in [59] corresponds to this situation. Its CFT dual consists of a circular quiver associated to a set of $N_{\rm D3}$ D3-branes, with $N_{\rm D3}$ as in (A.5), stretched between k' NS5-branes, as illustrated in figure 8. These D3-branes are thus winding D3-branes. At each interval of length π there are also $k_{\rm D5}$ D5-branes. The field theory associated to this brane configuration was studied in [59] and denoted as $C_\rho^{\hat\rho}(\mathrm{SU}(N), L)$, with the positive integer L refering to the number of winding D3-branes. These theories degenerate to the $T_\rho^{\hat\rho}(\mathrm{SU}(N))$ theories of [63] when $L = 0$. In the next subsection we illustrate the connection between the solution in [59] for $k' = 1$ and the Abelian T-dual solution under discussion. The value $k' = 1$ corresponds to the limiting case of $N_{\rm D3}$ D3-branes stretched between two NS5-branes that are identified.

A.2 Connection with ABEG geometries

As in the case of the non-Abelian T-dual solution, we expect that the Abelian T-dual, which also preserves $\mathcal{N} = 4$ SUSY, contains an AdS_4 factor and has SO(4) isometry, fits within the formalism described in section 3.2. Indeed, from eqs. (A.1), (A.2) we can read off the values

$$\lambda_1^2=\frac{\sqrt{(1-\sigma)}(1+\sigma)}{\sqrt{2}\beta}\quad \lambda_2^2=\frac{(1-\sigma)^{3/2}}{\sqrt{2}\beta}\quad \lambda^2=\frac{\sqrt{2}\sqrt{1-\sigma}}{\beta},\quad \frac{1}{\tilde{\rho}^2}=2\sqrt{2}\beta\sqrt{1-\sigma}(1+\sigma),$$
$$b_1=r-n\pi,\quad b_2=\frac{k}{2}r+c_0,\quad z=\sigma+i\beta r,\quad e^{2\Phi}=4\frac{1-\sigma}{k^2(1+\sigma)}, \tag{A.7}$$

where $d(c_0)=0$ and β and σ are defined as in section 5. In terms of the classification we find that the T-dual solution is given by

$$N_1=\frac{k^3}{128\beta^4}(1+\sigma),\qquad N_2=\frac{k}{32\beta^4}(1-\sigma),\qquad W=-\frac{k}{16\beta^2},\qquad X=0, \tag{A.8}$$
$$h_1=\frac{k}{4\beta}(1+\sigma),\qquad h_2=\frac{1}{2\beta}(1-\sigma),\qquad h_1^D=-\frac{1}{2}\left(c_0+\frac{k}{2}r\right),\qquad h_2^D=\frac{1}{2}r.$$

As discussed in [59], this solution does not fit however within the Ansatz of [58]. Clearly, even if there are NS5 and D5 branes located at $\sigma=\mp1$, which could be taken as boundaries of an infinite strip, the branes are smeared in r by construction, and h_1, h_2 do not exhibit logarithmic singularities at the locations of the branes. The authors of [58] showed in [59] how to solve this problem. They considered a distribution $(\delta_a,\hat{\delta}_b)$ of 5-branes that is repeated infinitely-many times along the strip with a period $2t$, such that $\delta_{a+p}-\delta_a=\hat{\delta}_{b+\hat{p}}-\hat{\delta}_b=2t$, where p and $\hat{p}$ are, correspondingly, the total numbers of D5 and NS5-branes stacks. The resulting h_1, h_2 are, by construction, periodic under $z\to z+2t$. This allows for the identification of points separated by the period $2t$, thus turning the strip into an annulus (and thus the linear quiver into a circular quiver) in the $e^{i\pi z/t}$ plane, with NS5 (D5) brane stacks along the inner (outer) boundaries. The smearing of the branes comes out as a result of taking the limit $t\to0$ combined with a far-from-the-boundaries approximation. h_1 and h_2 become then independent of r and non-singular.

The introduction of the period $2t$ on the gravity side induces the winding D3-branes on the dual quiver, in a way that we specify below. These branes do not end on the 5-branes and therefore do not contribute to the linking numbers. The corresponding circular quiver is then characterised by two partitions ρ and $\hat{\rho}$, together with the number of winding D3-branes. The $t\to0$ limit that yields the Abelian T-dual solution corresponds to a large number of these winding D3's, which are then identified with the $N_{\rm D3}$ in (A.5). In this approximation the number of D3-branes ending on 5-branes is negligible, and the brane picture depicted in figure 8 arises.[6]

For the sake of transparency, let us finally show that the IIB NS-sector derived in [59] for $k'=1$ NS5-branes matches our Abelian T-dual solution.[7] The Einstein frame metric and dilaton in [59] are

$$ds^2_{\rm IIB}=R^2g(y)^{1/4}\left[ds^2_{AdS_4}+yds^2_{S_1^2}+(1-y)ds^2_{S_2^2}\right]+R^2g(y)^{-3/4}\left[\frac{4t^2}{\pi^4}dx^2+dy^2\right],$$
$$e^{2\phi'}=\frac{k'}{k}\sqrt{\frac{1-y}{y}}, \tag{A.9}$$

[6] Indeed, in this approximation the period $2t$ is simply related to $N_{\rm D3}$ as $N_{\rm D3}=(\pi^2kk')/(32t^2)$.

[7] Up to a scaling factor and an S-duality transformation, given e.g. in (6.1) of [59] for $c=b=0$ and $a=d=-1$. Note also that [59] uses a non-standard form of the dilaton, $\phi'\equiv\Phi/2$.

where the AdS_4 space is taken to be of unit radius, $R^4 = \pi^4 kk'/t^2$ and $g(y) = y(1-y)$. If expressed in string frame by multiplying with $e^{\phi'} = e^{\Phi/2}$ and allowing for the coordinate change

$$y = \sin^2\left(\frac{\mu}{2}\right), \qquad x = \frac{1}{4}r, \tag{A.10}$$

the solution in equation (A.1) is reproduced for $k' = 1$, for which $r \in [0, \pi]$. It can also be easily checked that with this coordinate change the RR-sector in (A.2) corresponds to the one of [59], up to a gauge transformation.

A.3 Free energy

Using the results of the previous subsection, we can compute the free energy of the Abelian T-dual solution from W, h_1 and h_2 using expressions (6.1) and (6.2) (see [59]). Taking the differential area of the strip $d^2x = \beta dr d\sigma$ and integrating in $r \in [0, \pi]$, $\sigma \in [-1, 1]$, we find

$$S_{\rm eff} = \frac{k^2}{3\pi^2\beta^3}, \tag{A.11}$$

and, using the conserved charges in (A.5),

$$S_{\rm eff} = \frac{\sqrt{2}\pi}{3}\sqrt{k_{\rm D5}}N_{\rm D3}^{3/2}. \tag{A.12}$$

It can easily be checked that this is the free energy of the IIA reduction of the $AdS_4 \times S^7/\mathbb{Z}_k$ orbifold, with $N_{\rm D2} \to N_{\rm D3}$ and $k \to k_{\rm D5}$. It is shown in the main text that it agrees with the free energy of the non-Abelian T-dual solution in the $r \in [n\pi, (n+1)\pi]$ interval and $n \to \infty$.

B Relating Abelian and non-Abelian T-duality

In the previous appendix we discussed the relationship between the Abelian and non-Abelian T-dual AdS_4 spaces studied in this paper. In this appendix we complete this analysis and elucidate a general relationship between the geometries generated by acting on a round S^3 with Hopf fibre T-duality and SU(2) non-Abelian T-duality.

Consider a type II supergravity solution with global SO(4) isometry and NS sector that can be written as

$$ds^2 = ds^2(\mathcal{M}_7) + 4e^{2C}ds^2(S^3), \qquad B = 0, \qquad e^{\Phi} = e^{\Phi_0} \tag{B.1}$$

where x are coordinates on $\mathcal{M}_7$ only. Non-Abelian T-duality acting on such solutions was considered at length in [18–23]. It will be useful to parametrise the 3-sphere in two different ways, making manifest the two dualisation isometries

$$4ds^2(S^3_{\rm U(1)}) = d\theta^2 + \sin^2\theta d\phi^2 + (d\psi + \cos\theta d\phi)^2, \qquad 4ds^2(S^3_{\rm SU(2)}) = \left(\omega_1^2 + \omega_2^2 + \omega_3^3\right), \tag{B.2}$$

where ω_i are SU(2) left invariant 1-forms. The first of these is suitable for T-duality on ψ which, following [35], results in the dual NS sector

$$\begin{aligned} &ds^2_{\rm ATD} = ds^2(\mathcal{M}_7) + e^{-2C}dr^2 + e^{2C}ds^2(S^2), \\ &B_2^{\rm ATD} = r\,{\rm Vol}(S^2), \qquad\qquad e^{-\Phi_{\rm ATD}} = e^{C-\Phi_0}, \end{aligned} \tag{B.3}$$

where we have performed a gauge transformation on B_2 to put it in this form, and S^2 is the unit norm 2-sphere spanned by θ, ϕ. The second sphere parametrisation is suitable for SU(2) non-Abelian T-duality and leads to the dual NS sector

$$\begin{aligned} ds^2_{\text{NATD}} &= ds^2(\mathcal{M}_7) + e^{-2C}dr^2 + \frac{e^{2C}r^2}{r^2+e^{4C}}ds^2(S^2), \\ B_2^{\text{NATD}} &= \frac{r^3}{r^2+e^{4C}}\,\text{Vol}(S^2), \qquad\qquad e^{-\Phi_{\text{NATD}}} = \sqrt{r^2+e^{4C}}e^{C-\Phi_0}. \end{aligned} \tag{B.4}$$

Comparing eqs (B.3) and (B.4) one finds they obey the relation

$$\lim_{r\to\infty} ds^2_{\text{NATD}} = ds^2_{\text{ATD}}, \qquad \lim_{r\to\infty} B_2^{\text{NATD}} = B_2^{\text{ATD}}, \qquad \lim_{r\to\infty} e^{-\Phi_{\text{NATD}}} = re^{-\Phi_{\text{ATD}}}. \tag{B.5}$$

This has been observed in the previous appendix and before, for instance in [44], but what has not been addressed is whether such a relation holds also for the RR fluxes. We now address this by considering the massive IIA fluxes

$$\begin{aligned} F_0 &= m, \\ F_2 &= G_2, \\ F_4 &= G_4 + 8G_1\wedge\text{Vol}(S^3), \end{aligned} \tag{B.6}$$

however the following statements also hold when transforming from type IIB to IIA. Performing T-duality on the Hopf fibre as before leads to the dual fluxes

$$\begin{aligned} F_1^{\text{ATD}} &= -mdr, \\ F_3^{\text{ATD}} &= -dr\wedge G_2 - G_1\wedge\text{Vol}(S^2) \\ F_5^{\text{ATD}} &= -dr\wedge G_4 + e^{3C}\star_7 G_4\wedge\text{Vol}(S^2) \end{aligned} \tag{B.7}$$

while performing non-Abelian T-duality on the whole S^3 leads to

$$\begin{aligned} F_1^{\text{NATD}} &= -G_1 - mrdr, \\ F_3^{\text{NATD}} &= e^{3C}\star_7 G_4 - rdr\wedge G_2 - \frac{r^3}{r^2+e^{4C}}G_1\wedge\text{Vol}(S^2) + \frac{mr^2e^{4C}}{r^2+e^{4C}}dr\wedge\text{Vol}(S^2) \\ F_5^{\text{NATD}} &= -rdr\wedge G_4 + \frac{r^2e^{4C}}{r^2+e^{4C}}dr\wedge G_2\wedge\text{Vol}(S^2) + \frac{r^3e^{3C}}{r^2+e^{4C}}\star_7 G_4\wedge\text{Vol}(S^2) - e^{3C}\star_7 G_2. \end{aligned} \tag{B.8}$$

Comparing eqs (B.7) and (B.8), one sees that there is indeed a relation between the flux polyforms, namely

$$\partial_r(\lim_{r\to\infty} F^{\text{NATD}}) = F^{\text{ATD}}, \tag{B.9}$$

which $e^{-\Phi_{\text{NATD}}}$ clearly also obeys. Notice that we can dispense with the derivative by weighting the flux polyform by the dilaton, namely

$$\lim_{r\to\infty} e^{\Phi_{\text{NATD}}}F^{\text{NATD}} = e^{\Phi_{\text{ATD}}}F^{\text{ATD}}. \tag{B.10}$$

That this holds is actually not so surprising. As shown in [17, 68], under T-duality the fluxes transform in the combination $e^{\Phi}F$. Specifically the fluxes and MW Killing spinors are transformed by the same matrix Ω as

$$\begin{aligned} \epsilon_1 &= \epsilon_1^0, \\ \epsilon_2 &= \Omega\epsilon_2^0, \\ e^{\Phi}F &= e^{\Phi^0}F^0\Omega^{-1}, \end{aligned} \tag{B.11}$$

where 0 hat denotes the seed solution. For SU(2) non-Abelian T-duality performed on a round 3-sphere there exists a frame in which

$$\Omega^{\mathrm{NATD}} = \frac{1}{\sqrt{r^2+e^{4C}}}\left(\Gamma_{r12} + r\Gamma_r\right) \tag{B.12}$$

where the flat directions $1, 2$ span $e^{2C}S^2$ in the non-Abelian T-dual. Clearly

$$\lim_{r\to\infty}\Omega^{\mathrm{NATD}} = \lim_{r\to\infty}\left(\Omega^{\mathrm{NATD}}\right)^{-1} = \Gamma_r \tag{B.13}$$

which we recognise as Ω^{ATD}. This means that

$$\lim_{r\to\infty} e^{\Phi^0}F^0(\Omega^{\mathrm{NATD}})^{-1} = e^{\Phi^0}F^0(\Omega^{\mathrm{ATD}})^{-1} \tag{B.14}$$

and so eq (B.10) just reconciles this with the final expression in eq (B.11).

To conclude, we have observed that the Hopf fibre T-dual is related to the non-Abelian T-dual as

$$\lim_{r\to\infty}\begin{pmatrix} ds^2 \\ B_2 \\ e^{\Phi}F \\ \epsilon_{1,2}\end{pmatrix}_{\mathrm{NATD}} = \begin{pmatrix} ds^2 \\ B_2 \\ e^{\Phi}F \\ \epsilon_{1,2}\end{pmatrix}_{\mathrm{ATD}} \tag{B.15}$$

while the dilaton is related as

$$\lim_{r\to\infty} e^{-\Phi_{\mathrm{NATD}}} = re^{-\Phi_{\mathrm{ATD}}}. \tag{B.16}$$

As discussed below eq (2.15), it is easy to understand the r appearing in the dilaton at the level of the string frame supergravity actions, where this factor precisely cancels the change in the volume of the T-dual submanifold in the NS sector. In the RR sector, it is the combination $e^{\Phi}F$ that absorbs the volume change. The $r \to \infty$ limit of the non-Abelian T-dual thus reproduces the Abelian T-dual.

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3.5 The AdS_5 non-Abelian T-dual of Klebanov-Witten as a $\mathcal{N} = 1$ linear quiver from M5-branes

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The AdS_5 non-Abelian T-dual of Klebanov-Witten as a $\mathcal{N} = 1$ linear quiver from M5-branes

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Abstract: In this paper we study an AdS_5 solution constructed using non-Abelian T-duality, acting on the Klebanov-Witten background. We show that this is dual to a linear quiver with two tails of gauge groups of increasing rank. The field theory dynamics arises from a D4-NS5-NS5' brane set-up, generalizing the constructions discussed by Bah and Bobev. These realize $\mathcal{N} = 1$ quiver gauge theories built out of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ vector multiplets flowing to interacting fixed points in the infrared. We compute the central charge using a-maximization, and show its precise agreement with the holographic calculation. Our result exhibits n^3 scaling with the number of five-branes. This suggests an eleven-dimensional interpretation in terms of M5-branes, a generic feature of various AdS backgrounds obtained via non-Abelian T-duality.

Keywords: String Duality, AdS-CFT Correspondence, Conformal Field Models in String Theory

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Contents

1 Introduction

Non-Abelian T-duality [1], the generalization of the Abelian T-duality symmetry of String Theory to non-Abelian isometry groups, is a transformation between world-sheet field theories known since the nineties. Its extension to all orders in g_s and α' remains however a technically-hard open problem [2–8]. As a result, non-Abelian T-duality does not stand as a String Theory duality symmetry, as its Abelian counterpart does.

In the paper [9], Sfetsos and Thompson reignited the interest in this transformation by highlighting its potential powerful applications as a solution generating technique in supergravity. An interesting synergy between Holography (the Maldacena conjecture) [10] and non-Abelian T-duality was also pointed out. This connection was further exploited in [11]–[37]. These works have widely applied non-Abelian T-duality to generate new AdS backgrounds of relevance in different contexts. While some of the new solutions avoid previously existing classifications [11, 28, 31, 32], which has led to generalizations of existing families [38–41], some others provide the only known explicit solutions belonging to a given family [32, 33], which can be used to test certain conjectures, such as 3d-3d duality [42, 43]. Some of these works also put forward some ideas to define the associated holographic duals. Nevertheless, these initial attempts always encountered some technical or conceptual puzzle, rendering these proposals only partially satisfactory.

It was in the papers [44–46], where the field theoretical interpretation of non-Abelian T-duality (in the context of Holography) was first addressed in detail. One outcome of these works is that non-Abelian T-duality changes the dual field theory. In other words, that new AdS backgrounds generated through non-Abelian T-duality have dual CFTs different from those dual to the original backgrounds. This is possible because, contrary to its Abelian counterpart, non-Abelian T-duality has not been proven to be a String Theory symmetry.

The results in [44–46] open up an exciting new way to generate new quantum field theories in the context of Holography. In these examples the dual CFT arises in the low energy limit of a given Dp-NS5 brane intersection. This points to an interesting relation between AdS non-Abelian T-duals and M5-branes, that is confirmed by the n^3 scaling of the central charges.

Reversing the logic, the understanding of the field theoretical realization of non-Abelian T-duality brings in a surprising new way (using Holography!) to extract global information about the new backgrounds. Indeed, as discussed in the various papers [2–8], one of the long-standing open problems of non-Abelian T-duality is that it fails in determining global aspects of the dual background.

The idea proposed in [44] and further elaborated in [45, 46], relies on using the dual field theory to globally define (or *complete*) the background obtained by non-Abelian T-duality. In this way the Sfetsos-Thompson solution [9], constructed acting with non-Abelian T-duality on the $AdS_5 \times S^5$ background, was completed and understood as a superposition of Maldacena-Núñez solutions [47], dual to a four dimensional CFT. This provides a global definition of the background and also smoothes out its singularity. This idea was also put to work explicitly in [45] in the context of $\mathcal{N} = 4$ AdS_4 solutions. In this case the non-Abelian T-dual solution was shown to arise as a patch of a geometry discussed in [48–51], dual to the renormalization fixed point of a $T_\rho^{\hat{\rho}}(\mathrm{SU}(N))$ quiver field theory, belonging to the general class introduced by Gaiotto and Witten in [52].

In the two examples discussed in [44, 45] the non-Abelian T-dual solution arose as the result of zooming-in on a particular region of a completed and well-defined background. Remarkably, this process of zooming-in has recently been identified more precisely as a Penrose limit of a well-known solution. The particular example studied in the paper [46], a background with isometries $\mathbb{R}\times\mathrm{SO}(3)\times\mathrm{SO}(6)$, was shown to be the Penrose limit of a given Superstar solution [53]. This provides an explicit realization of the ideas in [44] that is clearly applicable in more generality.

In this paper we follow the methods in [44] to propose a CFT interpretation for the $\mathcal{N}=1$ AdS_5 background obtained in [12, 13, 28], by acting with non-Abelian T-duality on a subspace of the Klebanov-Witten solution [54]. We show that, similarly to the examples in [44, 45], the dual CFT is given by a linear quiver with gauge groups of increasing rank. The dynamics of this quiver is shown to emerge from a D4-NS5-NS5' brane construction that generalizes the Type IIA brane set-ups discussed by Bah and Bobev in [55], realizing $\mathcal{N}=1$ linear quivers built out of $\mathcal{N}=1$ and $\mathcal{N}=2$ vector multiplets that flow to interacting fixed points in the infrared. These quivers can be thought of as $\mathcal{N}=1$ twisted compactifications of the six-dimensional $(2,0)$ theory on a punctured sphere, thus providing a generalization to $\mathcal{N}=1$ of the $\mathcal{N}=2$ CFTs discussed in [56].

The results in this paper suggest that the non-Abelian T-dual solution under consideration could provide the first explicit gravity dual to an ordinary $\mathcal{N}=1$ linear quiver associated to a D4-NS5 brane intersection [55]. In this construction, the $\mathcal{N}=2$ SUSY D4-NS5 brane set-up associated to the Sfetsos-Thompson solution (see [44]) is reduced to $\mathcal{N}=1$ SUSY through the addition of extra orthogonal NS5-branes, as in [55]. The quiver that we propose does not involve the T_N theories introduced by Gaiotto [57], and is in contrast with the classes of $\mathcal{N}=1$ CFTs constructed in [58–61]. We support our proposal with the computation of the central charge associated to the quiver, which is shown to match exactly the holographic result. We also clarify a puzzle posed in [12, 13], where the non-Abelian T-dual background was treated as a solution in the general class constructed in [58, 59], involving the T_N theories, whose corresponding central charge was however in disagreement with the holographic result.

Before describing the plan of this paper, let us put the present work in a wider framework, discussing in some more detail the general ideas behind it.

1.1 General framework and organization of this paper

In the papers [12, 13], the non-Abelian T-dual of the Klebanov-Witten background was constructed. There, it was loosely suggested that the dual field theory could have some relation to the $\mathcal{N}=1$ version of Gaiotto's CFTs. Indeed, following the ideas in [60], the non-Abelian T-dual of the Klebanov-Witten solution could be thought of as a mass deformation of the non-Abelian T-dual of $AdS_5\times S^5/\mathbb{Z}_2$, as indicated in the following diagram,

$$\begin{tikzcd} AdS_5\times S^5/\mathbb{Z}_2 \arrow[r] \arrow[d, "\text{mass}"] & \text{NATD of } AdS_5\times S^5/\mathbb{Z}_2 \arrow[d, "\text{mass}"] \\ AdS_5\times T^{1,1} \arrow[r] & \text{NATD of } AdS_5\times T^{1,1}. \end{tikzcd}$$

Nevertheless, there were many unknowns and not-understood subtle issues when the papers [12, 13] were written. To begin with, the dual CFT to the non-Abelian T-dual of $AdS_5 \times S^5$ was not known, the holographic central charge of such background was not expressed in a way facilitating the comparison with the CFT result, the important role played by large gauge transformations [19, 25] had not been identified, etc. In hindsight, the papers [12, 13] did open an interesting line of research, but left various uncertainties and loose ends.

This line of investigations evolved to culminate in the works [44–46], that gave a precise dual field theoretical description of different backgrounds obtained by non-Abelian T-duality. This led to a field-theory-inspired *completion* or *regularization* of the non-Abelian T-dual backgrounds. Different checks of this proposal have been performed. Most notably, the central charge is a quantity that nicely matches the field theory calculation with the holographic computation in the completed (regulated) background.

In this paper we will apply the ideas of [44–46] and the field theory methods of [55] to the non-Abelian T-dual of the Klebanov-Witten background. A summary of our results is:

- We perform a study of the background and its quantized charges, and deduce the Hanany-Witten [62] brane set-up, in terms of D4 branes and two types of five-branes NS5 and NS5'.
- We calculate the holographic central charge. This requires a regularization of the background, particularly in one of its coordinates. The regularization we adopt here is a hard-cutoff. Whilst geometrically unsatisfactory, previous experience in [44] shows that this leads to sensible results, easy to compare with a field theoretical calculation.
- Based on the brane set-up, we propose a precise linear quiver field theory. This, we conjecture, is dual to the *regulated* non-Abelian T-dual background. We check that the quiver is at a strongly coupled fixed point by calculating the beta functions and R-symmetry anomalies.
- The quiver that we propose is a generalization of those studied in [55]. It can be thought of as a mass deformation of the $\mathcal{N} = 2$ quiver dual to the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$, that is constructed following the ideas in [44]. It is the presence of a flavor group in the CFT that *regulates* the space generated by non-Abelian T-duality.
- We calculate the field theoretical central charge applying the methods in [55]. We find precise agreement with the central charge computed holographically for the regulated non-Abelian T-dual solution.

In more detail, the present paper is organized as follows. In section 2, we summarize the main properties of the solution constructed in [12, 13]. We perform a detailed study of the quantized charges, with special attention to the role played by large gauge transformations. Our analysis suggests a D4, NS5, NS5' brane set-up associated to the solution, similar to

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that associated to the Abelian T-dual of Klebanov-Witten, studied in [63, 64]. In section 3 we summarize the brane set-up and $\mathcal{N}=1$ linear quivers of [55], which we use in section 4 for the proposal of a linear quiver that, we conjecture, is dual to the *regulated* version of the non-Abelian T-dual solution of $AdS_5 \times T^{1,1}$. We provide support for our proposal with the detailed computation of the (field theoretical) central charge which we show to be in full agreement with the (regulated) holographic result. We give an interpretation for the field theory dual to our background in terms of a mass deformation of the $\mathcal{N}=2$ CFT associated to the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$. This suggests the geometrically sensible way of completing our background. Section 5 contains a discussion where we further elaborate on the relation between our proposal and previous results in [12, 13]. We also resolve a puzzle raised there regarding the relation between the non-Abelian T-dual solution and the solutions in [59]. Concluding remarks and future research directions are presented in section 6. Detailed appendices complement our presentation. In appendix A, we explicitly calculate the differential forms showing that the non-Abelian T-dual solution fits in the classification of [65], for $\mathcal{N}=1$ SUSY spaces with an AdS_5-factor. Appendix B studies in detail the relation between the non-Abelian T-dual solution and its (Abelian) T-dual counterpart. Finally in appendix C we present some field theory results relevant for the analysis in section 4.

2 The non-Abelian T-dual of the Klebanov-Witten solution

In this section we summarize the Type IIA supergravity solution obtained after a non-Abelian T-duality transformation acts on the $T^{1,1}$ of the Klebanov-Witten background [54]. This solution was first derived in [12, 13]. It was later studied in [28] where a more suitable set of coordinates was used. More general solutions in Type IIA were constructed in [26] as non-Abelian T-duals of $AdS_5 \times Y^{p,q}$ Sasaki-Einstein geometries. Following our paper, the study of their dual CFTs appears to be a natural next step to investigate.

We start by introducing our conventions for the background and by summarizing the calculation of the holographic central charge of the $AdS_5 \times T^{1,1}$ solution.

2.1 The $AdS_5 \times T^{1,1}$ solution

The metric is given by,

$$\begin{aligned} ds^2 &= ds^2_{AdS_5} + L^2\, ds^2_{T^{1,1}}, \\ ds^2_{AdS_5} &= \frac{r^2}{L^2}\, dx^2_{1,3} + \frac{L^2}{r^2}\, dr^2, \\ ds^2_{T^{1,1}} &= \lambda_1^2(\sigma_{\hat{1}}^2 + \sigma_{\hat{2}}^2) + \lambda_2^2(\sigma_1^2 + \sigma_2^2) + \lambda^2(\sigma_3 + \cos\theta_1\, d\phi_1)^2, \end{aligned} \tag{2.1}$$

where $\lambda^2 = \frac{1}{9}$, $\lambda_1^2 = \lambda_2^2 = \frac{1}{6}$ and

$$\begin{aligned} &\sigma_{\hat{1}} = \sin\theta_1\, d\phi_1, && \sigma_{\hat{2}} = d\theta_1, \\ &\sigma_1 = \cos\psi\, \sin\theta_2\, d\phi_2 - \sin\psi\, d\theta_2, && \sigma_2 = \sin\psi\, \sin\theta_2\, d\phi_2 + \cos\psi\, d\theta_2, \\ &\sigma_3 = d\psi + \cos\theta_2\, d\phi_2. \end{aligned} \tag{2.2}$$

The background includes a constant dilaton and a self-dual RR five-form,

$$F_5 = \frac{4}{g_s\, L}\left[\mathrm{Vol}\big(AdS_5\big) - L^5\, \mathrm{Vol}\big(T^{1,1}\big)\right]. \tag{2.3}$$

The associated charge is given by

$$\frac{1}{2\,\kappa_{10}^2\, T_{D3}} \int_{T^{1,1}} F_5 = N_3\,. \tag{2.4}$$

Using that $2\,\kappa_{10}^2\, T_{Dp} = (2\pi)^{7-p}\, g_s\, \alpha'^{\frac{7-p}{2}}$ this leads to a quantization of the size of the space,

$$L^4 = \frac{27}{4}\,\pi\, g_s^2\, \alpha'^2\, N_3\,. \tag{2.5}$$

To calculate the holographic central charge of this background, we use the formalism developed in [28, 66]. Indeed, for a generic background and dilaton of the form,

$$ds^2 = a(r,\theta^i)\left[dx_{1,d}^2 + b(r)\, dr^2\right] + g_{ij}(r,\theta^i)\, d\theta^i\, d\theta^j, \quad \Phi(r,\theta^i), \tag{2.6}$$

we define the quantities $\hat{V}_{\rm int}$, $\hat{H}$ as,

$$\hat{V}_{\rm int} = \int d\theta^i \sqrt{\det[g_{ij}]\, e^{-4\Phi}\, a^d}\,, \qquad \hat{H} = \hat{V}_{\rm int}^2\,. \tag{2.7}$$

The holographic central charge for the $(d+1)$-dimensional QFT is calculated as,

$$c = \pi\, d^d\, \frac{b^{d/2}\hat{H}^{\frac{2d+1}{2}}}{G_{N,10}\,\big(\hat{H}'\big)^d}\,, \qquad G_{N,10} = 8\,\pi^6 g_s^2\, \alpha'^4. \tag{2.8}$$

Using these expressions for the background in eq. (2.1), we have

$$a = \frac{r^2}{L^2},\quad b = \frac{L^4}{r^4},\quad d = 3,\quad \sqrt{e^{-4\Phi}\, \det[g_{ij}]\, a^3} = g_s^{-2} L^2 r^3 \lambda\, \lambda_1^4\, \sin\theta_1\, \sin\theta_2\,. \tag{2.9}$$

After some algebra, we obtain the well-known result [67],

$$c_{KW} = \pi \frac{L^8}{108\pi^3 g_s^4 \alpha'^4} = \frac{27}{64} N_3^2\,. \tag{2.10}$$

We now study the action of non-Abelian T-duality on one of the SU(2) isometries displayed by the background in eq. (2.1). We use the notation and conventions in [28].

2.2 The non-Abelian T-dual solution

The NS-NS sector of the non-Abelian T-dual solution constructed in [12, 13, 28] is composed of a metric, a NS-NS two-form and a dilaton. Using the variables in [28], the metric reads,[1]

$$\begin{aligned} d\hat{s}^2 = \frac{r^2}{L^2} dx_{1,3}^2 + \frac{L^2}{r^2} dr^2 + L^2\lambda_1^2\big(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\big) + \frac{L^2}{Q}\Big[\lambda_1^4\,\big(\cos\chi\, d\rho - \rho\sin\chi\, d\chi\big)^2 \\ + \lambda^2\lambda_1^2\big(\sin\chi\, d\rho + \rho\cos\chi\, d\chi\big)^2 + \lambda^2\lambda_1^2\,\rho^2\sin^2\chi\,\big(d\xi + \cos\theta_1 d\phi_1\big)^2 + \rho^2 d\rho^2\Big]\,. \end{aligned} \tag{2.11}$$

[1] Henceforth we use the rescaling $\rho \longrightarrow \frac{L^2}{\alpha'}\rho$ so that all factors in the internal metric scale with L^2. We also substitute $\lambda_2 = \lambda_1$ for convenience.

The NS two-form is,

$$\begin{aligned} B_2 =& \frac{L^2\rho^2\sin\chi}{2Q}\Big[\big(\lambda^2-\lambda_1^2\big)\sin 2\chi\; d\xi\wedge d\rho + 2P\,\rho\; d\xi\wedge d\chi\Big] \\ &- \frac{L^2\lambda^2\cos\theta_1}{Q}\Big[\big(\lambda_1^4+\rho^2\big)\cos\chi d\rho\wedge d\phi_1 - \lambda_1^4\;\rho\sin\chi\; d\chi\wedge d\phi_1\Big]\,, \end{aligned} \tag{2.12}$$

and the dilaton is given by,[2]

$$e^{-2\hat{\Phi}} = \frac{L^6\;Q}{g_s^2\;\alpha'^3}\;. \tag{2.13}$$

For convenience we have defined the following functions,

$$Q = \lambda^2\lambda_1^4 + \rho^2 P\,, \qquad P = \lambda^2\cos^2\chi + \lambda_1^2\sin^2\chi = \lambda_1^2 + \big(\lambda^2-\lambda_1^2\big)\cos^2\chi\;. \tag{2.14}$$

This solution is supported by a set of RR fluxes which read,

$$\begin{aligned} F_2 &= -\frac{4\,L^4\lambda\,\lambda_1^4}{g_s\,\alpha'^{3/2}}\sin\theta_1\, d\theta_1\wedge d\phi_1\,, \\ F_4 &= -\frac{2\,L^6\lambda\,\lambda_1^4}{g_s\,\alpha'^{3/2}Q}\,\rho^2\sin\chi\;\sin\theta_1\, d\theta_1\wedge d\phi_1\wedge\Big[\big(\lambda^2-\lambda_1^2\big)\sin 2\chi\, d\xi\wedge d\rho + 2\,P\,\rho\, d\xi\wedge d\chi\Big] \\ &= B_2\wedge F_2\;. \end{aligned} \tag{2.15}$$

The higher rank RR fields which are related to the previous ones through $F_p = \big(-1\big)^{[p/2]}\star F_{10-p}$ read,

$$\begin{aligned} F_6 &= -\frac{4\,L^3}{g_s\,\alpha'^{3/2}}\,\rho\,\mathrm{Vol}_{AdS_5}\wedge d\rho\,, \\ F_8 &= -\frac{4\,L^5\lambda^2\lambda_1^4}{g_s\,\alpha'^{3/2}\,Q}\,\rho^2\;\sin\chi\,\mathrm{Vol}_{AdS_5}\wedge d\rho\wedge d\chi\wedge\big(d\xi+\cos\theta_1\, d\phi_1\big)\;. \end{aligned} \tag{2.16}$$

The associated RR potentials C_1 and C_3, defined through the formulas $F_2 = dC_1$ and $F_4 = dC_3 - H_3\wedge C_1$, are given by,

$$\begin{aligned} C_1 &= \frac{4\,L^4\,\lambda\,\lambda_1^4}{g_s\,\alpha'^{3/2}}\cos\theta_1\, d\phi_1\,, \\ C_3 &= \frac{2\,L^6\,\lambda\,\lambda_1^4}{g_s\,\alpha'^{3/2}Q}\,\rho^2\;\cos\theta_1\;\sin\chi\,\Big[\big(\lambda_1^2-\lambda^2\big)\;\sin 2\chi\, d\rho\wedge d\xi - 2\,P\,\rho\, d\chi\wedge d\xi\Big]\wedge d\phi_1 \\ &= B_2\wedge C_1\;. \end{aligned} \tag{2.17}$$

In the papers [12, 13] this solution of the Type IIA equations of motion was shown to preserve $\mathcal{N}=1$ supersymmetry. In the coordinates used in this paper the Killing vector ∂_ξ is dual to the R-symmetry of the CFT.

In appendix A we promote the background in eqs. (2.11)–(2.17) to a solution of eleven-dimensional supergravity. We show that this background fits in the classification of $\mathcal{N} =$

[2] As in the original paper [1], the dilaton needs to transform as well in order to fulfil the equations of motion.

1 AdS_5 solutions in M-theory of [65]. We write in detail the forms satisfying a set of differential relations and define the SU(2)-structure. The eleven dimensional lift suggests that this solution is associated to M5-branes wrapped on a spherical 2d manifold. We discuss this picture further in section 5.

As indicated, one goal of this paper is to propose a conformal field theory dual to the Type IIA non-Abelian T-dual solution. We will do this by combining different insights coming from the large ρ-asymptotics, the quantized charges and the calculation of field theoretical observables using the background.

2.2.1 Asymptotics

In complicated systems, like those corresponding to intersections of branes, it is often illuminating to consider the asymptotic behavior of the background. In the case at hand, for the background in eqs. (2.11)–(2.17), we consider the leading-order behavior of the solution, when $\rho \to \infty$. This allows us to *read* the brane intersection that in the decoupling limit and for a very large number of branes generates the solution.

Indeed, for $\rho \to \infty$, the leading behavior of the NS-fields is

$$
\begin{aligned}
ds^2 &\approx ds^2_{AdS_5} + L^2\,\lambda_1^2 \left[d\Omega_2^2(\theta_1,\phi_1) + \left(d\chi^2 + \frac{\lambda^2 \sin^2\chi}{P(\chi)} \left(d\xi + \cos\theta_1 d\phi_1 \right)^2 \right) + \frac{d\rho^2}{\lambda_1^2 P(\chi)} \right] , \\
B_2 &\approx -L^2 \rho \left[d\Omega_2(\chi,\xi) + \frac{\lambda^2 \cos\chi}{P(\chi)}\, d\Omega_2(\theta_1,\phi_1) - \lambda^2\, \cos\theta_1\, \partial_\chi \left(\frac{\cos\chi}{P(\chi)} \right) d\chi \wedge d\phi_1 \right] \\
&\quad + \frac{L^2 \sin\chi}{2\,P(\chi)} \left(\lambda^2 - \lambda_1^2 \right) \sin 2\chi\, d\xi \wedge d\rho \,, \qquad (2.18) \\
e^{-2\phi} &\approx \frac{L^6}{g_s^2 \alpha'^3}\, P(\chi)\, \rho^2 \,,
\end{aligned}
$$

where we have performed a gauge transformation in B_2, of the form $B_2 + d\Lambda_1$, with

$$
\Lambda_1 = L^2 \lambda^2 \rho \cos\theta_1 \left(\frac{\cos\chi}{P(\chi)} \right) d\phi_1 \,.
$$

Intuitively, this result suggests that we have two different types of NS-five branes. One type of five-branes (which we refer to as *NS*) extend along $\mathbb{R}^{1,3} \times S^2(\theta_1,\phi_1)$. The second type of five branes (referred to as *NS'*) extend along $\mathbb{R}^{1,3} \times \tilde{S}^2(\chi,\xi)$. To preserve SUSY, the spaces $S^2(\theta_1,\phi_1)$ and $\tilde{S}^2(\chi,\xi)$ are fibered by the monopole gauge field $A_1 = \cos\theta_1 d\phi_1$. This fibration is also reflected in the B_2-field, that contains a term that mixes the spheres.

The asymptotics of the RR-fields can be easily read from eq. (2.16). Indeed, the expression $F_6 = dC_5$, generates asymptotically $C_5 \approx \rho r^4 dx_{1,3} \wedge d\rho$. This suggests an array of D4 branes extended along the directions $\mathbb{R}^{1,3} \times \rho$. D6 branes appear due to the presence of the B_2-field, that blows up the D4 branes due to the Myers effect [68].

In summary, the asymptotic analysis suggests that the background in eqs. (2.11)–(2.17), is generated in the decoupling limit of an intersection of NS5-NS5'-D4 branes. This will be confirmed by the calculation of the quantized charges associated to this solution.

2.2.2 Quantized charges

In the papers [44, 45], the brane set-ups encoding the dynamics of the CFTs dual to the corresponding non-Abelian T-dual backgrounds were proposed after a careful analysis of the quantized charges. The charges that are relevant for the study of the non-Abelian T-dual of the Klebanov-Witten background are those related to $D4$, $D6$ and $NS5$ branes. Based on this analysis we will propose an array of branes, from which the dynamics of a linear quiver with gauge groups of increasing rank will be obtained.

For $D6$ branes the Page charge reads,

$$Q_{D6} = \frac{1}{2\,\kappa_{10}^2\,T_{D6}} \int_{(\theta_1,\phi_1)} F_2 = \frac{8\,L^4 \lambda\,\lambda_1^4}{g_s^2\;\alpha'^2} = \frac{2}{27}\frac{L^4}{g_s^2\,\alpha'^2} = N_6\,, \tag{2.19}$$

where we have absorbed an overall minus sign by choosing an orientation for the integrals. Imposing the quantization of the $D6$ charge, the AdS radius L is quantized in terms of N_6,

$$L^4 = \frac{27}{2}\,g_s^2\,\alpha'^2 N_6\ . \tag{2.20}$$

This relation differs from that for the original background, see eq. (2.5), which is a common feature already observed in the bibliography [19].

In turn, the Page charge associated to $D4$-branes vanishes,

$$Q_{D4} = \frac{1}{2\,\kappa_{10}^2\,T_{D4}} \int_{M_4} \big(F_4 - F_2\wedge B_2\big) = 0\ . \tag{2.21}$$

This charge becomes however important in the presence of large gauge transformations,

$$B_2 \to B_2 + \Delta B_2\,, \tag{2.22}$$

under which the Page charges transform as,

$$\Delta Q_{D4} = -\frac{1}{2\,\kappa_{10}^2\,T_{D4}} \int_{M_4} F_2\wedge \Delta B_2\,, \qquad \Delta Q_{D6} = 0\ . \tag{2.23}$$

Indeed, consider a four-manifold $M_4 = [\theta_1,\phi_1]\times\Sigma_2$, with the two-cycle given by $\Sigma_2 = [\chi,\xi]$.[3] Under a large gauge transformation of the form,

$$\Delta B_2 = -n\,\pi\,\alpha' \sin\chi\, d\chi\wedge d\xi\,, \qquad d\big[\Delta B_2\big] = 0\,, \tag{2.24}$$

the Page charges transform as

$$\Delta Q_{D4} = n\,N_6\,, \qquad \Delta Q_{D6} = 0\ . \tag{2.25}$$

The first relation shows that n units of D4-brane charge are induced in each D6-brane. Conversely, nN_6 D4-branes can expand in the presence of the B_2 field given by eq. (2.24) into N_6 D6-branes wrapped on Σ_2, through Myers dielectric effect. Consider now the (conveniently normalized) integral of the B_2 field, given by eq. (2.12), along the non-trivial

[3] Note that this 2-cycle vanishes at $\rho\to 0$, while at $\rho\to\infty$ it is *almost* a two sphere of finite size.

2-cycle $\Sigma_2 = [\chi, \xi]$. Following the paper [25], this must take values in the interval $[0, 1]$.[4] Imposing this condition implies that $|b_0| \leq 1$ with,

$$b_0 = \frac{1}{4\,\pi^2\,\alpha'} \int_{\Sigma_2} B_2 = -\frac{1}{\pi}\frac{L^2}{\alpha'}\left[\rho - \frac{\sqrt{2}}{6\sqrt{1+54\,\rho^2}}\tanh^{-1}\!\left(\frac{3\sqrt{2}\,\rho}{\sqrt{1+54\,\rho^2}}\right)\right] . \tag{2.26}$$

The asymptotic behavior of b_0 for small and large values of ρ is given by,

$$\begin{aligned} b_0 &= -\frac{48\,L^2\rho^3}{\pi\alpha'} + \mathcal{O}(\rho^5)\,, \qquad \rho \ll 1\,, \\ b_0 &= -\frac{L^2}{\alpha'}\frac{\rho}{\pi} + \mathcal{O}\left(\frac{1}{\rho}\right), \qquad \rho \gg 1\ . \end{aligned} \tag{2.27}$$

The expression given by eq. (2.26) is monotonically increasing for all $\rho \in [0, \infty)$, and takes the value $|b_0| = 1$ only once. In order to bring the function $|b_0(\rho)|$ back to the interval $[0, 1]$ we need to perform a large gauge transformation of the type defined in eq. (2.24), whenever $|b_0(\rho_n)| = n$, $n \in \mathbb{N}$. The number of D4-branes in the configuration then increases by a multiple of N_6, as implied by eq. (2.25), each time we cross the position $\rho = \rho_n$.

The form of the B_2 potential in eq. (2.12) suggests that it is also possible to take a different 2-cycle,

$$\Sigma_2' = [\theta_1, \phi_1]_{\chi=0}\ , \tag{2.28}$$

which is a rounded $S^2(\theta_1, \phi_1)$ at $\chi = 0$. As in the case analyzed above, large gauge transformations are needed as we move in ρ in order to render b_0 in the fundamental region, $b_0 \in [0, 1]$. This shift does not modify however the number of D4 or D6-branes, while it induces NS5-brane charge (we call these NS5' for later convenience) in the configuration.

Indeed, let us discuss the NS5-brane charges associated to the solution. Let us first consider the three-cycle,

$$\Sigma_3 = [\rho, \chi, \xi]\,, \tag{2.29}$$

built out of the first 2-cycle $\Sigma_2 = [\chi, \xi]$ and the ρ-coordinate. Taking into account the expression for the B_2 field given by eq. (2.12) one finds,

$$H_3\big|_{\Sigma_3} = L^2\left[\frac{(\lambda^2-\lambda_1^2)\,\rho^2}{2}\,\partial_\chi\!\left(\frac{\sin\chi\sin 2\chi}{Q}\right) - P\sin\chi\ \partial_\rho\!\left(\frac{\rho^3}{Q}\right)\right] d\rho\wedge d\chi\wedge d\xi\ . \tag{2.30}$$

The first term does not contribute to the charge, which reads,

$$Q_{NS5} = \frac{1}{4\,\pi^2\,\alpha'}\int_{(\rho,\chi,\xi)} H_3 = -\frac{1}{4\,\pi^2\,\alpha'}\,2\,\pi\,L^2\rho_n^3\int_0^\pi \frac{P}{Q}\,\sin\chi\,d\chi = b_0(\rho_n) = n\ . \tag{2.31}$$

This calculation shows that we have n $NS5$ branes for $\rho \in [0, \rho_n]$. If, on the other hand, we take the cycle defined by

$$\Sigma_3' = [\rho, \phi_1, \theta_1]_{\chi=0}, \tag{2.32}$$

we find that $\rho_n' = n\,\pi\,\alpha'/L^2$ and that a new NS5' brane is created each time we cross these values ρ_n' for $n = 1, 2, \ldots$.

[4] A physical interpretation of this condition in terms of a fundamental string action was presented in [35].

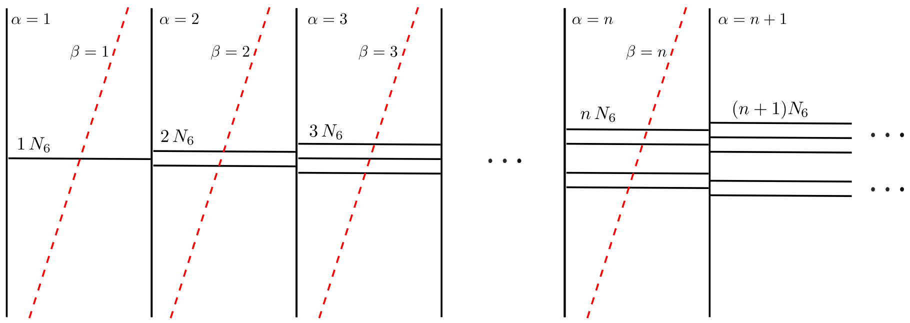

Figure 1. Brane set-up consistent with the quantized charges of the non-Abelian T-dual solution, consisting on $\alpha = 1, 2, \ldots, n+1$ NS5-branes (vertical black lines), $\beta = 1, 2, \ldots, n$ NS5'-branes (tilted red dashed lines) and $m\, N_6$ D4-branes (horizontal lines), where $m = 1, 2, \ldots, n+1$ changes by one each time a NS5-brane is crossed.

The conclusion of this analysis is that one can define *two types* of NS5-branes in the non-Abelian T-dual background: NS5-branes located at ρ_n and transverse to $\tilde{S}^2(\chi, \xi)$, and NS5'-branes located at $\rho'_n = n\,\pi\,\alpha'/L^2$ and transverse to $S^2(\theta_1, \phi_1)$. These branes are localized in the ρ direction, such that a NS5'-brane lies in between each pair of NS5-branes, as illustrated in figure 1. Further, as implied by eq. (2.25), N_6 D4-branes are created each time a NS5-brane is crossed. This brane set-up will be the basis of our proposed quiver in section 4, and will be instrumental in defining the dual CFT of the non-Abelian T-dual solution. As we will see, it will allow us to identify the global symmetries and the parameters characterizing the associated field theory.

Let us study now an important field theoretical quantity, calculated from the Type IIA solution, the central charge.

2.2.3 Central charge

In this section, we compute the holographic central charge associated to the non-Abelian T-dual solution in eqs. (2.11)–(2.17). This will be the main observable to check the validity of the $\mathcal{N}=1$ quiver proposed in section 4.

We must be careful about the following subtle point. The calculation of the quantity $\hat{V}_{\text{int}}$ in eq. (2.7), will involve an integral in the ρ-direction of the metric in eq. (2.11). The range of this coordinate is not determined by the process of non-Abelian T-duality (the global issues we referred to in the Introduction). If we take $0 \leq \rho < \infty$, we face the problem that the central charge will be strictly infinite. A process of *regularization* or *completion* of the background of eqs. (2.11)–(2.17) is needed. In this paper we choose to end the space with a hard cut-off, namely $0 \leq \rho \leq \rho_n$. We do know that this is geometrically unsatisfactory. Nevertheless, the field theoretical analysis of section 4 will teach us that a flavor group, represented by D6 branes added to the background of eqs. (2.11)–(2.17), should end the space in the correct fashion. Previous experience [44] tells us that the hard-cutoff used here does capture the result for the holographic central charge that is suitable to compare with the field theoretical one found in section 4.

We then proceed, by considering the metric in eq. (2.11), the dilaton in eq. (2.13) and eqs. (2.6)–(2.8). We obtain,

$$c_{\text{KWNATD}} = \frac{9\, L^6 \left(\rho_b^3 - \rho_a^3\right)}{64\, \pi^3\, \alpha'^3}\, N_6^2, \tag{2.33}$$

where we have integrated ρ between two arbitrary values $[\rho_a, \rho_b]$. We have also used the quantization condition of eq. (2.20). For $\rho \in \left[0, n\,\pi\,\alpha'/L^2\right)$ this gives

$$c^{(0,n)}_{\text{KWNATD}} = \frac{9}{64}\, n^3 N_6^2\,. \tag{2.34}$$

On the other hand, for $\rho \in \left[n\,\pi\,\alpha'/L^2,\, (n+1)\,\pi\,\alpha'/L^2\right)$ we obtain,

$$c^{(n,n+1)}_{\text{KWNATD}} = \frac{9}{64}\, N_6^2 \left(3\, n^2 + 3\, n + 1\right). \tag{2.35}$$

This becomes $c^{(n,n+1)}_{\text{KWNATD}} = \frac{27}{64}\, N_4^2$ in the large n limit, where $\rho'_n = \rho_n$ and we can use that $N_4 = nN_6$ in the $\left[n\,\pi\,\alpha'/L^2,\, (n+1)\,\pi\,\alpha'/L^2\right)$ interval. Interestingly, this expression coincides with the central charge of the Abelian T-dual of the Klebanov-Witten background, that we discuss in detail in appendix B. This is that of the original background — see eq. (2.10), with N_3 replaced by N_4,

$$c_{\text{KWATD}} = \frac{27}{64}\, N_4^2. \tag{2.36}$$

For completeness, we also reproduce in appendix C.3 this value of the central charge from the field theory, using a-maximization. This matching between the central charges of non-Abelian and Abelian T-duals was found in previous examples [44, 45].

Next, we review aspects of the $\mathcal{N} = 1$ quivers discussed in [55]. These will be the basis of the quiver proposed to describe the field theory associated to the non-Abelian T-dual solution. In section 4, the holographic result in eq. (2.34) will be found by purely field theoretical means.

3 Basics of Bah-Bobev 4d $\mathcal{N} = 1$ theories

In this section, we provide a summary of the results in [55], which will be instrumental for our proposal of a field theory dual to the background in eqs. (2.11)–(2.17).

3.1 $\mathcal{N} = 1$ linear quivers

In [55], Bah and Bobev introduced $\mathcal{N} = 1$ linear quiver gauge theories built out of $\mathcal{N} = 2$ and $\mathcal{N} = 1$ vector multiplets and ordinary matter multiplets. These theories were argued to flow to interacting 4d $\mathcal{N} = 1$ SCFTs in the infrared. They consist of products of $\ell - 1$ copies of SU(N) gauge groups, with either $\mathcal{N} = 1$ (shaded) or $\mathcal{N} = 2$ (unshaded) vector multiplets — see figure 2. Let n_1 be the number of $\mathcal{N} = 1$ vector multiplets and n_2 the number of $\mathcal{N} = 2$ vector multiplets. There are also $\ell - 2$ bifundamental hypermultiplets of SU(N) $\times$ SU(N), depicted in figure 2 as lines between the nodes, and two sets of N

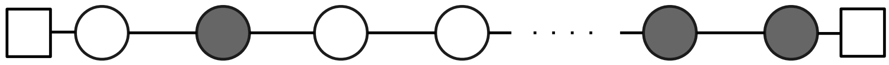

Figure 2. General linear quiver in [55]. Shaded (unshaded) circles represent SU(N), $\mathcal{N}=1$ ($\mathcal{N}=2$) vector multiplets. Lines between them represent bifundamentals of $\mathrm{SU}(N)\times\mathrm{SU}(N)$. The boxes at the two ends represent SU(N) fundamentals.

hypermultiplets transforming in the fundamental of the two end SU(N) gauge groups. Thus, there are in total $\ell-1=n_1+n_2$ gauge groups and ℓ matter multiplets. The total global symmetry is,

$$\mathrm{SU}(N)\times\mathrm{SU}(N)\times\mathrm{U}(1)^{\ell+n_2}\times\mathrm{U}(1)_R,$$

corresponding to the SU(N) flavor symmetries acting on the end hypermultiplets, the U(1) flavor symmetry acting on each of the ℓ hypermultiplets, the U(1) flavor acting on the chiral adjoint superfields (there are as many as $\mathcal{N}=2$ vector multiplets) and the R-symmetry. Out of these $\mathrm{U}(1)'s$ only a certain non-anomalous linear combination will survive in the IR SCFT. Similarly, the fixed point R-charge is computed through a-maximization [69] as a non-anomalous linear combination of the U(1)'s and $\mathrm{U}(1)_R$.

As shown in [55], it is convenient to assign a charge $\sigma_i=\pm1$ to each matter hypermultiplet, with the rule that $\mathcal{N}=1$ vector multiplets connect hypermultiplets with opposite sign, while $\mathcal{N}=2$ vector multiplets connect hypermultiplets with the same sign. Let p be the number of hypermultiplets with $\sigma_i=+1$ and $q=\ell-p$ those with $\sigma_i=-1$, and let us introduce the twist parameter z,

$$z=\frac{p-q}{\ell}\,. \tag{3.1}$$

Thus, $z=\pm1$ corresponds to a quiver with only $\mathcal{N}=2$ nodes, involving hypermultiplets of the same charge. $z=0$ corresponds in turn to a quiver with the same number of hypermultiplets of each type, so it includes the quiver with only $\mathcal{N}=1$ nodes. We will focus on $0\le z\le1$ ($q\le p$) without loss of generality. We also introduce $\kappa=(\sigma_0+\sigma_l)/2$, which can take values $\kappa=-1,0,+1$. This will later be associated to the type of punctures on the Riemann surface on which M5-branes are wrapped.

In a superconformal fixed point the a and c central charges can be computed from the 't Hooft anomalies associated to the R-symmetry [70],

$$a=\frac{3}{32}\left(3\,\mathrm{Tr}\,R^3-\mathrm{Tr}\,R\right),\qquad c=\frac{1}{32}\left(9\,\mathrm{Tr}\,R^3-5\,\mathrm{Tr}\,R\right), \tag{3.2}$$

where the R-symmetry is given by

$$R_\epsilon=R_0+\frac{1}{2}\,\epsilon\,\mathcal{F}\,, \tag{3.3}$$

and R_0 is the anomaly free R-symmetry, $\mathcal{F}$ is the non-anomalous global U(1) symmetry and ϵ is a number that is determined by a-maximization [69]. This was used in [55] to compute the a and c central charges associated to the general quiver represented in figure 2. Their values were shown to depend only on the set of parameters $\{\kappa,z,\ell,N\}$. It was then conjectured that all quivers with the same $\{\kappa,z,\ell,N\}$ should be dual to each

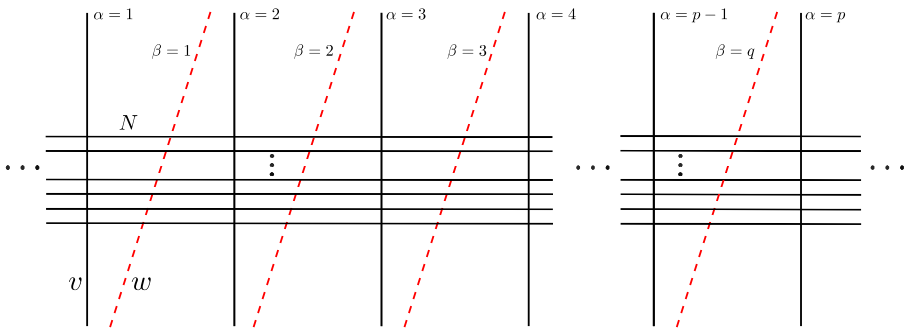

Figure 3. The brane set-up associated to the Bah-Bobev $\mathcal{N} = 1$ linear quivers. Vertical lines represent NS5-branes extended along $\{x_4, x_5\}$, denoted in [55] as v-branes, while diagonal lines represent the NS5'-branes extended along $\{x_7, x_8\}$, denoted as w-branes. The same number of D4-branes extended along the x_6 direction stretch between adjacent 5-branes.

other and flow to the same SCFT in the infrared. Moreover, for $\ell \to \infty$ the two central charges were shown to agree. Therefore, in this limit the quivers can admit holographic AdS duals. In section 4 we will provide a variation of these $\mathcal{N} = 1$ quivers for which this condition is satisfied, and argue that it is associated to the AdS_5 non-Abelian T-dual solution presented in section 2.

3.2 IIA brane realization and M-theory uplift

Interestingly, it was shown in [55] that the linear quivers discussed above have a natural description in terms of D4, NS5, NS5' brane set-ups that generalize the $\mathcal{N} = 2$ brane constructions in [56], and allow for an M-theory interpretation. The two types of NS5-branes in this construction are taken to be orthogonal to each other, explicitly breaking $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$. The specific locations of the branes involved are

- N coincident D4-branes extend along $\mathbb{R}^{1,3}$ and the x_6 direction.
- p non-coincident NS5-branes extend along $\mathbb{R}^{1,3} \times \{x_4, x_5\}$, and sit at $x_6 = x_6^\alpha$ for $\alpha = 1, \ldots, p$.
- q non-coincident NS5'-branes extend along $\mathbb{R}^{1,3} \times \{x_7, x_8\}$, and sit at $x_6 = x_6^\beta$ for $\beta = 1, \ldots, q$.

The corresponding brane set-up is depicted in figure 3, see also [55].

In this configuration, open strings connecting D4-branes stretched between two parallel NS5-branes are described at long distances and weak coupling by an $\mathcal{N} = 2$ SU(N) vector multiplet, while those connecting D4-branes stretched between perpendicular NS5 and NS5' branes are described by an $\mathcal{N} = 1$ SU(N) vector multiplet. In turn, open strings connecting adjacent D4-branes separated by a NS5-brane (NS5'-brane) are described at low energies by bifundamental hypermultiplets with charge $\sigma_i = 1$ ($\sigma_i = -1$). Finally, semi-infinite N D4-branes (or D6 branes) at both ends of the configuration yield two sets

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of hypermultiplets in the fundamental representation of SU(N). The resulting field theory is effectively four dimensional at low energies compared to the inverse size of the D4 along x_6. The effective gauge coupling behaves as $\frac{1}{g_4^2} \sim \frac{x_{6,n+1}-x_{6,n}}{g_s\sqrt{\alpha'}}$. Given that the 5-branes can be freely moved along the x_6 direction, the gauge couplings are marginal parameters. Rotations in the $v = x_4 + ix_5$ and $w = x_7 + ix_8$ planes of the NS5 and NS5' branes give a $\mathrm{U}(1)_v$ and a $\mathrm{U}(1)_w$ global symmetry, so that the IR fixed point R-symmetry and flavor U(1) are realized geometrically as linear combinations of them:

$$R_0 = \mathrm{U}(1)_v + \mathrm{U}(1)_w\,, \qquad \mathcal{F} = \mathrm{U}(1)_v - \mathrm{U}(1)_w\,. \tag{3.4}$$

Relying on similar $\mathcal{N}=2$ constructions in [56], it is possible to describe the previous system of intersecting branes at strong coupling in M-theory. The x_6 direction is combined with the M-theory circle x_{11} to form a complex coordinate $s = (x_6 + ix_{11})/R_{11}$ describing a Riemann surface Σ_2, which is a punctured sphere or, equivalently, a punctured cylinder. The uplift of this system yields,

- N M5-branes wrapping the cylinder, from the N D4-branes extended on x_6.
- p simple punctures (in the language of [57]) on the cylinder, coming from the p transversal M5-branes with flavor charge $\sigma_i = 1$.
- q simple punctures on the cylinder, coming from the q transversal M5-branes with flavor charge $\sigma_i = -1$.
- Two maximal punctures, coming from the stacks of N transversal M5-branes at both ends of the cylinder. They are also assigned $\sigma_0, \sigma_\ell = \pm 1$, from which the additional parameter $\kappa = (\sigma_0 + \sigma_\ell)/2$ is defined, taking values $\kappa = -1, 0, +1$.

The cylinder or sphere the M5-branes wrap can be viewed as a Riemann surface $\mathcal{C}_{g,n}$ of genus $g = 0$ and $n = p + q + 2$ punctures, so that $\Sigma_2 = \mathcal{C}_{0,n}$. This Riemann surface can be deformed by bringing some of the punctures close to each other (which corresponds to certain weak and strong coupling limits of the dual 6d $\mathcal{N} = (0, 2)$ A_{N-1} field theory living on the M5-branes) to a collection of higher-genus and less-punctured surfaces. The κ parameter is associated to the type of punctures on the $\mathcal{C}_{g,n}$ Riemann surface.

This closes our summary of the findings of the paper [55], that we will use in the next section. Let us now propose a dual CFT to our background in eqs. (2.11)–(2.17).

4 The non-Abelian T-dual of Klebanov-Witten as a $\mathcal{N}=1$ linear quiver

As we showed in section 2.2.2, the analysis of the quantized charges of the non-Abelian T-dual solution is consistent with a D4, NS5, NS5' brane set-up in which the number of D4-branes stretched between the NS5 and NS5' branes increases by N_6 units every time a NS5-brane is crossed. This configuration thus generalizes the brane set-ups discussed in the previous section and in [55].

In this section, inspired by the previous analysis, we will use the brane set-up depicted in figure 1 to propose a linear quiver dual to the background in eqs. (2.11)–(2.17). As a consistency check we will compute its central charge using a-maximization and show that it is in perfect agreement with the holographic study in section 2.2.3 and the result of eq. (2.34), in particular. We will show that the central charge also satisfies the well-known 27/32 ratio [72] with the central charge associated to the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$. This suggests defining our $\mathcal{N} = 1$ conformal field theory as the result of deforming by mass terms the $\mathcal{N} = 2$ CFT associated to the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$.

4.1 Proposed $\mathcal{N} = 1$ linear quiver

The quantized charges associated to the non-Abelian T-dual solution are consistent with a brane set-up, depicted in figure 1, in which D4-branes extend on $\mathbb{R}^{1,3} \times \{\rho\}$, NS5-branes on $\mathbb{R}^{1,3} \times S^2(\theta_1, \phi_1)$ and NS5'-branes on $\mathbb{R}^{1,3} \times \tilde{S}^2(\chi, \xi)$. This produces for $\rho \in [n\pi\alpha'/L^2, (n+1)\pi\alpha'/L^2]$, $n \to \infty$ and upon compactification, the brane set-up, depicted in Figure 7 in appendix B, associated to the Abelian limit of the solution.

We conjecture that, in a similar fashion, the non-Abelian T-dual background in eqs. (2.11)–(2.17), arises as the decoupling limit of a D4, NS5, NS5' brane intersection. As opposed to its Abelian counterpart, the precise way in which D-branes transform under non-Abelian T-duality has not been worked out in the literature. This would require analysing the transformation of the boundary conditions at the level of the sigma model (see [71] for some preliminary steps in this direction). Still, as stressed in the previous works [44, 45], similar assumptions based on the analysis of the quantized charges of the supergravity background have produced consistent successful outcomes. Given that the precise D4, NS5, NS5' brane intersection is not known prior to the near horizon limit, it is unclear, on the other hand, how the original D3-brane configuration associated to the Klebanov-Witten solution would be recovered. In fact, even after taking the near horizon limit it is unclear how the Klebanov-Witten background would be recovered from the background defined by eqs. (2.11)–(2.17), given that the original SU(2) symmetry used to construct it is no longer present.[5] These issues make non-Abelian T-duality substantially different from its Abelian counterpart, and underlie the fact that it can non-trivially change the dual CFT.

Coming back to our proposal, we would have an *infinite-length* quiver with (in the notation of section 3) $p = n$, $q = n$, $\ell = p+q = 2n$ and $z = (p-q)/\ell = 0$ with $n \to \infty$. The associated field theory would consist on $(2n-1)$ $\mathcal{N} = 1$ vector multiplets and matter fields connecting them. However, this infinitely-long quiver *does not* describe a four dimensional field theory (its central charge is strictly infinite, among other problematic aspects). This is the same issue that we discussed when calculating the holographic central charge in section 2.2.3. Some regularization is needed and, as we will see, the field theory precisely provides the way to do this.

Elaborating on the ideas in [44], we propose to study this quiver for finite n, *completing* it as shown in figure 4. The proposed field theory has the following characteristics:

[5]This is related to the well-known non-invertibility of non-Abelian T-duality, noticed in the early works [2–8].

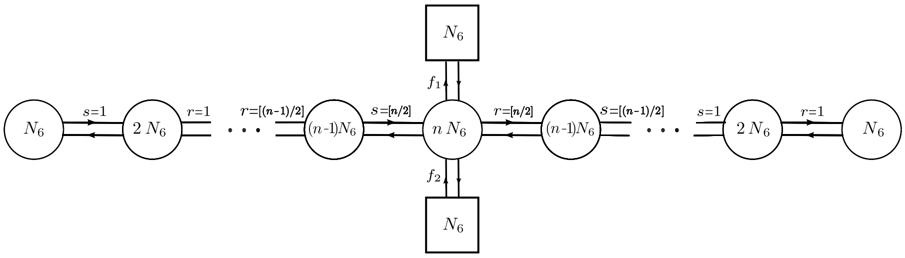

Figure 4. Linear quiver proposed as dual to the non-Abelian T-dual solution. There are two matter fields $Q_j, \tilde{Q}_j$ in the bifundamental and anti-bifundamental of each pair of nodes, associated to a 5-brane connecting adjacent D4-stacks, with a total number of $j = 1, \ldots, n-1$ hypermultiplets $H_j = (Q_j, \tilde{Q}_j)$ at each side of the quiver. We label $r = 1, \ldots, [n/2]$ the $\sigma_j = +1$ hypermultiplets corresponding to NS5-branes and $s = 1, 2, \ldots, [n/2]$ the $\sigma_j = -1$ hypermultiplets from NS5'-branes, assuming an alternating distribution of both types of 5-branes. This configuration comes from a re-ordering of the branes in figure 1 that is consistent with Seiberg self-duality and the vanishing of the beta functions and R-symmetry anomalies. The squares in the middle of the quiver denote flavor groups corresponding either to semi-infinite D4-branes ending on the NS5 and NS5' branes or to D6-branes transversal to the D4-branes. They *complete* the quiver at finite n. We choose $\sigma_{f_1} = -\sigma_{f_2}$ for the corresponding fundamental hypermultiplets.

- It is strongly coupled. This is in correspondence with the fact that it should be dual to an AdS solution whose internal space is smooth in a large region and reduces to our non-Abelian T-dual background in eqs. (2.11)–(2.17) in some limit.
- The field theory is self-dual under Seiberg duality. This can be quickly seen, by observing that each node is at the self-dual point (with $N_f = 2N_c$).
- The beta function and the R-symmetry anomalies vanish, in correspondence with the SO(2, 4) isometry of the background and the number of preserved SUSYs.
- The central charge calculated by field theoretical means coincides (for long enough quivers) with the holographic result of eq. (2.34).
- The quiver can be thought of as a mass deformation of the $\mathcal{N} = 2$ quiver dual to the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$.

Below, we show that the field theory represented in figure 4 has all these characteristics. As it happens in the paper [44], the completion we propose with the flavor groups has the effect of ending the space at a given finite value in the ρ direction.

4.2 β-functions and R-symmetry anomalies

In this section we study the β-functions and the anomalies associated to the linear quiver proposed in figure 4. This analysis clarifies that the quantum field theory flows to a conformal fixed point in the infrared.

In a supersymmetric gauge theory, the β-function for a coupling constant g is given by the well-known Novikov-Shifman-Vainshtein-Zakharov (NSVZ) formula [73], which can be

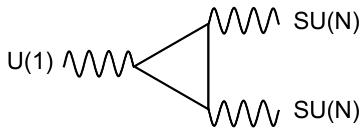

Figure 5. The R-symmetry anomaly.

written in terms of the number of colors, N_c, the number of flavors, N_{f_q}, and the anomalous dimensions for the matter fields, γ_q, as

$$\beta_g \sim 3\, N_c - \sum_q N_{f_q}\, (1 - \gamma_q)\ . \tag{4.1}$$

Here, we considered the Wilsonian beta function. The denominator in the NSVZ formula is not relevant for us (see [74] for a nice explanation of this). Another important quantity is the R-symmetry anomaly, given by the correlation function of three currents and represented by the Feynman diagram in figure 5. The anomaly is given by the relation,

$$\langle \partial^\mu J^5_\mu \rangle \sim \Delta\Theta\, F\, \tilde{F}\,, \qquad \Delta\Theta = \sum R_f\, T(R_f)\,, \tag{4.2}$$

where R_f is the R-charge of the fermions in the multiplet. In the case of an SU(N) gauge group

$$T(R_f) = \begin{cases} 2N, & \text{for fermions in the adjoint representation} \\ 1, & \text{for fermions in the fundamental representation} \end{cases} \tag{4.3}$$

Moreover, at the conformal point, one should take into account the relation between the physical dimension of a gauge invariant operator $\mathcal{O}$ (with engineering dimension $\Delta_{\mathcal{O}}$) and its R-charge $R_{\mathcal{O}}$,

$$\dim\, \mathcal{O} = \Delta_{\mathcal{O}} + \frac{\gamma_{\mathcal{O}}}{2} = \frac{3}{2}\, R_{\mathcal{O}}\ . \tag{4.4}$$

In the appendix C, we present details of these calculations for the well-known example of the Klebanov-Witten CFT. Readers unfamiliar with that example can study the details in appendix C and then come back to the more demanding calculation presented below.

Let us now analyze the quiver depicted in figure 4. We propose for the anomalous dimensions and R-charges of the matter fields and gauginos the same values as in the Klebanov-Witten CFT,

$$\gamma_Q = \gamma_{\tilde{Q}} = -\frac{1}{2}\,, R_Q = R_{\tilde{Q}} = \frac{1}{2}, \quad R(\lambda) = 1. \tag{4.5}$$

Notice that in our proposal only one bifundamental field runs in each arrow. We call them Q or $\tilde{Q}$ depending on the direction of the arrow. We find, substituting in eq. (4.1) for the nodes with rank kN_6,

$$\beta_k \sim 3\, k\, N_6 - \Big((k+1)\, N_6 + (k-1)\, N_6\Big)\left(1 + \frac{1}{2}\right) = 0\,, \qquad k = 1, \ldots, n\,. \tag{4.6}$$

The first term reflects the contribution of the gauge multiplets and the second that of the matter fields. For the anomaly we find,

$$\Delta\theta_k = 2\,k\,N_6 + 2\Big((k+1)\,N_6 + (k-1)\,N_6\Big)\Big(-\frac{1}{2}\Big) = 0\,, \qquad k = 1,\ldots,n\,. \tag{4.7}$$

The first term indicates the contribution of the gauginos and the second one the contribution of the fermions in the $Q,\tilde{Q}$ multiplets.

These calculations indicate that both R-symmetry anomalies and beta functions are vanishing. Indeed, they belong to the same anomaly multiplet. Also, notice that the large anomalous dimensions indicate that the CFT is strongly coupled. With this numerology, we calculate that

$$\dim\, Q = \dim\, \tilde{Q} = \frac{3}{4}. \tag{4.8}$$

This allows for the presence of superpotential terms involving four matter multiplets, like the ones proposed in [55]. Let us move now to the calculation of the central charge.

4.3 Field-theoretical central charge

In this section we compute the central charge of the quiver depicted in figure 4 at the fixed point, using the a-maximization procedure [69].

As recalled in section 3, the a and c central charges can be computed from the $\mathcal{N}=1$ R-symmetry t'Hooft anomalies of the fermionic degrees of freedom of the theory,

$$a(\epsilon) = \frac{3}{32}\left(3\,\mathrm{Tr}\,R_\epsilon^3 - \mathrm{Tr}\,R_\epsilon\right), \quad c(\epsilon) = \frac{1}{32}\left(9\,\mathrm{Tr}\,R_\epsilon^3 - 5\,\mathrm{Tr}\,R_\epsilon\right). \tag{4.9}$$

The R-symmetry is given by $R_\epsilon = R_0 + \frac{1}{2}\,\epsilon\,\mathcal{F}$. Assigning charges $R_0(Q_j) = R_0(\tilde{Q}_j) = 1/2$ to the chiral multiplet scalars, we have that

$$R_\epsilon(Q_j) = R_\epsilon(\tilde{Q}_j) = \frac{1}{2}\left(1 + \epsilon\,\sigma_j\right),$$

and, for the fermions

$$R_\epsilon(\psi_j) = R_\epsilon(\tilde{\psi}_j) = \frac{1}{2}\left(-1 + \epsilon\,\sigma_j\right).$$

Now, we can compute the linear contribution to the anomaly coming from the hypermultiplet $H_j = (Q_j,\tilde{Q}_j)$, whose chiral fields transform in the fundamental of a gauge group with rank N_a and in the anti-fundamental of another gauge group with rank N_b, and vice-versa:

$$\mathrm{Tr}\,R_\epsilon(H_j) = N_a\,N_b\Big(R_\epsilon(\psi_j) + R_\epsilon(\tilde{\psi}_j)\Big) = N_a\,N_b(\epsilon\,\sigma_j - 1)\,. \tag{4.10}$$

The cubic contribution is

$$\mathrm{Tr}\,R_\epsilon^3(H_j) = N_a\,N_b\Big(R_\epsilon^3(\psi_j) + R_\epsilon^3(\tilde{\psi}_j)\Big) = 2N_a\,N_b\left[\frac{1}{2}(\epsilon\,\sigma_j - 1)\right]^3. \tag{4.11}$$

In turn, the linear and cubic anomaly contributions from an $\mathcal{N}=1$ vector multiplet V_t are given by,

$$\mathrm{Tr}\,R_\epsilon(V_t) = \mathrm{Tr}\,R_\epsilon^3(V_t) = N_a^2 - 1\,, \tag{4.12}$$

where we have used that $R_\epsilon(\lambda) = R_0(\lambda) = 1$ for the gaugino.

We now consider the completed quiver in figure 4. Hypermultiplets with $\sigma_j = +1$ and $\sigma_j = -1$ (transforming in the bifundamental of gauge groups of ranks N_j, N_{j+1}) alternate along the quiver, and $\sigma_{f_1} = -\sigma_{f_2}$. In this way all nodes are equipped with $\mathcal{N} = 1$ vector multiplets. Moreover, we have $z = 0$ exactly, as well as $\kappa = 0$. The total linear contribution of the hypermultiplets is then:

$$\begin{aligned}
\mathrm{Tr}\, R_\epsilon(H) &= \sum_{j=1,\mathrm{left}}^{n-1} \mathrm{Tr}\, R_\epsilon(H_j) + \sum_{j=1,\mathrm{right}}^{n-1} \mathrm{Tr}\, R_\epsilon(H_j) + \sum_{i=1}^{2} \mathrm{Tr}\, R_\epsilon(H_{f_i}) \\
&= N_6^2 \Bigg\{ \sum_{j=1}^{n-1} j\,\big(j+1\big)\big(\epsilon\,\cancelto{0}{(\sigma_{j,\mathrm{left}} + \sigma_{j,\mathrm{right}})} - 1\big) + n \sum_{i=1}^{2} \big(\epsilon\,\sigma_{f_i} - 1\big) \Bigg\} \\
&= N_6^2 \Bigg\{ -2 \sum_{j=1}^{n-1} j\,\big(j+1\big) + n\big(\epsilon\,\cancelto{0}{(\sigma_{f_1} + \sigma_{f_2})} - 2\big) \Bigg\} \\
&= N_6^2 \Bigg\{ -\frac{2}{3}\, n\big(n^2 - 1\big) - 2n \Bigg\} = N_6^2 \Bigg\{ -\frac{2}{3} n^3 - \frac{4}{3} n \Bigg\} \\
&\approx -\frac{2}{3} n^3\, N_6^2 + \mathcal{O}(n)\ .
\end{aligned} \tag{4.13}$$

In the last line the approximation of a *long quiver* (large n) has been used. Similarly, the total cubic contribution of the hypermultiplets can be readily computed to be,

$$\begin{aligned}
\mathrm{Tr}\, R_\epsilon^3(H) &= \sum_{j=1,\mathrm{left}}^{n-1} \mathrm{Tr}\, R_\epsilon^3(H_j) + \sum_{j=1,\mathrm{right}}^{n-1} \mathrm{Tr}\, R_\epsilon^3(H_j) + \sum_{i=1}^{2} \mathrm{Tr}\, R_\epsilon^3(H_{f_i}) \\
&= N_6^2 \Bigg[\sum_{j=1}^{n-1} j\,\big(j+1\big) \frac{1}{4} \Big(\big(\epsilon\,\sigma_{j,\mathrm{left}} - 1\big)^3 + \big(\epsilon\,\sigma_{j,\mathrm{right}} - 1\big)^3 \Big) + n\, N_6^2 \sum_{i=1}^{2} \frac{1}{4} \big(\epsilon\,\sigma_{f_i} - 1\big)^3 \Bigg] \\
&= \frac{N_6^2}{4} \Bigg[-2(1 + 3\epsilon^2) \sum_{j=1}^{n-1} j\,\big(j+1\big) + n \sum_{i=1}^{2} \big(\epsilon\,\sigma_{f_i} - 1\big)^3 \Bigg] \\
&= \frac{N_6^2}{12} \Bigg[-2\big(1 + 3\epsilon^2\big) n^3 - (12\epsilon^2 + 4) n \Bigg] \approx -\frac{1}{6}\, n^3\, N_6^2\, \big(1 + 3\,\epsilon^2\big) + \mathcal{O}(n),
\end{aligned} \tag{4.14}$$

where long quivers have been considered in the last expression. In turn, recalling that each node appears twice in the quiver depicted in figure 4, with the exception of the central one, the trace anomaly coming from the $\mathcal{N} = 1$ vector multiplets becomes,

$$\begin{aligned}
\mathrm{Tr}\, R_\epsilon(V) = \mathrm{Tr}\, R_\epsilon^3(V) &= 2 \sum_{t=1}^{n-1} \mathrm{Tr}\, R_\epsilon(V_t) + \mathrm{Tr}\, R_\epsilon(V_n) = 2 \sum_{t=1}^{n-1} \big(t^2 N_6^2 - 1\big) + \big(n^2 N_6^2 - 1\big) \\
&= \frac{N_6^2}{3} \big(2n^3 + n\big) - 2(n-1) \approx \frac{2}{3}\, n^3\, N_6^2 + \mathcal{O}(n)\ .
\end{aligned} \tag{4.15}$$

From this result we see that $\mathrm{Tr}\, R_\epsilon(V) \approx -\mathrm{Tr}\, R_\epsilon(H)$ in the large n limit, so that the overall linear trace anomaly is of order $n\, N_6^2$ at most. Putting all these expressions together we

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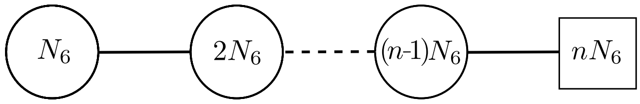

Figure 6. Completed quiver associated to the non-Abelian T-dual of $AdS_5 \times S^5$. Each line represents a hypermultiplet of $\mathcal{N}=2$ SUSY.

find, for the exact charges in eq. (4.9),

$$\begin{aligned} a(\epsilon) &= \frac{3N_6^2}{64}\left\{3(1-\epsilon^2)n^3 + 2(1-3\epsilon^2)n - \frac{4}{N_6^2}(2n-1)\right\}, \\ c(\epsilon) &= \frac{N_6^2}{64}\left\{9(1-\epsilon^2)n^3 + 2(5-9\epsilon^2)n - \frac{8}{N_6^2}(2n-1)\right\}. \end{aligned} \tag{4.16}$$

From these expressions we see that $a(\epsilon)$ is clearly maximized for $\epsilon = 0$, as expected for the $\mathcal{N}=1$ fixed point [55]. The superconformal central charges are thus found to be

$$\begin{aligned} a_{\mathcal{N}=1} &\equiv a(\epsilon=0) = \frac{3}{64}\left\{(3n^3+2n)N_6^2 - 4(2n-1)\right\}, \\ c_{\mathcal{N}=1} &\equiv c(\epsilon=0) = \frac{1}{64}\left\{(9n^3+10n)N_6^2 - 8(2n-1)\right\}. \end{aligned} \tag{4.17}$$

They give, in the large n limit,

$$c_{\mathcal{N}=1} \approx a_{\mathcal{N}=1} \approx \frac{9}{64}\, n^3\, N_6^2 + \mathcal{O}(n)\,. \tag{4.18}$$

This final result matches the holographic calculation given by eq. (2.34). This provides a non-trivial check of the validity of the linear quiver in figure 4 as dual to the background in eqs. (2.11)–(2.17). It is noteworthy that the agreement with the holographic result occurs in the large number of nodes limit, $n \to \infty$.

A further non-trivial check of the validity of our proposed quiver is that the central charge given by (4.18) and that associated with the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$ satisfy the same *27/32 relation* [72], that is,

$$c_{\mathcal{N}=1} = \frac{27}{32}\, c_{\mathcal{N}=2}, \tag{4.19}$$

as the central charges of the corresponding theories prior to dualization. Indeed, the quiver associated to the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$ can be obtained by modding out by $\mathbb{Z}_2$ the quiver describing the non-Abelian T-dual of $AdS_5 \times S^5$, constructed in [44] and depicted in figure 6. This quiver was completed at finite n by a flavor group with gauge group $SU(nN_6)$. It thus satisfies the condition to be conformal (preserving $\mathcal{N}=2$ SUSY), i.e. that the number of flavors is twice the number of colors at each node. Modding out by $\mathbb{Z}_2$ results in the same quiver in figure 4, but built out of $2n$ $\mathcal{N}=2$ vector and matter multiplets. Taking the central charge, computed in [44], for the non-Abelian T-dual of $AdS_5 \times S^5$ and doubling it, we obtain the central charge of the non-Abelian T-dual of $AdS_5 \times S^5/\mathbb{Z}_2$

$$c_{NATD\, AdS_5\times S^5/\mathbb{Z}_2} \approx 2 \times \frac{1}{12} n^3 N_6^2 + \mathcal{O}(n), \tag{4.20}$$

and we find that eq. (4.19) indeed holds with $c_{\mathcal{N}=1}$ as in eq. (4.18) and $c_{\mathcal{N}=2}$ as in eq. (4.20). We have checked in appendix C.2 that the same result (4.20) is reproduced using a-maximization. The a-charge is maximized for $\epsilon = \frac{1}{3}$, as previously encountered in [55].

Further, one can check that also at finite n, $a_{\mathcal{N}=1}$ and $c_{\mathcal{N}=1}$ satisfy the relation [72],[6]

$$a_{\mathcal{N}=1} = \frac{9}{32}\big(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\big)\,, \qquad c_{\mathcal{N}=1} = \frac{1}{32}\big(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\big) \tag{4.21}$$

with the $a_{\mathcal{N}=2}$, $c_{\mathcal{N}=2}$ exact central charges of the $\mathcal{N}=2$ quiver. The explicit expressions for $a_{\mathcal{N}=2}$ and $c_{\mathcal{N}=2}$ are given in eq. (C.11) in appendix C.2. This precisely defines our dual CFT as the result of deforming by mass terms the CFT dual to the Sfetsos-Thompson solution modded by $\mathbb{Z}_2$.

The material presented in this section makes very precise the somewhat loose ideas proposed in the works [12, 13]. In particular, we have identified the concrete relation via a RG-flow between the non-Abelian T-dual of $AdS_5 \times S^2/\mathbb{Z}_2$ and the non-Abelian T-dual of the Klebanov-Witten solution. Notice that here, we are providing precisions about the CFT dual to the non-Abelian T-dual backgrounds. This more precise information is matched by the regularized form of the non-Abelian T-dual solution.

The diagram in the Introduction section summarizes the connections between the UV and IR field theories discussed in this section. We repeat it here for the perusal of the reader.

$$\begin{tikzcd} AdS_5 \times S^5/\mathbb{Z}_2 \arrow[r] \arrow[d, "\text{mass}"] & \text{NATD of } AdS_5 \times S^5/\mathbb{Z}_2 \arrow[d, "\text{mass}"] \\ AdS_5 \times T^{1,1} \arrow[r] & \text{NATD of } AdS_5 \times T^{1,1} \end{tikzcd}$$

As a closing remark, an explicit flow (triggered by a VEV) between the $\mathcal{N}=1$ and the $\mathcal{N}=2$ non-Abelian T-dual backgrounds was constructed in [35]. It should be interesting to use the detailed field theoretical picture developed above and in [44], to be more precise about various aspects of this RG-flow.

5 Solving the INST-BBBW puzzle

The non-Abelian T-dual of the Klebanov-Witten background was first written in [12, 13] (INST). Further, in that paper an attempt was made to match the non-Abelian T-dual background with a Bah, Beem, Bobev and Wecht (BBBW) solution [59]. This matching was feasible assuming a particular split of the metric into a seven-dimensional and a four-dimensional internal space (see below). The formula in [59] for the central charge of BBBW solutions led however to $a \sim c \sim 0 + O(N)$ for the non-Abelian T-dual solution, in blatant disagreement with the holographic result. This was the puzzle that the authors of [12, 13] pointed out. In this section we present its resolution. We start by summarizing the most relevant aspects of the work [59].

[6] We would like to thank Nikolay Bobev for suggesting this to us.

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In the work of Bah, Beem, Bobev and Wecht new $\mathcal{N} = 1$ AdS_5 solutions in M-theory were constructed, describing the fixed points of new $\mathcal{N} = 1$ field theories associated to M5-branes wrapped on complex curves. The central charges of these SCFTs were computed using the six dimensional anomaly polynomial and a-maximization, and were shown to match, in the large number of M5-branes limit, the holographic results.

The solutions constructed in [59] were obtained by considering M-theory compactified on a deformed four-sphere. In principle, this compactification leads to an SO(5)-gauged supergravity in seven dimensions. Following the ideas in [47], BBBW searched for their solutions in the seven dimensional gravity theory (a $\mathrm{U}(1)^2$ truncation of the full SO(5) theory) discussed in [75]. They proposed a background consisting of a metric, two gauge fields $A_\mu^{(i)}$ and two scalars $\lambda^{(i)}$, of the form

$$\begin{aligned} ds_7^2 &= e^{2f(r)}[dx_{1,3}^2 + dr^2] + e^{2g(r)} d\Sigma_k(x_1, x_2), \\ F^{(1)} &= \frac{p}{8g-8} vol\Sigma_k, \quad F^{(2)} = \frac{q}{8g-8} vol\Sigma_k, \quad \lambda^{(i)}(r). \end{aligned} \tag{5.1}$$

They then searched for 'fixed point' solutions, namely, those where $\frac{d}{dr}\lambda^{(i)} = \frac{d}{dr}g = 0$ and $f \sim -\log r$, leading to backgrounds of the form $AdS_5 \times \Sigma_k$. They found general solutions depending on four parameters (N, κ, z, g). For excitations with wavelength longer than the size of Σ_k, these are dual to four dimensional CFTs. In the dual CFT N is the number of M5-branes, $\kappa = \pm 1, 0$ is the curvature of the 2d Riemann surface that they are wrapping, g is its genus and z is the so-called 'twisting parameter', defined as $z = \frac{(p-q)}{2(g-1)}$ from the integer numbers p, q that indicate the twisting applied to the M5-branes. The holographic central charge computed in [59] depends on these parameters, and reads

$$c = a = N^3(1-g)\left[\frac{1 - 9z^2 + \kappa(1+3z^2)^{3/2}}{48z^2}\right]. \tag{5.2}$$

BBBW completed their analysis deriving various of their formulas, in particular the holographic central charge, using purely 4d CFT arguments. Their CFTs are combinations of Gaiotto's T_N-theories, conveniently gauged and connected with other T_N factors, with either $\mathcal{N} = 1$ or $\mathcal{N} = 2$ vector multiplets (shaded and unshaded T_N's in the same line as what we explained in section 3).

The key-point to be kept in mind after this discussion is that these results were obtained in the context of a *compactification* of eleven-dimensional supergravity to seven dimensions.

Let us now come back to the paper [12, 13]. The matching of the non-Abelian T-dual solution with a BBBW geometry assumed that the seven dimensional part of the metric in (5.1) was $AdS_5 \times S^2(\theta_1, \phi_1)$ and that the internal space contained the coordinates $[\rho, \chi, \xi, x_{11}]$. Also, the authors of [12, 13] chose the parameters $\kappa = z = 1$ for such matching. Using the formula (5.2) in BBBW for the central charge they then found that at leading order the central charge vanished.

What was not-correct in the analysis of [12, 13] was the assumption that the non-Abelian T-dual solution could be obtained from a *compactification* of M-theory on a deformed four-sphere (and hence be in the BBBW class of solutions). In fact, inspecting the

BPS equations of BBBW — eq. (3.10) of [59] — one finds that a fixed point solution does not exist for the set of values $\kappa = |z| = 1$. Even more, the generic solution that BBBW wrote in their eq. (3.8) is troublesome for those same values.

A parallel argument can be made by comparing the BBBW and non-Abelian T-dual solutions in the language of the paper [76]. Indeed, the comparison in the appendix C of [76], shows that these solutions fit in their formalism in section 4.2 for values of parameters that are incompatible. Either BBBW is fit *or* the non-Abelian T-dual solution is, for a chosen set of parameters.

The resolution to this problem is that the non-Abelian T-dual background should instead be thought of as providing a *non-compactification* of eleven dimensional supergravity. Strictly speaking, our coordinate ρ runs in $[0, \infty]$, the four manifold is non-compact. In our calculation of the central charge, we *assumed* that the ρ-coordinate was bounded in $[0, n\pi\frac{\alpha'}{L^2}]$, but this hard cut-off, as we emphasized, is not a geometrically satisfactory way of bounding a coordinate. There should be another, more general solution, that contains our non-Abelian T-dual metric in a small patch of the space (for small values of ρ), and closes the ρ-coordinate at some large value $\rho_n = n\pi\frac{\alpha'}{L^2}$. But this putative new metric, especially its behaviour near ρ_n, will differ considerably from the one obtained via non-Abelian T-duality. Below, we will comment more about this putative solution.

Let us close with some field theoretical remarks. The class of CFTs studied by BBBW [59] are quite different from those studied by Bah and Bobev in [55]. Their central charges are different, and the first involve Gaiotto's T_N theories while the second do not. In the same line, our CFT discussed in section 4 is a generalization, but strictly different, of the theories in [55], and is certainly different from those in [59].

The quiver we presented in section 4 encodes the dynamics of a solution in Type IIA/M-theory where the ρ-coordinate is bounded in a geometrically sounding fashion. The addition of the flavor groups in our quiver encode the way in which the ρ-coordinate should be ended. Indeed, in analogy with what was observed in [44, 45], we expect the metric behaving like that of D6 branes close to the end of the space. In M-theory language, we expect to find a puncture on the Riemann surface, representing the presence of flavor groups in the dual CFT. We will be slightly more precise about this in the Conclusions section.

6 Conclusions and future directions

Let us briefly summarize the main achievements of this paper.

After discussing details of the Type IIA solution obtained by non-Abelian T-duality applied on the Klebanov-Witten background, we carefully studied its quantized charges and holographic central charge (section 2). We lifted the solution to M-theory and showed by explicit calculation of the relevant differential forms that the background has SU(2)-structure and fits the classification of [65].

Based on the quantized charges, we proposed a brane set-up (section 4) and a precise quiver gauge theory, generalizing the class of theories discussed by Bah and Bobev in [55] (and summarized in our section 3). This quiver was used to calculate the central charge, one of the important observables of a conformal field theory at strong coupling. Indeed, in section 4, we showed the precise agreement of this observable, computed by field theoretical means, with the holographic central charge. We also showed that the quiver has a strongly coupled IR-fixed point. Finally, section 5, solves a puzzle raised in previous bibliography. Various appendices discuss technical points in detail. In particular, relations of the non-Abelian T-dual of the Klebanov-Witten background and the more conventional T-dual, details about the dual field theory, etc, are carefully explained there.

To close this paper let us state the most obvious and natural continuation of our work. As we discussed, the holographic central charge calculation in section 2 was done for a *regulated* version of the Type IIA background. Indeed, the integral over the internal space was taken to range in a finite interval for the ρ-coordinate. We introduced a hard-cutoff, but emphasized that this form of regularization is not rigorous from a geometric viewpoint. Fortunately, the dual CFT provides a rationale to regulate the space. The flavor groups SU(N_6) that end our quiver field theory (see figure 4), will be reflected in the Type IIA background by the presence of flavor branes that will backreact and end the geometry, solving the Einstein's equations. In eleven dimensions, the same effect will be captured by punctures on the S^2 that the M5 branes are wrapping. A phenomenon like this was at work in the papers [44, 45].

The formalism to backreact these flavor D6 branes is far-less straightforward in the present case, as the number of isometries and SUSY is less than in the cases of [44, 45]. Qualitatively one may think of defining the completed solution by deforming with mass terms the superposition of $\mathcal{N} = 2$ Maldacena-Nunez solutions [47] used in [44] to complete the Sfetsos-Thompson background. This would give rise to a superposition of $\mathcal{N} = 1$ MN solutions defining the completed non-Abelian T-dual solution. It is unclear however in which precise way this superposition would solve the (very non-linear) PDEs associated to $\mathcal{N} = 1$ solutions [65, 77]. We see two possible paths to follow:

- In the paper [77], Bah rewrote the general M-theory background of [65] in terms of a new set of coordinates that are more useful to discuss the addition of punctures on the Riemann surface. In the type IIA language the new solutions found using Bah's non-linear and coupled PDEs should represent the addition of the flavor D6 branes argued above. The equations need to be solved close to the singularity (the puncture or the flavor D6 brane) and then numerically matched with the rest of the non-Abelian T-dual background.

- In [76] generic backgrounds in *massive* Type IIA were found with an AdS_5 factor in the metric and preserving eight SUSYs. For the particular case in which the internal space contains a Riemann surface of constant curvature, the involved set of non-linear and coupled PDEs simplifies considerably. One of the solutions, for the case in which the massive parameter vanishes, is the one studied in this paper — named INST in [76]. Since the paper [76] and some follow-up works have discussed ways of ending these spaces by the addition of D6 and D8 branes, we could consider these technical developments together with the ideas discussed above.

Finding a *completed* or *regularized* solution would provide the first example for a background dual to a CFT like that discussed in section 4. The natural following steps would be to extend the formalism to discuss the situations for a cascading QFT. In fact, the precise knowledge of the CFT we have achieved in this paper can be used to improve the understanding and cure the singularity structure of the backgrounds written in the first paper in [12, 13], in [20], etc. We reserve these problems to be discussed in forthcoming publications.

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A Connection with the GMSW classification

In this appendix we prove that the uplift of the non-Abelian T-dual of the Klebanov-Witten solution fits in the classification of $\mathcal{N} = 1$ AdS_5 backgrounds in M-theory of GMSW [65].

A.1 Uplift of the non-Abelian T-dual solution

The eleven dimensional uplift of the non-Abelian T-dual solution consists of metric and 4-form flux. The metric is given by

$$ds^2_{11} = e^{-\frac{2\Phi}{3}}\, ds^2_{IIA} + e^{\frac{4\Phi}{3}}\left(dx_{11} + C_1\right)^2 , \tag{A.1}$$

where x_{11} stands for the 11^{th} coordinate, ds^2_{IIA} is the ten dimensional metric, given by eq. (2.11), Φ is the dilaton, given by eq. (2.13), and C_1 is the RR potential given in eq. (2.17). The eleven dimensional four-form field, F_4^M, is derived from $F_4^M = dC_3^M$, where

$$C_3^M = C_3 + B_2 \wedge dx_{11} , \tag{A.2}$$

and C_3 and B_2 are given by (2.17) and (2.12), respectively. The final expression for F_4^M is given by

$$\begin{aligned} F_4^M = &-\frac{\alpha'\,\lambda^2\left(L^4\,\lambda_1^4 + \alpha'^2\,\rho^2\right)}{Q}\cos\chi\, d\Omega_2(\theta_1,\phi_1)\wedge d\rho\wedge dx_{11} \\ &+ \frac{\alpha'\,L^4\,\lambda_1^4\,\lambda^2}{Q}\,\rho\,\sin\chi\, d\Omega_2(\theta_1,\phi_1)\wedge d\chi\wedge dx_{11} \end{aligned}$$

$$+\frac{4\,\alpha'^{3/2}\,L^4\,\lambda_1^4\,\lambda\left(\lambda^2-\lambda_1^2\right)}{g_s\,Q}\,\rho^2\,\cos\chi\,\sin^2\chi\,d\Omega_2(\theta_1,\phi_1)\wedge d\rho\wedge d\xi$$
$$+\frac{4\,\alpha'^{3/2}\,L^4\,\lambda_1^4\,\lambda\,P}{g_s\,Q}\,\rho^3\,d\Omega_2(\theta_1,\phi_1)\wedge d\Omega_2(\chi,\xi)$$
$$+\frac{4\,\alpha'^{3/2}\,L^4\,\lambda_1^4\,\lambda^3\,\mathcal{S}}{Q^2}\,\rho^2\,\cos\theta_1\,d\Omega_2(\chi,\xi)\wedge d\rho\wedge d\phi_1$$
$$+\frac{\alpha'^3\,\lambda^2\,\mathcal{S}}{Q^2}\,\rho^2\,\sin\chi\left(d\xi+\cos\theta_1\,d\phi_1\right)\wedge d\rho\wedge d\chi\wedge dx_{11}\,, \tag{A.3}$$

where for the sake of clarity we have defined,

$$\mathcal{S}\equiv Q-2\,L^4\,\lambda_1^4\left(\lambda^2+\lambda_1^2\right)-2\,\alpha'^2\,\rho^2\,\lambda_1^2\,. \tag{A.4}$$

In the large n limit this expression takes the simpler form ($\lambda^2=1/9$, $\lambda_1^2=1/6$, $\alpha'=g_s=1$),

$$F_4^M\approx\frac{L^4}{27}(\rho-n\pi)\,d\Omega_2(\theta_1,\phi_1)\wedge d\Omega_2(\chi,\xi)\,, \tag{A.5}$$

which tells us that the M5-branes sourcing this flux are transversal to both squashed two-spheres $S^2(\theta_1,\phi_1)$ and $\tilde{S}^2(\chi,\xi)$. These are associated to the global isometries $\mathrm{U}(1)_w$ and $\mathrm{U}(1)_v$, whose product lies in the Cartan of both the local R-symmetry and the non-anomalous flavor symmetry.

A.2 Review of GMSW

Before matching the previous solution within the classification in [65], let us briefly review the most general $\mathcal{N}=1$ eleven dimensional solutions with an AdS_5 factor found in that paper. These solutions are described by a metric of the form,

$$ds_{11}^2=e^{2\Lambda}\left[ds^2_{AdS_5}+ds^2_{M_4}+\frac{e^{-6\Lambda}}{\cos^2\zeta}\,dy^2+\frac{\cos^2\zeta}{9\,m^2}\left(d\tilde{\psi}+\tilde{\rho}\right)^2\right]. \tag{A.6}$$

Here $\tilde{\psi}$ is the R-symmetry direction, $\tilde{\rho}$ is a one-form defined on M_4, whose components depend on both the M_4 coordinates and y, and Λ and ζ are functions also depending on the M_4 coordinates and y. The coordinate y is related to the warping factor Λ and the function ζ through,

$$2\,m\,y=e^{3\Lambda}\,\sin\zeta\,, \tag{A.7}$$

with m being the inverse radius of AdS_5.

The four-dimensional manifold M_4 admits an SU(2) structure which is characterized by a $(1,1)$-form J and a complex $(2,0)$-form Ω. The SU(2) structure forms, together with the frame components K^1 and K^2, defined as,

$$K^1\equiv\frac{e^{-3\Lambda}}{\cos\zeta}\,dy\,,\qquad K^2\equiv\frac{\cos\zeta}{3\,m}\left(d\tilde{\psi}+\tilde{\rho}\right), \tag{A.8}$$

must satisfy the following set of differential conditions dictated by supersymmetry,

$$e^{-3\Lambda}\,d\big(e^{3\Lambda}\,\sin\zeta\big)=2\,m\,\cos\zeta\,K^1\,, \tag{A.9}$$
$$e^{-6\Lambda}\,d\big(e^{6\Lambda}\,\cos\zeta\,\Omega\big)=3\,m\,\Omega\wedge\big(-\sin\zeta\,K^1+i\,K^2\big)\,, \tag{A.10}$$
$$e^{-6\Lambda}\,d\big(e^{6\Lambda}\,\cos\zeta\,K^2\big)=e^{-3\Lambda}\star G+4\,m\left(J-\sin\zeta\,K^1\wedge K^2\right), \tag{A.11}$$
$$e^{-6\Lambda}\,d\big(e^{6\Lambda}\,\cos\zeta\,J\wedge K^2\big)=e^{-3\Lambda}\,\sin\zeta\,G+m\left(J\wedge J-2\,\sin\zeta\,J\wedge K^1\wedge K^2\right). \tag{A.12}$$

In the above formulas, $\star$ stands for Hodge duality in the six-dimensional space spanned by M_4 and the one-forms K^1 and K^2. G is an eleven-dimensional four-form whose components lie along the six-dimensional space that is transverse to AdS_5,[7]

$$\begin{aligned} G = &- \partial_y e^{-6\,\Lambda}\,\widehat{\mathrm{vol}}_4 - \frac{e^{-9\,\Lambda}}{\cos\zeta}\left(\hat{\star}_4 d_4 e^{6\,\Lambda}\right)\wedge K^1 - \frac{\cos^3\zeta}{3\,m}\left(\hat{\star}_4\partial_y\tilde{\rho}\right)\wedge K^2 \\ &- \left[\frac{e^{3\,\Lambda}}{3\,m}\cos^2\zeta\,\hat{\star}_4 d_4\tilde{\rho} + 4\,m\,e^{-3\,\Lambda}\,\hat{J}\right]\wedge K^1\wedge K^2\ . \end{aligned} \tag{A.13}$$

In this expression the hatted quantities are referred to the four-dimensional metric $\hat{g}^{(4)}_{\mu\nu} = e^{6\,\Lambda}\,g^{(4)}_{\mu\nu}$. Finally, d_4 is the exterior derivative on the four-dimensional space that is transverse to AdS_5 and K^1, K^2.

A.3 Recovering the non-Abelian T-dual from GMSW

Let us now find the explicit map between the GMSW geometry and the lifted non-Abelian T-dual geometry. In order to do this we first identify the functions Λ and ζ according to,

$$e^{6\,\Lambda} = 4\,y^2 + \frac{q}{9}\,, \qquad \cos\zeta = \sqrt{\frac{q}{36\,y^2+q}}\,, \tag{A.14}$$

where q is a function of the coordinates on M_4 and y, determined below. We also take

$$\tilde{\rho} = -\frac{1}{6\,q}\,dw - \frac{1-12\,q}{12\,q}\,\cos\theta_1\,d\phi_1\ . \tag{A.15}$$

Then the one-forms K^1 and K^2 read,

$$K^1 = \frac{3}{\sqrt{q}}\,dy\,, \qquad K^2 = \frac{1}{3\,m}\sqrt{\frac{q}{36\,y^2+q}}\left[d\tilde{\psi} - \frac{1}{6\,q}\,dw - \frac{1-12\,q}{12\,q}\,\cos\theta_1\,d\phi_1\right]\ . \tag{A.16}$$

Moreover, we define an orthogonal frame for the four-dimensional space M_4,

$$\begin{aligned} e^1 &= \frac{1}{\sqrt{6}}\,\sin\theta_1\,d\phi_1\,, & e^2 &= \frac{1}{\sqrt{6}}\,d\theta_1 \\ e^3 &= \frac{1}{18}\sqrt{\frac{36\,q-1}{36\,y^2\,q+q^2}}\,dz\,, & e^4 &= \frac{1}{18}\sqrt{\frac{36\,q-1}{36\,y^2\,q+q^2}}\left(dw + \frac{1}{2}\,\cos\theta_1\,d\phi_1\right)\ . \end{aligned} \tag{A.17}$$

In the above expressions, q can be thought of as a function of z and y through the relation,

$$z - 162\,y^2 - \frac{36\,q-1}{12} - \frac{1}{12}\,\ln\left(36\,q-1\right) = 0\ . \tag{A.18}$$

Solving this equation for q one finds,[8]

$$q = \frac{1}{36}\left[1 + \mathrm{ProductLog}\left(e^{12\,(z-162\,y^2)}\right)\right]\ . \tag{A.19}$$

[7]There is a sign difference between the first term in the second line of (A.13) and the corresponding term in eq. (2.50) of [65], that is due to our different conventions for Hodge duality.

[8]With ProductLog($\mathcal{Z}$) we mean the solution of the equation $\mathcal{Z} = \mathcal{W}\,e^{\mathcal{W}}$ in terms of $\mathcal{W}$.

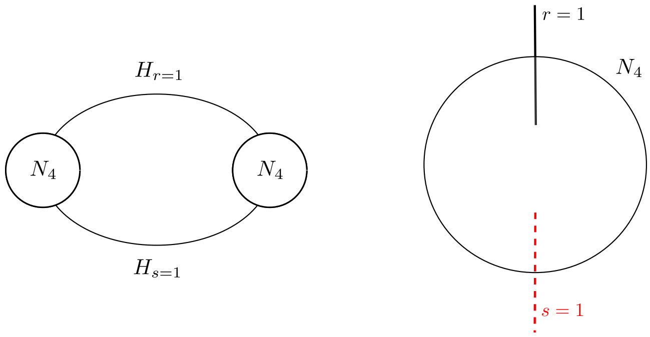

Figure 7. Circular quiver associated to the Abelian T-dual solution and corresponding brane set-up. There are N_4 D4-branes stretched between the NS5 and NS5' branes. NS5 and NS5'-branes are represented by transversal black and red dashed lines, respectively.

From the above frame one can construct the forms J and Ω of the SU(2) structure on M_4 as,

$$J = e^1 \wedge e^2 + e^3 \wedge e^4 \,, \qquad \Omega = e^{i\,\tilde\psi}\left(e^1 + i\,e^2\right) \wedge \left(e^3 + i\,e^4\right) \,. \tag{A.20}$$

Both the metric and the 4-form flux associated to our solution are then obtained after identifying,[9]

$$y = \frac{\rho\,\cos\chi}{6}\,, \qquad w = 9\,x_{11} + \frac{\xi}{6}\,, \qquad \tilde\psi = \xi \tag{A.21}$$

and

$$q = \frac{1}{36} + \frac{3}{2}\rho^2\,\sin^2\chi \,. \tag{A.22}$$

One can also check that with the above definitions the constraints (A.9)–(A.12), proving that the solution of appendix A.1 fits into the class of solutions found in [65], are satisfied.

B The Abelian T-dual of the Klebanov-Witten solution

The Abelian T-dual, Type IIA description, of the Klebanov-Witten theory is particularly useful for the study of certain properties of this theory [63, 64]. One interesting aspect is that the field theory can directly be read from the D4, NS5, NS5' brane set-up associated to this solution. We have depicted both the brane set-up and the associated quiver in figure 7.

In this appendix we discuss some aspects of this description that are relevant for the understanding of the CFT interpretation of the non-Abelian T-dual solution, the main objective of this work.

B.1 Background

The paper [63] considered an Abelian T-duality transformation along the Hopf-fiber direction of the $T^{1,1}$. This dualization gives rise to a well-defined string theory background.

[9] We take $L = m = \alpha' = g_s = 1$ for convenience. There is a minus overall sign between G, from (A.13), and F_4, from (A.3), due to our different conventions.

It is however a typical example of *Supersymmetry without supersymmetry* [78], being the low energy supergravity background non-supersymmetric. Since our ultimate goal in this section will be to compare with the non-Abelian T-dual solution, which is only guaranteed to be a well-defined string theory background at low energies, we will instead dualize along the ϕ_2 azimuthal direction of the $T^{1,1}$. This preserves the $\mathcal{N}=1$ supersymmetry of the Klebanov-Witten solution, and can be matched directly with the non-Abelian T-dual solution in the large ρ limit.

We start by rewriting the Klebanov-Witten metric in terms of the T-duality preferred frame, in which ϕ_2 does only appear in the form $d\phi_2$ and just in one vielbein,

$$\begin{aligned}
&e^{x^\mu} = \frac{r}{L} dx^\mu\,, \qquad e^r = \frac{L}{r} dr\,, \qquad e^1 = L\,\lambda_1\, d\theta_1\,, \qquad e^2 = L\,\lambda_1\,\sin\theta_1\, d\phi_1\,,\\
&e^{\hat{1}} = L\,\lambda_2\, d\theta_2\,, \qquad e^{\hat{2}} = L\,\lambda\,\lambda_2\,\frac{\sin\theta_2}{\sqrt{P(\theta_2)}}\left(d\psi + \cos\theta_1\, d\phi_1\right),\\
&e^3 = e^C\left(d\phi_2 + \tilde{A}_1\right),
\end{aligned} \tag{B.1}$$

where $e^{2C} = L^2 P(\theta_2)$ with $P(\theta_2) = \lambda^2\cos^2\theta_2 + \lambda_2^2\sin^2\theta_2$, and we have introduced the connection

$$\tilde{A}_1 = \frac{\lambda^2\cos\theta_2}{P(\theta_2)}\left(d\psi + \cos\theta_1\, d\phi_1\right).$$

The Klebanov-Witten metric thus reads

$$\begin{aligned}
ds^2 = ds^2_{AdS_5} + L^2\Bigg[\lambda_1^2\, d\Omega_2^2(\theta_1,\phi_1) + \lambda_2^2\left(d\theta_2^2 + \frac{\lambda^2\sin^2\theta_2}{P(\theta_2)}\left(d\psi + \cos\theta_1 d\phi_1\right)^2\right)\\
+ P(\theta_2)\left(d\phi_2 + \frac{\lambda^2\cos\theta_2}{P(\theta_2)}\left(d\psi + \cos\theta_1 d\phi_1\right)\right)^2\Bigg].
\end{aligned} \tag{B.2}$$

A U(1) T-duality performed on the ϕ_2 direction trades the vielbein e^3 for $\hat{e} = \alpha' e^{-C} d\phi_2$, and generates a NS-NS 2-form $B_2 = \alpha'\tilde{A}_1 \wedge d\phi_2$. The NS-NS sector for the dual solution is then given by:[10]

$$\begin{aligned}
ds^2_{\rm ATD} &= ds^2_{AdS_5} + L^2\lambda_1^2\left[d\Omega_2^2(\theta_1,\phi_1) + \left(d\theta_2^2 + \frac{\lambda^2\sin^2\theta_2}{P(\theta_2)}\left(d\psi + \cos\theta_1 d\phi_1\right)^2\right) + \frac{d\phi_2^2}{\lambda_1^2 P(\theta_2)}\right],\\
B_2^{\rm ATD} &= -\frac{L^2\lambda^2\cos\theta_2}{P(\theta_2)}\left(d\phi_2\wedge d\psi + \cos\theta_1\, d\phi_2\wedge d\phi_1\right),\\
e^{-2\Phi_{\rm ATD}} &= \frac{L^2}{g_s^2\alpha'} P(\theta_2)\,.
\end{aligned} \tag{B.3}$$

We can see in the metric the geometrical realization of the U(1) R-symmetry in the ψ direction. We can also see that it agrees with the asymptotic form of the metric of the non-Abelian T-dual solution, given by the first equation in (2.18), under the replacements

$$\chi \to \theta_2\,, \qquad \xi \to \psi\,, \qquad \rho \to \phi_2\,. \tag{B.4}$$

[10] We rescale $\phi_2 \to \frac{L^2}{\alpha'}\phi_2$, so that the metric of the internal space scales with L^2. We also use that $\lambda_2 = \lambda_1$ for later comparison with the NATD solution.

The B_2 fields do also agree, once a gauge transformation of parameter

$$\Lambda = -L^2 \cos\theta_2\, \phi_2 \Big(d\psi + \frac{\lambda^2 \cos\theta_1}{P(\theta_2)} d\phi_1 \Big) \tag{B.5}$$

is performed, giving rise to

$$\begin{aligned} B_2 \;=\; & -L^2 \phi_2 \left[d\Omega_2(\theta_2,\psi) + \frac{\lambda^2 \cos\theta_2}{P(\theta_2)}\, d\Omega_2(\theta_1,\phi_1) - \lambda^2\, \cos\theta_1\, \partial_{\theta_2} \left(\frac{\cos\theta_2}{P(\theta_2)} \right) d\theta_2 \wedge d\phi_1 \right] \\ & + \frac{L^2 \sin\theta_2}{2\, P(\theta_2)} \left(\lambda^2 - \lambda_1^2\right)\, \sin 2\theta_2\, d\psi \wedge d\phi_2 \,. \end{aligned} \tag{B.6}$$

We will use this expression for the B_2-field in the remaining of this section. As in [44, 45], the two dilatons satisfy $e^{-2\Phi_{NATD}} \approx \rho^2\, e^{-2\Phi_{\rm ATD}}$ for large ρ (after re-absorbing the scaling factors in $\rho \to \frac{\alpha'}{L^2}\rho$). As explained in [44, 45], this relation has its origin in the different measures in the partition functions of the non-Abelian and Abelian T-dual sigma models.

Finally, the RR fields are:

$$\begin{aligned} F_4 &= \frac{4\, L^4\, \lambda\, \lambda_1^4}{g_s\, \alpha'^{1/2}}\, \sin\theta_1 \sin\theta_2\; d\theta_1 \wedge d\phi_1 \wedge d\theta_2 \wedge d\psi \,, \\ F_6 &= \frac{4\, L}{g_s\, \alpha'^{1/2}}\, {\rm Vol}_{AdS_5} \wedge d\phi_2 \,. \end{aligned} \tag{B.7}$$

One can check that, as in [45], for large ρ the fluxes polyforms satisfy

$$e^{\Phi_{NATD}} F_{NATD} \approx e^{\Phi_{\rm ATD}} F_{\rm ATD} \,. \tag{B.8}$$

The previous relations show that the non-Abelian T-dual solution reduces in the $\rho \to \infty$ limit to the Abelian T-dual one. This connection between non-Abelian and Abelian T-duals was discussed previously in examples where the dualization took place on a round S^3 [44, 45]. Our results show that it extends more generally. It is worth stressing however that in this case the relation is more subtle globally. Indeed, the relations in eq. (B.4) identify $\xi \in [0, 2\pi]$ with $\psi \in [0, 4\pi]$. The reason for this apparent mismatch is that the dualization on ϕ_2 generates a bolt singularity in the metric, and this must be cured by setting $\psi \in [0, 2\pi]$, such that the bolt singularity reduces to the coordinate singularity of $\mathbb{R}^2$ written in polar coordinates. Once this is taken into account the ranges of both coordinates also agree. As encountered in [12, 13], the dualization has enforced a $\mathbb{Z}_2$ quotient on ψ. Our Abelian T-dual is thus describing the Klebanov-Witten theory modded by $\mathbb{Z}_2$. This is consistent with the brane set-up that is implied by the quantized charges of the background, as we now show.

B.2 Quantized charges and brane set-up

The background fluxes of the Abelian T-dual solution support $D4$ and $NS5$-brane charges. The Page charge for the $D4$ branes is given by:

$$Q_{D4} = \frac{1}{2\, \kappa_{10}^2\, T_{D4}} \int_{M_4} F_4 = \frac{2}{27} \frac{L^4}{\pi\, g_s^2\, \alpha'^2} = N_4 \,. \tag{B.9}$$

Imposing the quantization of this charge we find that the radius L is related to the number of $D4$ branes through the formula:

$$L^4 = \frac{27}{2}\,\pi\, g_s^2\, \alpha'^2\, N_4\ . \tag{B.10}$$

We find a factor of 2 of difference with respect to the original background. This is due to the change in the periodicity of the ψ direction from $[0, 4\pi]$ to $[0, 2\pi]$.

In turn, the charge of $NS5$ branes is calculated from:

$$Q_{NS5} = \frac{1}{4\,\pi^2\,\alpha'}\int_{M_3} H_3\,. \tag{B.11}$$

As in section 2.2.2, we can define two 3-cycles: $\Sigma_3 = [\phi_2, \theta_2, \psi]$ and $\Sigma_3' = [\phi_2, \theta_1, \phi_1]_{\theta_2=0}$. Taking M_3 to be any of these cycles we find that there are two units of NS5, or NS5', charge. This is consistent with a brane picture of two alternating NS5, NS5' branes, transverse to either of the two 2-cycles $\tilde{S}^2(\theta_2, \psi)$, $S^2(\theta_1, \phi_1)$, located along the compact ϕ_2-direction. This is the brane set-up discussed in [64], describing the Klebanov-Witten theory modded by $\mathbb{Z}_2$ in Type IIA. The general $\mathbb{Z}_k$ case is depicted in figure 9 of appendix C.3. Note that, as discussed in [64], the positions of the branes in the ϕ_2-circle are not specified by the geometry, so generically we can only think that they define four intervals in the ϕ_2-circle.[11] The same number of D4-branes are stretched between each pair of NS5, NS5' branes since even if large gauge transformations are required as we pass the value $\phi_2 = \pi L^2/\alpha'$, the D4-brane charge does not change in the absence of F_2-flux.

Coming back to section 2.2.3, the relation found there between the central charges of the non-Abelian and Abelian T-dual solutions helps us understand now the connection between ρ and ϕ_2 globally. The computation in that section showed that the central charges agree when $\rho \in [n\pi\frac{L^2}{\alpha'}, (n+1)\pi\frac{L^2}{\alpha'}]$ and n is sent to infinity. This is consistent with the $\rho \to \infty$ limit that must be taken at the level of the solutions. Furthermore, it clarifies why globally the ρ direction is identified, through the replacements in (B.4), with $\phi_2 \in [0, 2\pi\frac{L^2}{\alpha'}]$. This is just implied by the $\mathbb{Z}_2$ quotient enforced by the Abelian T-duality transformation.

C Some field theory elaborations

In this appendix we discuss some aspects of the field theory analysis presented in section 4. We start with the calculation of the beta functions and anomalies for the Klebanov-Witten CFT.

C.1 A summary of the Klebanov-Witten CFT

The field content of the Klebanov-Witten theory consists on a $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge group with bifundamental matter fields A_1, A_2 and B_1, B_2, transforming in the $(N, \bar{N})$

[11] In [64] it was argued that the different orderings correspond to different phases in the Kähler moduli space of the orbifold singularity. This is interpreted in the field theory side in terms of Seiberg duality [79, 80], so the corresponding theories should flow to the same CFT in the infrared.

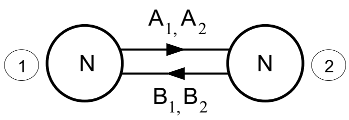

Figure 8. The KW quiver.

and $(\bar{N},N)$ representations of SU(N), respectively. This theory is represented by the quiver depicted in figure 8. The anomalous dimensions of the matter fields are,

$$\gamma_{A_i}=\gamma_{B_i}=-\frac{1}{2}\,, \tag{C.1}$$

and thus the physical dimensions and the R-charges are given by,

$$\begin{aligned} \dim(A_i)=\dim(B_i)=1-\frac{1}{4}=\frac{3}{4}\,,\\ R_{[A]}=R_{[B]}=\frac{1}{2}\,,\qquad R_{\Psi_A}=R_{\Psi_B}=-\frac{1}{2}\,. \end{aligned} \tag{C.2}$$

Substituting in eq. (4.1) we see that the β-functions for the couplings g_1 and g_2 vanish:

$$\beta_i\sim 3\,N-2\,N\left[1-\left(-\frac{1}{2}\right)\right]=0\,,\qquad i=1,2\,. \tag{C.3}$$

We can also check the vanishing of the anomaly,

$$\Delta\theta_i=2\,N+2\,(2\,N)\left(-\frac{1}{2}\right)=0\,, \tag{C.4}$$

where we took into account that the R-charge of the gaugino is 1 while that of the two Weyl fermions is $-1/2$.

We hope that this has prepared the reader unfamiliar with these formalities to understand the material in our section 4.

C.2 Central Charge of the $\mathcal{N}=2$ UV CFT

In this appendix we compute the central charge of the $\mathcal{N}=2$ quiver associated to the non-Abelian T-dual of $AdS_5\times S^5/\mathbb{Z}_2$, using a-maximization. We obtain that the central charge is maximized for $\epsilon=\frac{1}{3}$, as for the equal rank quivers considered in [55]. Furthermore, we show that the result of this calculation leads, consistently, to the holographic central charge given by eq. (4.20).

We consider the $\mathbb{Z}_2$-reflection of the quiver of figure 6 and take $\sigma_i=+1$ for all hypermultiplets, including the ones associated with the flavor groups. We then find for the trace anomalies ($N\equiv N_6$):

$$\begin{aligned} \mathrm{Tr}\,R_\epsilon(H)&=2\sum_{j=1}^{n-1}\mathrm{Tr}\,R_\epsilon(H_j)+\sum_{i=1}^{2}\mathrm{Tr}\,R_\epsilon(H_{f_i})\\ &=2N^2\sum_{j=1}^{n-1}j\,(j+1)\,(\epsilon\,\sigma_j-1)+n\,N^2\sum_{i=1}^{2}(\epsilon\,\sigma_{f_i}-1) \end{aligned}$$

$$= N^2 \left[\frac{2}{3} (n^3 - n)(\epsilon - 1) + 2n(\epsilon - 1) \right]$$
$$= N^2 \left[\frac{2}{3} n^3 + \frac{4}{3} n \right] (\epsilon - 1) \approx \frac{2}{3} n^3 N^2 (\epsilon - 1) + \mathcal{O}(n) , \tag{C.5}$$

as well as

$$\begin{aligned}
\operatorname{Tr} R_\epsilon^3(H) &= 2 \sum_{j=1}^{n-1} \operatorname{Tr} R_\epsilon^3(H_j) + \sum_{i=1}^{2} \operatorname{Tr} R_\epsilon^3(H_{f_i}) \\
&= 2\, N^2 \sum_{j=1}^{n-1} j\,(j+1)\, \frac{(\epsilon - 1)^3}{4} + n\, N^2 \sum_{i=1}^{2} \frac{(\epsilon\, \sigma_{f_i} - 1)^3}{4} \\
&= \frac{N^2}{4} \left[\frac{2}{3} (n^3 - n)(\epsilon - 1)^3 + 2n(\epsilon - 1)^3 \right] \\
&= N^2 \left[\frac{1}{6} n^3 + \frac{1}{3} n \right] (\epsilon - 1)^3 \approx \frac{1}{6} n^3 N^2 (\epsilon - 1)^3 + \mathcal{O}(n) .
\end{aligned} \tag{C.6}$$

For the $\mathcal{N} = 2$ vector multiplets ($\mathcal{N} = 1$ vector + chiral adjoint) the non-anomalous R-charge $R_\epsilon = R_0 + \epsilon\, \mathcal{F}/2$ is obtained from the R-charge for the gaugino, $R_0(\lambda) = 1$, plus the non-anomalous flavor charge of the fermion in the chiral adjoint $\mathcal{F}(\psi_j) = (-1)(\sigma_{j-1} + \sigma_j)$, being $R_0(\psi_j) = 0$. We thus have:

$$\begin{aligned}
\operatorname{Tr} R_\epsilon(V_j) &= (N_j^2 - 1) \left(1 - \frac{1}{2}\, \epsilon\, (\sigma_{j-1} + \sigma_j) \right) , \\
\operatorname{Tr} R_\epsilon^3(V_j) &= (N_j^2 - 1) \left(1 - \frac{1}{8}\, \epsilon^3\, (\sigma_{j-1} + \sigma_j)^3 \right) .
\end{aligned} \tag{C.7}$$

These are summed up easily for all $\sigma_j = +1$:

$$\begin{aligned}
\operatorname{Tr} R_\epsilon(V) &= 2 \sum_{j=1}^{n-1} \operatorname{Tr} R_\epsilon(V_j) + \operatorname{Tr} R_\epsilon(V_n) = \left[2 \sum_{j=1}^{n-1} (j^2 N^2 - 1) + (n^2 N^2 - 1) \right] (1 - \epsilon) \\
&= \left[\frac{1}{3} (2n^3 + n)\, N^2 - (2n - 1) \right] (1 - \epsilon) \approx \frac{2}{3} n^3 N^2 (1 - \epsilon) + \mathcal{O}(n) .
\end{aligned} \tag{C.8}$$

The cubic term follows most readily:

$$\operatorname{Tr} R_\epsilon^3(V) = \left[\frac{1}{3} (2n^3 + n)\, N^2 - (2n - 1) \right] (1 - \epsilon^3) \approx \frac{2}{3} n^3 N^2 (1 - \epsilon^3) + \mathcal{O}(n) . \tag{C.9}$$

We thus see that both linear contributions (C.5) and (C.8) from the hypermultiplets and vector multiplets cancel at leading order, so that

$$\operatorname{Tr} R_\epsilon \equiv \operatorname{Tr} R_\epsilon(H) + \operatorname{Tr} R_\epsilon(V) \approx \mathcal{O}(n) .$$

Now both $a(\epsilon)$ and $c(\epsilon)$ charges can be computed exactly,

$$\begin{aligned}
a(\epsilon) &= \frac{3}{64} (1 - \epsilon) \left\{ \left[3n^3 (1 + \epsilon)^2 + 2(1 + 3\epsilon) n \right] N_6^2 - 2(2n - 1) \Big(2 + 3\epsilon(1 + \epsilon) \Big) \right\} , \\
c(\epsilon) &= \frac{1}{64} (1 - \epsilon) \left\{ \left[9n^3 (1 + \epsilon)^2 + 2(5 + 9\epsilon) n \right] N_6^2 - 2(2n - 1) \Big(4 + 9\epsilon(1 + \epsilon) \Big) \right\} ,
\end{aligned} \tag{C.10}$$

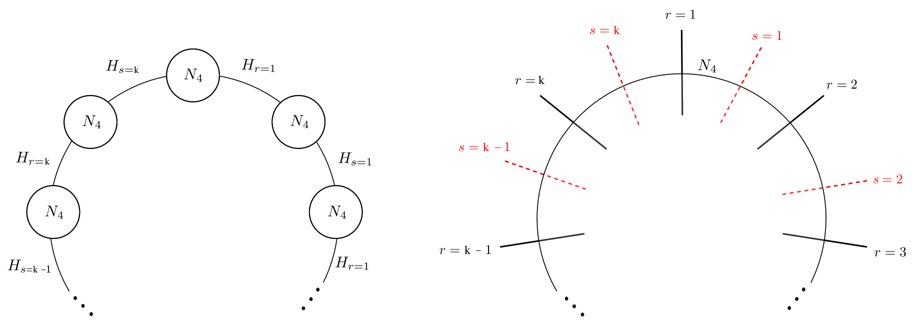

Figure 9. Circular quiver associated to the KW theory modded by $\mathbb{Z}_k$, and corresponding brane set-up. There are N_4 D4-branes stretched between $p=k$ NS5-branes, labeled by $r=1,\ldots,k$ (as for the corresponding hypermultiplets) and $q=k$ NS5'-branes labeled by $s=1,\ldots,k$. NS5 and NS5'-branes are represented by transversal black and red dashed lines, respectively.

and $a(\epsilon)$ is maximized for $\epsilon=1/3$, yielding the superconformal charges:

$$\begin{aligned} a_{\mathcal{N}=2} &\equiv a(\epsilon=1/3) = \frac{1}{24}\left\{(4n^3+3n)N_6^2 - 10n+5\right\}, \\ c_{\mathcal{N}=2} &\equiv c(\epsilon=1/3) = \frac{1}{6}\left\{(n^3+n)N_6^2 - 2n+1\right\}. \end{aligned} \tag{C.11}$$

In the long quiver approximation, we recover the holographic result

$$c_{\mathcal{N}=2} \approx a_{\mathcal{N}=2} \approx \frac{1}{6}\, n^3\, N_6^2 + \mathcal{O}(n)\,, \tag{C.12}$$

as expected. It is noteworthy that $\epsilon=1/3$ is the value of ϵ predicted in [55] for $\mathcal{N}=2$ quivers with nodes of the same rank.

C.3 Central charge of the Klebanov-Witten theory modded by $\mathbb{Z}_k$

In this appendix we include, for completeness, the field theory calculation of the central charge of the Klebanov-Witten theory, using a-maximization. We will center in the more general case in which the theory is modded by $\mathbb{Z}_k$. The computation of the field theoretical central charge in this example is very illustrative of the a-maximization technique used throughout the paper.

In this case we have, in the Type IIA description, $p=k$ NS5-branes and $q=k$ NS5'-branes, and $\ell=p+q=2k$ hypermultiplets connecting ℓ $\mathcal{N}=1$ vector multiplets [64]. The first and the last nodes are made to coincide, as depicted in Figure 9. We closely follow the field-theoretical computation of the central charge for the linear quiver proposed in section 4.3. We just need to take $N_a=N_b=N_4$ for the bifundamentals in (4.10) for all the $\ell=2k$ nodes. This yields the linear contribution for the hypermultiplets:

$$\begin{aligned} \mathrm{Tr}\, R_\epsilon(H) &\equiv \sum_{j=1}^{\ell} \mathrm{Tr}\, R_\epsilon(H_j) = \sum_{r=1}^{k} \mathrm{Tr}\, R_\epsilon(H_r) + \sum_{s=1}^{k} \mathrm{Tr}\, R_\epsilon(H_s) \\ &= \ell\, N_4^2\big(z\,\epsilon-1\big) \overset{z=0}{=} -2\,k\,N_4^2\,, \end{aligned}$$

where we have used $z = (p-q)/\ell$. Similarly, the cubic contribution is given by

$$\mathrm{Tr}\, R_\epsilon^3(H) = \frac{\ell}{4}\, N_4^2 \Big(z\,(\epsilon^3 + 3\,\epsilon) - 3\,\epsilon^2 - 1 \Big)\Big|_{z=0} = -\frac{k}{2}\, N_4^2\,(3\,\epsilon^2 + 1)\ .$$

Contributions from $\mathcal{N} = 1$ vector multiplets are computed straightforwardly to be:

$$\mathrm{Tr}\, R_\epsilon(V) = \mathrm{Tr}\, R_\epsilon^3(V) = \ell\,(N_4^2 - 1) = 2\,k\,(N_4^2 - 1)\,.$$

We can now use (4.9) to get

$$c(\epsilon) = \frac{\ell}{128}\Big(27\, N_4^2\,(1-\epsilon^2) - 16\Big)\,, \tag{C.13}$$

which, upon a-maximization for $\epsilon = 0$, yields the fixed point central charge (for large N_4):

$$c \approx \frac{27}{64}\, k\, N_4^2\,, \tag{C.14}$$

which coincides, as expected, with the holographic value (given by eq. (2.36) for $k = 1$). This expression is valid for any $k \geq 1$, i.e. no large $\ell = 2k$ limit has been assumed.

Note that in the absence of flavor groups it is not possible to define σ_0, σ_ℓ, and neither $\kappa = \sigma_0 + \sigma_\ell$, as we have done for the linear quivers discussed in section 3. Still, the result in (C.14) agrees with the central charge of a Bah-Bobev type of linear quiver (see eq. (3.20) in [55]) for $\kappa = 0$ and large ℓ. Indeed, even if there is no clear definition for κ in this case, the uplift of the circular brane set-up is interpreted as M5-branes wrapping a torus ($\kappa = 0$) with minimal punctures, as the gauging of the end flavor groups of the linear quiver corresponds in M-theory to gluing the two left-over maximal punctures, closing up the Riemann surface.

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3.6 $\text{Mink}_3 \times S^3$ solutions of type II supergravity

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$\mathrm{Mink}_3 \times S^3$ solutions of type II supergravity

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Abstract

We initiate the classification of supersymmetric solutions of type II supergravity on $\mathbb{R}^{1,2} \times S^3 \times M_4$. We find explicit local expressions for all backgrounds with either a single Killing spinor or two of equal norm, up to PDE's. We show that the only type II $\mathrm{AdS}_4 \times S^3$ solution is the known $\mathcal{N} = 4$ AdS_4 background obtained from the near-horizon limit of intersecting D2–D6 branes. Various known branes and intersecting brane systems are recovered, and we obtain a novel class of $\mathbb{R}^{1,2} \times S^2 \times S^3$ solutions in IIA.

1. Introduction

The advent of the AdS-CFT correspondence has led to significant interest in the construction of Anti-de Sitter string backgrounds in various dimensions and with various amounts of supersymmetry. One of the most famous AdS_4 backgrounds is the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ solution. The discovery of this solution far pre-dates the correspondence [1], however it was not realised how it fit into the holographic paradigm until the works of [2] and [3]. A plethora of other such AdS_4 classes and explicit examples have been found using (in some cases vastly) different methods

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and exhibiting different amounts of supersymmetry: consider the very incomplete list of [4–14] for $\mathcal{N}=1$, [15–21] for $\mathcal{N}=2$, [22] for $\mathcal{N}=3$ and [23–26] for $\mathcal{N}=4$. Solutions with $\mathcal{N}>4$ where recently classified in [27], they are very restricted.

One of the more prominent methods of finding AdS_d backgrounds is to find bosonic solutions with an AdS_d factor to the supersymmetry constraints, which also satisfy the Bianchi identities. As a consequence of various integrability theorems, such solutions automatically solve the equations of motions. The Killing spinor equations reduce to constraints on the internal manifold, which can then be solved by means of G-structure and generalised geometrical techniques. The literature usually approaches this problem by assuming an AdS_d from the start. However we are also interested in solutions of relevance to flux compactifications and the broader definition of holography that includes non-conformal solutions. As such we shall consider assume Minkowski factor, in this case Mink_3, so that our results are more broadly applicable.

Finding Minkowski solutions using G-structure techniques [28–31] or otherwise is by now quite a mature program, see [32–35] for some recent examples. Usually the aim is to preserve minimal or even no supersymmetry for phenomenological reason which makes the problem in general quite hard. We shall take inspiration from [36] and assume the existence of an S^3 factor in the metric. This will necessarily mean that we are dealing with at least $\mathcal{N}=2$ which is of less phenomenological interest, however with these solutions classified it should then be possible to systematically break some (or even all) of this symmetry by deforming the S^3.

In this paper we classify all supersymmetric solutions of Type II supergravity on $\mathbb{R}^{1,2}\times S^3\times M_4$, under the assumption that the seven-dimensional internal Killing spinors have equal norms and that the physical fields of the solution respect the $ISO(1,2)\times SO(4)$ isometry subgroup. Our classification is quite detailed, going as far as to give explicit local expressions the metric, fluxes and dilaton in terms of simple (Laplace-like) PDE's. As we shall see, solutions in this class are generically $\mathcal{N}=2$, from the Minkowski perspective, and support a $SU(2)$ R-symmetry realised geometrically as one factor of the $SO(4)\simeq SU(2)_+\times SU(2)_-$ isometry group manifold of S^3 – the remaining $SU(2)$ factor is a "flavour" under which the Killing spinors are uncharged.[1] This may sound strange as there is no 3d superconformal algebra with $SU(2)_R$, but this only matters for solutions where $\mathbb{R}^{1,2}$ is part of a AdS_4 factor so that $SO(2,3)$ is realised. Ultimately our results end up side stepping this issue as in general their is either an enhancement of the R-symmetry to $SO(4)$ via the emergence of an additional $S^{2,3}$ factor, or an enhancement of the Minkowski factor to dimensions where $SU(2)_R$ is a necessary part of the superconformal algebra.

The classification recovers various well-known intersecting brane systems listed in [40] and some of their U-duals and some of their S-duals. New classes we find include a pure NS $\mathbb{R}^{1,2}\times S^3\times S^2\times\mathbb{R}$ vacuum, its U-dual in IIB, the cone over $\mathbb{R}^{1,2}\times S^3\times S^3$, and a novel class of $\mathbb{R}^{1,2}\times S^2\times S^3\times\Sigma_2$ solutions in massless and massive IIA.

One of our main results is that the only compact $\mathrm{AdS}_4\times S^3\times M_3$ solution of type II supergravity is the known $\mathcal{N}=4$ solution of type IIA on a foliation of $\mathrm{AdS}_4\times S^3\times S^2$ over an interval which is the near-horizon limit of the D2–D6 brane system. The required $SO(4)_R$ is realised with one $SU(2)$ from each sphere, and not the S^3 alone. Indeed this is to be expected as if the 3-sphere does realise two $SU(2)$ R-symmetries there would be two sets of $\mathcal{N}=2$ spinors transforming in the (2, 1) and (1, 2) of $SO(4)$ – there is no $\mathcal{N}=4$ super-conformal algebra in 3d

[1] As such all classes we present are compatible with performing non-Abelian T-duality [45–47] on this $SU(2)$ whilst preserving $SU(2)_R$ [48].

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with Q-generators that transform in this fashion. So it seems likely that the only avenue left open for holographic duals of $\mathcal{N}=4$ is to seek $\mathrm{AdS}_4 \times S^2 \times S^2$ solution like [23,24], but in massive IIA.

Our other main result is the discovery of a new class of $\mathcal{N}=4$ solutions on $\mathbb{R}^{1,2} \times S^2 \times S^3 \times \Sigma_2$ preserving an $SO(4)$ R-symmetry but no AdS_4. These generically have all possible IIA fluxes turned on and can be divided into cases either in massless or massive IIA at which point solutions are in one to one correspondence with a single PDE on σ_2. In particular the massless solutions are governed by a 3d cylindrical Laplace equation with axial symmetry. These classes look very promising both for finding compact Mink_3 solutions, but also possibly solutions that asymptote to AdS.

Let us now describe the outline of the paper: in order to solve the supersymmetry constraints, we will make use of the reformulation of the Killing spinor equations in terms of so-called pure spinor equations. Such pure spinor equations were first used for backgrounds of the form $M_{10} = \mathbb{R}^{1,3} \times M_6$, where it was shown that they are related to integrability constraints of generalised almost complex structures on the internal space M_6 [30]. For backgrounds of the form $M_{10} = \mathbb{R}^{1,2} \times M_7$, the pure spinor equations were constructed in [31] (see also [37]). Next, we decompose $M_7 = S^3 \times M_4$, leading to pure spinor equations on the internal M_4. We explain this setup in detail in section 2. The resulting supersymmetry constraints vary significantly, depending on whether the theory at hand is type IIA or type IIB. We will solve the supersymmetry constraints as well as the Bianchi identities for IIB backgrounds in section 3 and for IIA backgrounds in section 4. In section 5, we then show that there is a unique solution with a warped AdS_4 factor, obtained from the D2–D6 system. In addition to the case where the internal Killing spinors have equivalent norm, in section 6 we examine all backgrounds in the case where one of the Killing spinors vanishes, i.e., $\epsilon_2 = 0$. In this case, there is no need to distinguish between IIA and IIB; we demonstrate that all such backgrounds are pure NSNS and give the solutions. In the appendix, we discuss conventions and identities used, a mild extension of the $3+7$ pure spinor equation construction (including the non-equivalent norm case), and a discussion on similar backgrounds from an M-theory perspective.

2. Mink_3 with an S^3 factor

We are interested in solutions to type II with at least a three-dimensional external Minkowski component, with the fluxes respecting the three-dimensional Poincaré invariance:

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,2}) + ds^2(M_7)\,, \quad F = f + e^{3A}\mathrm{Vol}_3 \wedge \star_7 \lambda(f)\,, \tag{2.1}$$

where the RR flux f is a polyform on M_7 and the warp factor A and the dilaton Φ are functions on M_7.[2] Moreover, we take the NSNS 3-form H to be internal as well. The Killing spinors for $\mathcal{N}=1$ supersymmetric solutions decompose as

$$\epsilon_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \zeta \otimes \chi_1\,, \quad \epsilon_2 = \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \otimes \zeta \otimes \chi_2\,, \tag{2.2}$$

where ζ is a Majorana spinor of $Spin(1,2)$ and $\chi_{1,2}$ are Majorana spinors of $Spin(7)$ and where the upper (lower) signs are taken in IIA (IIB). Following [31], we define two real seven-dimensional bispinors $\Phi_\pm$ in terms of $\chi_{1,2}$:

[2] We work in the democratic formalism. Other conventions can be found in appendix A.

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$$\Phi_+ + i\Phi_- = 8e^{-A}\chi_1 \otimes \chi_2^\dagger \,, \tag{2.3}$$

where the subscript $+/-$ refers to the even/odd forms in the decomposition of the polyform. The conditions for unbroken $\mathcal{N}=1$ supersymmetry are equivalent to

$$d_H(e^{2A-\Phi}\Phi_\pm) = 0 \,, \tag{2.4a}$$

$$d_H(e^{3A-\Phi}\Phi_\mp) + e^{3A} \star_7 \lambda(f) = 0 \,, \tag{2.4b}$$

$$(\Phi_\pm \wedge \lambda(f))\big|_{\text{Top}} = 0 \,, \tag{2.4c}$$

as long as the norms of spinors $\chi_{1,2}$ are equal,[3] which leads to

$$|\chi^1|^2 = |\chi^1|^2 = e^A \tag{2.5}$$

The assumption of equal norm is a global requirement for AdS$_4$ (see footnote 6 in section 5), and a local requirement for the existence of calibrated D-branes or O-planes (see section 6), however this is not a requirement in general – rather we view this as a well-motivated simplifying assumption.

Next, we require that the internal space can be decomposed locally as $M_7 = S^3 \times M_4$, and in order to ensure that compactification leads to an $SO(4)$ global symmetry we insist that the fluxes respect the $SO(4)$ isometry. As a result, the metric and fluxes decompose further as

$$\begin{aligned} &ds^2(M_7) = e^{2C}ds^2(S^3) + ds^2(M_4) \,, \quad f = G_\mp + e^{3C}\text{Vol}(S^3) \wedge G_\pm \,, \\ &H = H_3 + H_0 e^{3C}\text{Vol}(S^3) \,. \end{aligned} \tag{2.6}$$

We decompose the 7d spinors in the same fashion in terms of a single[4] pseudoreal (i.e., $(\xi^c)^c = -\xi$) Killing spinor ξ on S^3, and two pseudoreal spinors $\eta_{1,2}$ on M_4:

$$\chi_i = e^{\frac{A}{2}}(\xi \otimes \eta_i + \xi^c \otimes \eta_i^c) = e^{\frac{A}{2}}\xi^a \otimes \eta_i^a \,, \qquad i = 1,2 \tag{2.7}$$

which is the most general parameterisation consistent with an $S^3 \times M_4$ product and the Majorana condition.[5] Note that we do not restrict the $Spin(4)$ spinors η_i to be chiral and we normalise $\eta_{1,2}^\dagger \eta_{1,2} = 1$. The Killing spinors on S^3 satisfy the Killing spinor equation

$$\nabla_\alpha \xi = \frac{1}{2} i \nu \sigma_\alpha \xi \,, \quad \nu = \pm 1 \,, \tag{2.8}$$

which preserves two supercharges for each of $\nu = \pm 1$. We will not make a choice of ν so we can establish whether any solutions are independent of this choice – the S^3 of such a solution would preserve 4 supercharges. As explained in Appendix C a spinor on S^3 defines a doublet

[3] In [31], [37] an additional constraint that was imposed in order to derive (2.4) was that the external component of the NSNS 3-form flux is trivial; unlike in four dimensions, this is not enforced by Poincaré invariance. It turns out that this second assumption is redundant though, as is shown in appendix D: if $|\chi_1|^2 = |\chi_2|^2$ and spacetime does not admit a cosmological constant, then supersymmetry enforces that the external NSNS flux vanishes.

[4] As explained in Appendix C, there are two independent types of Killing spinors on S^3, ξ_+ and ξ_- – however they cannot be mapped to each other using the $SO(4)$ invariants of the fluxes or the Killing spinor equations. This is all that appears when one decompose $M_7 = S^3 \times M_4$, so if one were to include terms like $\xi_+ \otimes \eta_+$ and $\xi_- \otimes \eta_-$ then reduced the 7d spinor conditions to 4d ones you would find that $\eta_\pm$ never mix. So setting one of $\eta_\pm$ to zero excludes no solutions in our analysis.

[5] One might imagine it was possible to construct a more general 7d spinor from two 4d spinors like $\xi \otimes \eta + \xi^c \otimes \tilde{\eta}$. But if one then adds the Majorana conjugate to this the resulting spinor can be put in the form of (2.7) by redefining η, $\tilde{\eta}$.

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$$\xi^a = \begin{pmatrix} \xi \\ \xi^c \end{pmatrix} \tag{2.9}$$

which is charged under one $SU(2)$ factor of $SO(4) = SU(2)_+ \times SU(2)_-$, depending on the sign of ν – ξ^a is a singlet under the action of the second $SU(2)$. As such, a generic solution with $\text{Mink}_3 \times S^3$ will have an R-symmetry $SU(2)_R$ and an additional global flavour symmetry $SU(2)_F$. Such solutions preserve at least $\mathcal{N} = 2$ supersymmetry from the 3d perspective, so 4 real supercharges – indeed, the 10d Killing spinors may be written as

$$\epsilon_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \zeta^a \otimes (\xi^a \otimes \eta^1 + \xi^{ac} \otimes \eta^{1c}), \quad \epsilon_2 = \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \otimes \zeta^a \otimes (\xi^a \otimes \eta^2 + \xi^{ac} \otimes \eta^{2c}),$$

where ζ^a is a doublet of Killing spinors on $\mathbb{R}^{1,2}$, that allow the 10d spinors to be invariant under $SU(2)_R$ transformations. However we only need to solve an $\mathcal{N} = 1$ sub-sector, because the part of the Killing spinor which couples to ζ^1 is mapped to the part coupling to ζ^2 under the action of $SU(2)_R$ – so if you solve one part, the other is guaranteed. If a solution ends up being independent of ν then there is a copy of (2.2) for each sign and supersymmetry is doubled to $\mathcal{N} = 4$ – there are two $SU(2)$ R-symmetries, but they do not appear as a product so do not form $SO(4)_R$ – as we shall see, this only happen in a small number of special cases.

Using the gamma matrix decomposition (A.2), the seven-dimensional bispinor (2.3) decomposes as

$$\chi_1 \otimes \chi_2^\dagger = (\xi^a \otimes \xi^b)_+ \wedge (\eta^a \otimes \eta^b) + (\xi^a \otimes \xi^b)_+ \wedge (\hat{\gamma}\eta^a \otimes \eta^b). \tag{2.10}$$

Here, $\hat{\gamma}$ is the four-dimensional chirality matrix and the $\pm$ subscripts again refer to even and odd form components. We see that the components are in fact matrices and that the seven-dimensional bispinor is constructed as the trace of the product of the components.

The S^3 component leads to the bispinor matrix

$$\xi^a \otimes \xi^{b\dagger} = \frac{1}{2}\left((1 - ie^{3C}\text{Vol}(S^3)) + \frac{1}{2}\left(e^C K_i - \frac{\nu}{2} i e^{2C} dK_i\right)(\sigma^i)_{ab}\right), \tag{2.11}$$

where K_i is a vielbein defining a trivial structure on S^3 (see appendix A).

The M_4 component leads to the bispinor matrix

$$(\eta_1^a \otimes \eta_2^{b\dagger})_\pm = \begin{pmatrix} \psi_\pm^1 & \psi_\pm^2 \\ \mp(\psi_\pm^2)* & \pm(\psi_\pm^1)* \end{pmatrix}, \quad (\hat{\gamma}\eta_1^a \otimes \eta_2^{b\dagger})_\pm = \begin{pmatrix} \psi_{\hat{\gamma}\pm}^1 & \psi_{\hat{\gamma}\pm}^2 \\ \mp(\psi_{\hat{\gamma}\pm}^2)* & \pm(\psi_{\hat{\gamma}\pm}^1)* \end{pmatrix}, \tag{2.12}$$

where

$$\psi^1 = 4\eta^1 \otimes \eta^{2\dagger}, \quad \psi^2 = 4\eta^1 \otimes \eta^{2c\dagger}, \quad \psi_{\hat{\gamma}}^1 = 4\hat{\gamma}\eta^1 \otimes \eta^{2\dagger}, \quad \psi_{\hat{\gamma}}^2 = 4\hat{\gamma}\eta^1 \otimes \eta^{2c\dagger} \tag{2.13}$$

Since the matrix entries are somewhat involved, we refer to appendix B for details. Plugging both components (2.11), (2.12) into the seven-dimensional bispinors (2.10), it follows that

$$\begin{aligned}\Phi_+ &= \text{Re}\psi_+^1 - e^{3C}\text{Vol}(S^3) \wedge \text{Im}\psi_{\hat{\gamma}-}^1 + \frac{e^C}{2}(K_1 \wedge \text{Re}\psi_{\hat{\gamma}-}^2 + K_2 \wedge \text{Im}\psi_{\hat{\gamma}-}^2 + K_3 \wedge \text{Re}\psi_{\hat{\gamma}-}^1), \\ &- \frac{e^{2C}}{4}(K_1 \wedge K_2 \wedge \text{Im}\psi_+^1 + K_1 \wedge K_3 \wedge \text{Re}\psi_+^2 + K_2 \wedge K_3 \wedge \text{Im}\psi_+^2), \end{aligned} \tag{2.14}$$

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$$\Phi_- = \text{Im}\psi_-^1 - e^{3C}\text{Vol}(S^3)\wedge\text{Re}\psi^1_{\hat{\gamma}+} + \frac{e^C}{2}(K_1\wedge\text{Im}\psi^2_{\hat{\gamma}_+} - K_2\wedge\text{Re}\psi^2_{\hat{\gamma}+} + K_3\wedge\text{Im}\psi^1_{\hat{\gamma}_+}),$$

$$+\frac{e^{2C}}{4}(K_1\wedge K_2\wedge\text{Re}\psi_-^1 - K_1\wedge K_3\wedge\text{Im}\psi_-^2 + K_2\wedge K_3\wedge\text{Re}\psi_-^2).$$

At this point, the IIA and IIB supersymmetry equations diverge, and we shall relegate their explicit form to the relevant sections.

With our set up, a solution to the supersymmetry equations is a solution to the equation of motion if and only if it satisfies the Bianchi identities [4] [38] [39]. These are given by $d_H F = dH = 0$ away from localised sources. By definition, a localised (magnetic) source manifests itself in the Bianchi identity of some field strength F as $dF = Q\delta^n(x)$ and hence in such cases F is discontinuous. Loosely speaking, a localised source corresponds physically to an extended object (such as a brane) located at a submanifold of the ten-dimensional spacetime $\mathcal{S}\subset M_{10}$ which is pointlike in some of the local coordinates. The standard approach to obtaining backgrounds, which we follow as well, is to first solve the supersymmetry equations by introducing local coordinates, and then afterwards determine the physically sensible range of these local coordinates by examining the obtained geometry and fluxes. The presence of localised sources is signified by discontinuities of not just the fluxes, but of the spacetime geometry as well, precisely at the location of the sources. Therefore, it is possible to obtain solutions with localised sources even when making use of the Bianchi identities with no sources: one examines possible discontinuities in the geometry and fluxes and determines whether or not such discontinuities are associated with localised sources or not by comparing them with the divergent behaviour of known extended objects.

Making use of the flux decomposition (2.1), (2.6), the Bianchi identities thus reduce to

$$\begin{aligned} d_{H_3}\big(e^{3A+3C}\star_4\lambda(G_\pm)\big) = d_{H_3}\big(e^{3C}\star_4\lambda(G_\mp)\big) &= 0\,,\\ d_{H_3}(G_\pm) = d_{H_3}\big(e^{3C}G_\mp\big) &= 0\,,\\ dH_3 &= 0\,. \end{aligned} \tag{2.15}$$

This is after imposing $H_0 = 0$, which turns out to be a requirement for every solution to the supersymmetry equations that we obtain.

2.1. Summary of obtained backgrounds

As the rest of the paper is somewhat technical, let us summarise our results here. We find a number of well-known backgrounds, as well as some new ones.

In type IIB with internal Killing spinors of equal norm, we find:

1. The intersecting D3–D7 system with metric (3.27), fluxes (3.25) and scalar field constraints (3.26).
2. The D5-brane with metric (3.37), fluxes (3.35) and scalar field constraints (3.36).
3. A generalization of the D5-brane generated by U-duality. The metric is given by (3.43), the fluxes by (3.41), scalar field constraints by (3.42).
4. A new background on the cone over $\mathbb{R}^{1,2}\times S^3\times S^3_{\text{sq}}$, with S^3_{sq} a generically squashed three-sphere admitting an $SU(2)\times U(1)$ isometry group. For the unsquashed limit, the metric is given by (3.59), the fluxes by (3.57), the scalar field constraints by (3.58). In the generic squashed case, the metric and dilaton are given by (3.75), the fluxes by (3.74). We note that the more general squashed case can be obtained from the unsquashed case by a duality chain.

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In type IIA with internal Killing spinors of equal norm, we find:

1. The intersecting D4–D8 system with metric (4.14), fluxes (4.12) and scalars constraints (4.11), (4.13).
2. The intersecting D2–D6 system with metric (4.24), fluxes (4.22) and scalar constraints (4.21), (4.23).
3. A generalization of the D4–D8 system generated by U-duality. The metric is given by (4.30), the fluxes by (4.29), and scalar constraints by (4.27).
4. A class of new backgrounds. The metric contains an $\mathbb{R}^{1,2}\times S^3\times S^2$ factor, with various warpings, and is given by (4.42). The warp factors are constrained by various PDE, given in (4.43). In general, all fluxes are turned on and are given by (4.44). This new class of backgrounds contains a subset with a $U(1)$ isometry. In this case, T-dualising along the isometry direction leads to the new IIB backgrounds outlined above, with generic squashing.

In addition, we find two more backgrounds when setting $\epsilon_2=0$. These backgrounds are pure NS, and as such, can be found in both type IIA and type IIB. We find:

1. The NS5-brane, with metric (6.18), flux (6.16) and the scalar constraints (6.17).
2. A pure NS background on $\mathbb{R}^{1,2}\times\mathbb{R}\times S^3\times S^3$, dual to the new (unsquashed) IIB background. The metric is given by (6.22), the flux by (6.21). All scalars are determined up to constant factors.

3. Mink$_3$ with an S^3 factor in IIB

The type IIB supersymmetry equations are obtained by plugging the decomposed seven-dimensional bispinors (2.14) into the seven-dimensional supersymmetry constraints (2.4). This leads to the following constraints on the four-dimensional bispinors

$$d_{H_3}(e^{2A-\Phi}\mathrm{Re}\psi^1_+)=0\,, \tag{3.1a}$$

$$d_{H_3}(e^{3A+2C-\Phi}\psi^2_-)+2i\nu e^{3A+C-\Phi}\psi^2_{\hat\gamma+}=0\,, \tag{3.1b}$$

$$d_{H_3}(e^{2A+2C-\Phi}\psi^2_+)+2i\nu e^{2A+C-\Phi}\psi^2_{\hat\gamma-}=0\,, \tag{3.1c}$$

$$d_{H_3}(e^{3A+2C-\Phi}\mathrm{Re}\psi^1_-)-2\nu e^{3A+C-\Phi}\mathrm{Im}\psi^1_{\hat\gamma+}=0\,, \tag{3.1d}$$

$$d_{H_3}(e^{2A+2C-\Phi}\mathrm{Im}\psi^1_+)+2\nu e^{2A+C-\Phi}\mathrm{Re}\psi^1_{\hat\gamma-}=0\,, \tag{3.1e}$$

$$d_{H_3}(e^{2A+3C-\Phi}\mathrm{Im}\psi^1_{\hat\gamma-})+e^{2A+3C-\Phi}H_0\mathrm{Re}\psi^1_+=0, \tag{3.1f}$$

while the fluxes are determined by

$$d_{H_3}(e^{3A-\Phi}\mathrm{Im}\psi^1_-)+e^{3A}\star_4\lambda(G_+)=0\,, \tag{3.2a}$$

$$d_{H_3}(e^{3A+3C-\Phi}\mathrm{Re}\psi^1_{\hat\gamma+})-e^{3A+3C-\Phi}H_0\mathrm{Im}\psi^1_-+\nu e^{3A+3C}\star_4\lambda(G_-)=0 \tag{3.2b}$$

and must additionally satisfy the pairing equation

$$\Big(\mathrm{Im}\psi^1_{\hat\gamma-}\wedge\lambda(G_-)-\mathrm{Re}\psi^1_+\wedge\lambda(G_+)\Big)\Big|_4=0\,. \tag{3.3}$$

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In order to solve these, we will first examine the 0-form conditions. These are given by

$$(\psi^2_{\hat{\gamma}})_0 = (\text{Im}\psi^1_{\hat{\gamma}})_0 = H_0(\text{Re}\psi^1_{\hat{\gamma}})_0 = 0\,. \tag{3.4}$$

We solve the first two of these in Appendix B, which leads to a spinor ansatz depending on 6 real functions with support on M_4

$$\alpha,\ \ a_1,\ \ b_1,\ \ \lambda_1,\ \ \lambda_2\ \ ,\lambda_3 \tag{3.5}$$

subject to the constraint

$$a_1^2 + b_1^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1. \tag{3.6}$$

The third 0-form constraint, which is unique to IIB, still needs to dealt with. After making use of (B.12), it reduces to

$$H_0(a_1 \cos\frac{\alpha}{2} + b_1 \sin\frac{\alpha}{2}) = 0\,. \tag{3.7}$$

Here, as well as in IIA, the solutions depend drastically on the behaviour of α. We can distinguish between three different cases: $\alpha = 0$, $\alpha = \frac{1}{2}\pi$, and generic $\alpha \in (0, \pi), \alpha \neq \frac{1}{2}\pi$. Let us reiterate that we introduced α in (B.5) by defining

$$\eta_1 = \cos\left(\frac{\alpha}{2}\right)\eta + \sin\left(\frac{\alpha}{2}\right)\hat{\gamma}\eta\,, \tag{3.8}$$

where η is a locally defined non-chiral spinor, where the chiral components are normalised. Note that the non-chirality is crucial: it ensures that η can be used to define the local trivial structure (i.e., the vielbein) via (B.3). In the case that $\alpha = 0$, the 4d internal Killing spinors $\eta_1 = \eta$ are such that the chiral components of η_1 have equal norm. In the case that $\alpha = \pi/2$, we see that η_1 becomes chiral. It turns out that we can treat this case together with $\alpha \neq 0$, but find no such solutions. Thus we separate our solutions into two branches.

Branch I: Here $\alpha = 0$. The only non-trivial zero form is $a_1 H_0 = 0$, which a priori can be solved in two ways. However, we shall see in the next section that only $H_0 = 0$ is consistent with the higher form conditions. In order to solve (B.6) we parametrise

$$a_1 = \sin\beta,\ \ b_1 = \cos\beta\sin\delta,\ \ \lambda_1 = y_1\cos\beta\cos\delta,\ \ \lambda_1 = y_2\cos\beta\cos\delta,\ \ \lambda_3 = y_3\cos\beta\cos\delta, \tag{3.9}$$

with

$$y_1 = \sin\theta\cos\phi,\quad y_2 = \sin\theta\sin\phi,\quad y_3 = \cos\theta. \tag{3.10}$$

Branch II: Here $0 < \alpha < \pi$. Note that $\alpha = \pi$ is equivalent to $\alpha = 0$, which is easiest to see by sending $\eta \to \hat{\gamma}\eta$ in (B.5). We choose to parametrise

$$\begin{aligned}
&a_1 = \cos\beta\sin(\delta - \frac{\alpha}{2})\,,\quad b_1 = \cos\beta\cos(\delta - \frac{\alpha}{2})\\
&\lambda_1 = -\cos\beta\cos\delta y_1\quad ,\quad \lambda_2 = -\sin\beta y_3\\
&\lambda_3 = -\sin\beta y_2
\end{aligned}$$

where y_i is defined in (3.10). This ensures (B.6) and all but the first equation of (3.7) are solved, which becomes

$$H_0\cos\beta\sin\delta = 0\,. \tag{3.12}$$

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3.1. Branch I: solutions with $\alpha = 0$

In order to solve branch I, it is convenient to first examine a number of lower form conditions that follow from (3.1). To do this it is useful to first rotate the canonical frame of (B.3) such that

$$\begin{aligned} v_1 &\to \sin\phi w_2 + \cos\phi(\cos\theta v_1 + \sin\theta(\cos\delta w_1 - \sin\delta v_2)) \\ v_2 &\to \cos\delta v_2 + \sin\delta w_1 \\ w_1 &\to \cos\theta(\cos\delta w_1 - \sin\delta v_2) - \sin\theta v_1 \\ w_2 &\to \cos\phi w_2 - \sin\phi(\cos\theta v_1 + \sin\theta(\cos\delta w_1 - \sin\delta v_2))\,. \end{aligned} \tag{3.13}$$

Making use of these, one finds that the supersymmetry equations imply

$$\sin\beta H_0 = 0\,, \tag{3.14a}$$

$$d(e^{2A+2C-\Phi}\cos\beta\cos\delta) - 2\nu e^{2A+C-\Phi}\cos\beta v_2 = 0\,, \tag{3.14b}$$

$$d(e^{2A+3C-\Phi}\sin\beta(\cos\delta v_2 + \sin\delta w_1)) - e^{2A+3C-\Phi}H_0\cos\beta\cos\delta v_1\wedge w_2 = 0, \tag{3.14c}$$

$$\cos\beta(e^C\cos\delta d\theta + 2\nu\sin\delta v_1) = \cos\beta(e^C\cos\delta\sin\theta d\phi - 2\nu\sin\delta w_2) = 0\,, \tag{3.14d}$$

$$\begin{aligned} &d(e^{3A+2C-\Phi}\sin\beta(\cos\delta w_1 - \sin\delta v_2)) + 2\nu e^{3A+C-\Phi}(\sin\beta v_2\wedge w_1 + \sin\delta\cos\beta v_1\wedge w_2) \\ &+ e^{3A+2C-\Phi}\cos\beta\cos\delta(d\theta\wedge w_2 + \sin\theta d\phi\wedge v_1) = 0, \end{aligned} \tag{3.14e}$$

which is not a compete list. The first thing to establish is how to solve (3.14a) – if we set $\sin\beta = 0$, one needs to set $\cos\delta = 0$ to solve (3.14c), but since $\nu = \pm 1$, (3.14b) leads to a contradiction.

The next conditions we consider are (3.14d). For $\cos\beta \neq 0$ we see that either $\sin\delta = d\theta = d\phi = 0$, or $0 < \sin\delta < \frac{\pi}{2}$ in which case (θ, ϕ) define local coordinates on a 2-sphere. We are ignoring $\cos\beta = 0$ because, as should be clear from, (3.9), this is a subcase of $\sin\delta = d\theta = d\phi = 0$. Let us now prove that $0 < \sin\delta < \frac{\pi}{2}$ is not possible: Since $H_0 = 0$ we can solve (3.14b)–(3.14c) by introducing local coordinates x and $\rho = e^{2A+2C-\Phi}\cos\beta\cos\delta$ such that

$$\nu v_2 = \frac{1}{2}\sqrt{\frac{\cos\delta}{\rho\cos\beta}}e^{-A+\frac{\Phi}{2}}d\rho,\ \ w_1 = \frac{\sqrt{\cos\beta}\cos^{3/2}\delta}{2\nu\rho^{3/2}\sin\delta}e^{A-\frac{\Phi}{2}}(2\nu\cot\beta dx - \sec\beta e^{-2A+\Phi}\rho d\rho).$$

We can also rewrite (3.14e) as

$$d(e^{3A+2C-\Phi}\sin\beta(\cos\delta w_1 - \sin\delta v_2)) + 2\nu e^{3A+C-\Phi}(\sin\beta v_2\wedge w_1 - \sin\delta\cos\beta v_1\wedge w_2) = 0,$$

using (3.14d). The key point here is that $v_2,\ w_1$ only have legs in $(\rho,\ x)$ while $v_1,\ w_2$ sit orthogonal to this with legs in $(\theta,\ \phi)$ only. This means that the equation above, cannot be solved as there is a $\text{Vol}(S^2)$ term whose coefficient is non-vanishing. Thus we can conclude in general that $\sin\delta = d\theta = d\phi = 0$. Plugging this back into (4.1), one finds that nothing depends on the specific values these parameters take so we can set

$$H_0 = \theta = \phi = \delta = 0\,, \tag{3.15}$$

without loss of generality, leaving one undetermined function β.

We are now ready to write the supersymmetry conditions that follow when $\alpha = 0$, however we find it helpful to perform a second rotation of the canonical vielbein by considering

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$$v_1 + i w_2 \to e^{-i\beta} w, \quad v_2 - i w_1 \to -i v, \tag{3.16}$$

to ease presentation. The necessary and sufficient conditions for supersymmetry in the $\alpha = 0$ branch are

$$\begin{aligned}
&H_0 = d(e^{2A-\Phi}\sin\beta) = d(e^{A+C-\frac{1}{2}\Phi}\sqrt{\cos\beta}) - \nu e^{A-\frac{\Phi}{2}}\sqrt{\cos\beta}\, v_2 \\
&\quad = d(e^{2A+3C-\Phi}\sin\beta\, v_2) = 0\,, \\
&d(e^{3A+2C-\Phi} w) + 2\nu e^{3A+C-\Phi} w\wedge v_2 \\
&\quad = d(e^{3A+2C-\Phi}\sin\beta\, v_1) + 2\nu e^{3A+C-\Phi}\sin\beta\, v_1\wedge v_2 = 0\,, \\
&d(e^{2A+2C-\Phi} v_1\wedge w) - 2\nu e^{2A+C-\Phi} v_1\wedge w\wedge v_2 = 0\,, \\
&d(e^{2A+2C-\Phi}\sin\beta\, w_1\wedge w_2) - 2\nu e^{2A+C-\Phi}\sin\beta\, w_1\wedge w_2\wedge v_2 + e^{2A+2C-\Phi}\cos\beta\, H_3 = 0\,, \\
&H_3 + 2\beta\wedge w_1\wedge w_2 = d(e^{4A-\Phi}\cos\beta)\wedge w_1\wedge w_2\wedge v_2 = 0\,, \\
&d(e^{3A-\Phi}\cos\beta\, v_1) - d(e^{3A-\Phi}\sin\beta\, v_1\wedge w_1\wedge w_2) + e^{3A-\Phi}\cos\beta\, v_1\wedge H_3 - e^{3A}\star_4\lambda(G_+) \\
&\quad = 0\,, \\
&d(e^{3A+3C-\Phi}\cos\beta\, v_1\wedge v_2) - e^{3C+3A}\star_4\lambda(G_-) = 0\,, \\
&\Big((\sin\beta + \cos\beta\, w_1\wedge w_2)\wedge v_2\wedge\lambda(G_-) - (\sin\beta + \cos\beta\, w_1\wedge w_2)\wedge\lambda(G_+)\Big)\Big|_4 = 0.
\end{aligned} \tag{3.17}$$

We can simplify this system further, but not without making assumptions about β. We now proceed to study the systems that follow from different values of β, we find that the physical interpretation is quite different in each case.

3.1.1. Subcase: $\beta = 0$

Upon setting $\beta = 0$ in (3.17) one can show that the supersymmetry conditions reduce to

$$H_3 = H_0 = d(e^{-\Phi})\wedge w_1\wedge w_2 = d(e^{-4A})\wedge v_2\wedge w_1\wedge w_2 = 0\,, \tag{3.18a}$$

$$d(e^{-A} v_1)\wedge w = d(e^{A+C-\frac{1}{2}\Phi}) - \nu e^{A-\frac{1}{2}\Phi} v_2 = d(e^{A} w) = 0\,, \tag{3.18b}$$

$$e^{3A}\star_4\lambda(G_+) = d(e^{3A-\Phi} v_1), \quad e^{3A+3C}\star_4\lambda(G_-) = d(e^{3A+3C-\Phi} v_1\wedge v_2), \tag{3.18c}$$

$$\lambda(G_-)\wedge v_2\wedge w_1\wedge w_2 - \lambda(G_+)\wedge w_1\wedge w_2 = 0. \tag{3.18d}$$

We can solve (3.18b) by using it to define a vielbein in terms of local coordinates ψ, x_1, x_2 and

$$\rho = e^{A+C-\frac{1}{2}\Phi} \tag{3.19}$$

such that

$$\begin{aligned}
&v_1 = e^{A}(d\psi + V)\,, \quad v_2 = \nu e^{-A+\frac{1}{2}\Phi} d\rho\,, \quad w = e^{-A}(dx_1 + i dx_2)\,, \\
&V = f_1(x_1, x_2)dx_1 + f_2(x_1, x_2)dx_2\,.
\end{aligned} \tag{3.20}$$

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From (3.18a) we see there is no NSNS flux, ∂_ψ is an isometry and $A = A(\rho, x_1, x_2)$, $\Phi = \Phi(x_1, x_2)$. We now have enough information to calculate the fluxes. First we find

$$\begin{aligned}
\star_4 G_- &= -e^{A}\left(\partial_{x_1}(e^{-\Phi})w_1 + \partial_{x_2}(e^{-\Phi})w_2\right)\wedge v_1 \wedge v_2 - e^{3A-\Phi}\left(\partial_{x_1} f_2 - \partial_{x_2} f_1\right) v_1 \\
\star_4 G_+ &= -e^{3A}\left(\partial_{x_1}(e^{4A-\Phi})w_1 \wedge v_2 + \partial_{x_2}(e^{4A-\Phi})w_2 \wedge v_2 + \nu e^{-\frac{1}{2}\Phi}\partial_\rho e^{4A-\Phi} v_2 \wedge v_1\right) \\
&\quad - e^{3A-\Phi}\left(\partial_1 f_2 - \partial_2 f_1\right) w_1 \wedge w_2 \,.
\end{aligned} \tag{3.21}$$

We can then use coordinate dependence of the physical fields and local expression for the vielbein (3.20) to take the Hodge dual in (3.18c) arriving at

$$G_- = (\partial_{x_2} f_1 - \partial_{x_1} f_2)e^{4A-\Phi}(d\psi + V) + \partial_{x_2}(e^{-\Phi})dx_1 - \partial_{x_1}(e^{-\Phi})dx_2, \tag{3.22}$$

$$\begin{aligned}
G_+ = -\nu e^{3A-\frac{3}{2}\Phi}\Big(&\partial_{x_2}(e^{-4A+\Phi})dx_1 \wedge d\rho - \partial_{x_1}(e^{-4A+\Phi})dx_2 \wedge d\rho \\
&- e^{-\Phi}\partial_\rho(e^{-4A+\Phi})dx_1 \wedge dx_2 - \nu e^{\Phi}(\partial_{x_2} f_1 - \partial_{x_1} f_2)(d\psi + V)\wedge d\rho\Big).
\end{aligned} \tag{3.23}$$

Plugging this into (3.18d) we find $(\partial_{x_2} f_1 - \partial_{x_1} f_2) = 0$ which mean that V is closed and so we can locally fix

$$V = 0\,, \tag{3.24}$$

with a shift $\psi \to \psi - \eta$ for $d\eta = V$, without loss of generality. Taking this into account the ten-dimensional fluxes are

$$\begin{aligned}
F_1 &= \partial_{x_2}(e^{-\Phi})dx_1 - \partial_{x_1}(e^{-\Phi})dx_2, \quad F_5 = d\psi \wedge d(e^{4A-\Phi}) \wedge \text{Vol}_3 \\
&\quad - \nu\rho^3\left(\partial_{x_2}(e^{-4A+\Phi})dx_1 \wedge d\rho - \partial_{x_1}(e^{-4A+\Phi})dx_2 \wedge d\rho - e^{-\Phi}\partial_\rho(e^{-4A+\Phi})dx_1 \wedge dx_2\right) \\
&\quad \wedge \text{Vol}(S^3).
\end{aligned} \tag{3.25}$$

The final thing we need to do is impose the Bianchi identities, which away from localised sources rise to the PDEs

$$(\partial_{x_1}^2 + \partial_{x_2}^2)e^{-\Phi} = 0\,, \quad \frac{e^{-\Phi}}{\rho^3}\partial_\rho(\rho^3\partial_\rho e^{-4A+\Phi}) + (\partial_{x_1}^2 + \partial_{x_2}^2)(e^{-4A+\Phi}) = 0\,. \tag{3.26}$$

The local form of the metric is then

$$\begin{aligned}
ds^2 &= \frac{1}{\sqrt{fH}}ds^2(\mathbb{R}^{1,3}) + \sqrt{\frac{H}{f}}\left(d\rho^2 + \rho^2 ds^2(S^3)\right) + \sqrt{fH}\left(dx_1^2 + dx_2^2\right)\,, \\
H &= e^{-4A+\Phi}\,, \quad f = e^{-\Phi}
\end{aligned} \tag{3.27}$$

This corresponds to the intersecting D3–D7 brane system, where the D3-branes are embedded in the D7-branes [40].

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3.1.2. Subcase: $\beta = \frac{\pi}{2}$

Setting $\beta = \frac{\pi}{2}$ in (3.17) leads to the following necessary and sufficient conditions for unbroken supersymmetry

$$d(e^{2A-\Phi}) = d(e^{2A+3C-\Phi} v_2) = H_3 = H_0 = G_- = 0\,, \tag{3.28a}$$

$$d(e^{3A+2C-\Phi} u_i) + 2\nu e^{3A+C-\Phi} u_i \wedge v_2 = 0\,, \tag{3.28b}$$

$$d(e^{2A+2C-\Phi} \epsilon_{ijk} u_j \wedge u_k) - 2\nu e^{2A+C-\Phi} \epsilon_{ijk} u_j \wedge u_k \wedge v_2 = 0\,, \tag{3.28c}$$

$$e^{3A} \star_4 \lambda(G_+) = d(e^{3A-\Phi} v_1 \wedge w_1 \wedge w_2). \tag{3.28d}$$

Here we have introduced the notation

$$u = (v_1, w_1, w_2), \tag{3.29}$$

both to ease notation and to stress that the vielbeine u_i obey a cyclic property. Exploiting this property will be very helpful in solving this system and other systems we shall encounter which mirror this behaviour, so we will be very explicit in our derivation here, but less so elsewhere. The first thing to note is that the combination $(3.28\text{c})_i + e^{-A}\epsilon_{ijk}u_j \wedge (3.28\text{b})_k$ leads to

$$\epsilon_{ijk}\Big(d(e^{2A+C-\frac{1}{2}\Phi}) - \nu e^{2A-\frac{1}{2}\Phi} v_2\Big) \wedge u_j \wedge u_k = 0\,. \tag{3.30}$$

This implies that the 1-form in large brackets is zero. This can be seen by writing it as $X_j u^j + X_0 v_2$ for some functions X_j, noting that the vielbeine $u_{1,2,3}$ are independent, and then considering the resulting constraints for $i = 1, 2, 3$. Next by examining $(3.28\text{c})_i + e^{-A} u_j \wedge (3.28\text{b})_k$ and $(3.28\text{c})_i - e^{-A} \wedge (3.28\text{b})_j \wedge u_k$ for cyclic permutations of $(i, j, k) = (1, 2, 3)$ one realises that

$$d(e^{-A} u_i) \wedge u_j = 0\,, \quad i \neq j, \tag{3.31}$$

which implies that $d(e^{-A}u_i)$ has no leg in u_i, it is then not hard to see that since (3.28b) has no $\epsilon_{ijk} u_j \wedge u_k$ term it is in fact zero. So we can conclude without loss of generality that

$$d(e^{-A} u_i) = d(e^{2A+C-\frac{1}{2}\Phi}) - \nu e^{2A-\frac{1}{2}\Phi} v_2 = 0\,, \tag{3.32}$$

which imply (3.28b)–(3.28c) without further constants. We can then solve these conditions by using them to define the vielbeine in terms of local coordinates as

$$v_1 = e^A dx_1\,, \quad w_1 = e^A dx_2\,, \quad w_2 = e^A dx_3\,, \quad v_2 = \nu e^{-2A+\frac{1}{2}\Phi} d\rho\,, \quad \rho = e^{2A+C-\frac{1}{2}\Phi}\,. \tag{3.33}$$

We can now solve (3.28a), which in fact just tells us that

$$e^{\Phi} = e^{2A}, \tag{3.34}$$

up to rescaling g_s and that e^{2A} is a function of ρ only, making ∂_{x_i} all isometry directions parameterising either $\mathbb{R}^3$ or T^3 locally.

The only non-trivial flux is the RR 3-form

$$F_3 = -\nu \rho^3 \partial_\rho (e^{-4A}) \text{Vol}(S^3) \tag{3.35}$$

and its Bianchi identity, $dF_3 = 0$, imposes that

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$$e^{-4A} = c_1 + \frac{c_2}{\rho^2}, \quad dc_i = 0 \,. \tag{3.36}$$

This is the warp factor of a D5-brane or O5-hole, depending on the sign of c_2 (see for example [41]). Indeed the metric locally takes the form

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,5}) + e^{-2A}\Big(d\rho^2 + \rho^2 ds^2(S^3)\Big) \tag{3.37}$$

As we will see, this is a subcase of the solution in the next section.

3.1.3. Subcase: generic β

For generic $0 < \beta < \frac{\pi}{2}$ we are free to divide by the trigonometric functions in (3.17). Using $\sin\beta \neq 0$ it is possible to show that supersymmetry requires

$$d(e^{2A-\Phi}\sin\beta) = d(e^{-\Phi}\cos\beta) = de^A \wedge v_2 = 0, \tag{3.38a}$$

$$d(e^{-A}v_1) = d(e^{2A+C-\frac{\Phi}{2}}\sqrt{\sin\beta}) - \nu e^{2A-\frac{\Phi}{2}}\sqrt{\sin\beta}\, v_2 = d(e^{-A} w \csc\beta) = 0 \,, \tag{3.38b}$$

$$H_3 + 2d\beta \wedge w_1 \wedge w_2 = d\beta \wedge v_2 \wedge w_1 \wedge w_2 = 0 \,, \tag{3.38c}$$

$$e^{3A} \star_4 \lambda(G_+) = d(e^{3A-\Phi}\cos\beta v_1) + d(e^{3A-\Phi}\sin\beta v_1 \wedge w_1 \wedge w_2) - e^{3A-\Phi}\cos\beta H_3 \wedge v_1,$$

$$e^{3A+3C} \star_4 \lambda(G_-) = d(e^{3A+3C-\Phi}\cos\beta v_1 \wedge v_2), \tag{3.38d}$$

$$\lambda(G_-) \wedge v_2 \wedge (\sin\alpha + \cos\alpha w_1 \wedge w_2) - \lambda(G_+) \wedge (\sin\alpha + \cos\alpha w_1 \wedge w_2) = 0 \,, \tag{3.38e}$$

by following the same line of reasoning as in the previous subsection. First we solve (3.38b) by using it to define the vielbeine on M_4 locally

$$v_1 = e^A dx_1, \quad w = e^A \sin\beta(dx_2 + i dx_3), \quad v_2 = \nu e^{-A} d\rho, \quad \rho = e^{A+C} \,, \tag{3.39}$$

where we have used the first of (3.38a) to simplify these somewhat. Next (3.38a) is solved when

$$e^\Phi = e^{2A}\sin\beta, \quad \cot\beta = c e^{2A}, \quad dc = 0 \,, \tag{3.40}$$

with $A = A(\rho)$, $\beta = \beta(\rho)$. As a result, ∂_{x_i} are isometries. We then use (3.39) to take the Hodge dual of (3.38d), (3.38e) arriving at the fluxes

$$F_3 = -\nu\rho^3\partial_\rho(e^{-4A})\mathrm{Vol}(S^3), \quad H = 2e^{2A}\partial_\rho\beta\sin^2\beta d\rho \wedge dx_2 \wedge dx_3, \tag{3.41}$$

$$F_5 = \mathrm{Vol}_3 \wedge dx_1 \wedge d(e^{2A}\cot\beta) + \nu e^{-2A}\rho^3\big(\sin(2\beta)\partial_\rho A - \partial_\rho\beta\big) dx_2 \wedge dx_3 \wedge \mathrm{Vol}(S^3),$$

which solve (3.38e) without restriction. The Bianchi identities impose that

$$e^{-4A} = c_1 + \frac{c_2}{\rho^2}\,, \quad dc_i = 0 \,, \tag{3.42}$$

which is again the warp factor of a D5-brane or O5-hole. However, in this case the metric takes the local form

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,3}) + e^{2A}\sin\beta^2 ds^2(T^2) + e^{-2A}\Big(d\rho^2 + \rho^2 ds^2(S^3)\Big) \tag{3.43}$$

where T^2 is spanned by (x_2, x_3). This generalises the solution in the previous section by introducing an additional warping factor for a T^2 submanifold, thus breaking $SO(1,5)$ Lorentz

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symmetry and leading to more general fluxes. In fact, this solution can be generated from the D5-brane solution of the previous section via "G-structure rotation" [42] which is formally a U-duality [43].

3.2. Branch II: α non-zero solutions

For the second branch with $0 < \alpha < \pi$, we begin by studying the lower form conditions that follow from (3.1). Here we find it useful to rotate the canonical frame of (B.3) as

$$v \to -\sin w_1 + \cos\phi(\cos\theta v_1 - \sin\theta w_2) + i v_2, \tag{3.44}$$

$$w \to \cos\phi w_1 + \sin\phi(\cos\theta v_1 - \sin\theta w_2) + i(\cos\theta w_2 + \sin\theta v_1). \tag{3.45}$$

We then find the following necessary, but not sufficient, conditions for supersymmetry

$$\cos\beta \sin\delta H_0 = 0\ , \tag{3.46a}$$

$$e^C \cos\alpha \sin\beta d\theta - 2\nu(\cos\beta\cos\delta v_1 + \sin\beta\sin\alpha w_1) = 0\ , \tag{3.46b}$$

$$e^C \cos\alpha \sin\beta \sin\theta d\phi - 2\nu(\cos\beta\cos\delta w_1 - \sin\alpha\sin\beta v_1) = 0\ , \tag{3.46c}$$

$$d(e^{2A+2C-\Phi}\cos\alpha\sin\beta) - 2\nu e^{2A+C-\Phi}(\cos\beta\cos\delta w_2 - \sin\beta v_2), \tag{3.46d}$$

$$d(e^{2A+3C-\Phi}(\sin\alpha\sin\beta w_2 + \sin(\alpha-\delta)\cos\beta v_2)) + e^{2A+3C-\Phi}H_0\sin\beta\cos\alpha v_1\wedge w_1 = 0, \tag{3.46e}$$

$$\begin{aligned}
&d(e^{3A+2C-\Phi}(\sin\beta\sin\alpha v_2 - \cos\beta\sin(\alpha-\delta)w_2))\\
&+2\nu e^{3A+C-\Phi}\cos\beta(\sin\delta w_2\wedge v_2 + \cos(\alpha-\delta)v_1\wedge w_1)\\
&-e^{3A+2C-\Phi}\big(d\theta\wedge(\cos\beta\sin(\alpha-\delta)v_1 + \sin\beta w_1)\\
&+\sin\theta d\phi\wedge(\cos\beta\sin(\alpha-\delta)w_1 - \sin\beta v_1)\big) = 0\ .
\end{aligned} \tag{3.46f}$$

First we note that if either θ or ϕ become constant or if $\cos\alpha = 0$ then (3.46b), (3.46c) require that $\sin\beta = \cos\delta = 0$ which makes $(\theta,\ \phi)$ drop out of (3.11) entirely and the final line of (3.46f) vanishes (setting $\sin\theta = 0$ leads to the same conclusion). In this case we can conclude that we can set

$$H_0 = \theta = \phi = (\delta - \frac{\pi}{2}) = 0 \tag{3.47}$$

without loss of generality, which we study in section 3.2.1.

If we assume $\sin\theta$ and $\cos\alpha$ don't vanish, then (θ, ϕ) are local coordinates on a 2-sphere and we can take $\rho = e^{2A+2C-\Phi}\cos\alpha\sin\beta$ as a local coordinate. We can then use (3.46b)–(3.46d) to rewrite (3.46f) as

$$\begin{aligned}
&d(e^{3A+2C-\Phi}(\sin\beta\sin\alpha v_2 - \cos\beta\sin(\alpha-\delta)w_2))\\
&+2\nu e^{3A+C-\Phi}\cos\beta(\sin\delta w_2\wedge v_2 - \cos(\alpha-\delta)v_1\wedge w_1) = 0,
\end{aligned} \tag{3.48}$$

which we can then use to fix some of the free functions. First we note that if we solve (3.46a) with $\sin\delta = 0$, then (3.48) fixes $\cos\beta = 0$ – this is because $\sin\beta\sin\alpha v_2 - \cos\beta\sin\alpha w_2$ is parallel to $d\rho$ in this limit and so cannot generate the $\text{Vol}(S^2)$ factor that comes from $v_1 \wedge w_1$. Next, for $H_0 =$

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0 and for generic values of (α, β, δ) we can use (3.46b)–(3.46e) to locally define the vielbein on M_4 by introducing another local coordinate such that $dx = e^{2A+3C-\Phi}(\sin\alpha \sin\beta w_2 + \sin\alpha - \delta \cos\beta v_2)$, but then we must once more set the $v_1 \wedge w_1$ term in (3.48) to zero which fixes either $\cos\beta = 0$ or $\cos(\alpha - \delta) = 0$. Thus, for $H_0 = 0$ and a priori generic $(\theta, \phi, \alpha, \beta, \delta)$, we end up with just two cases. Firstly $\cos\beta = 0$, which solves (3.46a) and makes the δ dependence of (3.11) drop out such that we can set without loss of generality

$$\delta = 0\,, \quad \beta = \frac{\pi}{2}\,. \tag{3.49}$$

We shall examine this case in detail in section 3.2.2 where we find that it contains no solution. Secondly $\cos(\alpha - \delta) = 0$, such that we can set without loss of generality

$$\delta = \alpha + \frac{\pi}{2}\,, \tag{3.50}$$

which we shall study in section 3.2.3, finding a new class of solution.

There is one final option one can consider for $H_0 = 0$, by taking both $\cos\alpha$ and $\sin\theta$ non-vanishing – one can tune the values of (α, δ, β) such that $(\sin\alpha \sin\beta w_2 + \sin(\alpha - \delta)\cos\beta v_2)$ becomes parallel to $d\rho$ and so can no longer be used to introduce a local coordinate. This requires fixing

$$\tan\beta = \sqrt{\frac{\cos\delta \sin(\delta - \alpha)}{\sin\alpha}}. \tag{3.51}$$

It will then be (3.48) that will be used to define the final vielbein direction, which will necessarily be fibred over S^2. We shall examine this possibility in section 3.2.4.

3.2.1. Subcase $\beta = 0$

For $\beta = 0$ the supersymmetry conditions reduce to

$$d(e^{2A-\Phi}) = d(e^{2A+3C-\Phi}\cos\alpha v_2) = H_3 = H_0 = 0\,, \tag{3.52a}$$

$$d(e^{3A+2C-\Phi}\cos\alpha u_i) + 2\nu e^{3A+C-\Phi}(u_i \wedge v_2 + \frac{1}{2}\sin\alpha \epsilon_{ijk} u_j \wedge u_k) = 0\,, \tag{3.52b}$$

$$d(e^{2A+2C-\Phi}(\sin\alpha u_i \wedge v_2 + \frac{1}{2}\epsilon_{ijk} u_j \wedge u_k)) - \nu e^{2A+C-\Phi}\cos\alpha \epsilon_{ijk} u_j \wedge u_k \wedge v_2 = 0\,, \tag{3.52c}$$

$$e^{3A} \star_4 \lambda(G_+) = d(e^{3A-\Phi}\cos\alpha v_1 \wedge w_1 \wedge w_2), \quad e^{3A+3C} \star_4 \lambda(G_-) = -d(e^{3A+3C-\Phi}\sin\alpha), \tag{3.52d}$$

$$(\cos\alpha v_2 \wedge \lambda(G_-) + \sin\alpha v_1 \wedge v_2 \wedge w_1 \wedge w_2 \lambda(G_+))\big|_4 = 0, \tag{3.52e}$$

where as usual $u = (v_1, w_1, w_2)$. Note that this system reduces to that of section 3.1.2 when $\sin\alpha = 0$, and that (3.52b) imposes that $\cos\alpha \neq 0$, so we can take $0 < 2\alpha < \pi$. By adding linear combinations of wedge products of (3.52a), (3.52b) and the vielbein to (3.52b), it is then possible to derive enough independent 2-form conditions to establish that

$$d(e^{A-C}\sin\alpha) \wedge v_2 = d\alpha \wedge v_2 = d(e^{A+C}\cos\alpha) - \nu e^A v_2 = d(e^{-A}\sec\alpha u_i) \wedge u_j = 0, \;\; i \neq j. \tag{3.53}$$

This is sufficient to establish that e^{2A}, e^{2C} and α are functions of a single local coordinate ρ, which v_2 is parallel to – specifically

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$$\rho = e^{A+C}\cos\alpha\,, \quad v_2 = \nu e^{-A} d\rho\,. \tag{3.54}$$

The final condition in (3.53) implies that $d(e^{-A}\sec\alpha u_1) \propto u_2 \wedge u_3$ and cyclic permutations, however plugging this into (3.52b) and (3.52b) we realise we can without loss of generality take

$$u_i = \frac{c_1}{2} e^A \cos\alpha \tilde{K}_i\,, \quad \tilde{K}_i + \frac{\nu}{2}\epsilon_{ijk}\tilde{K}^j \wedge \tilde{K}^k\,, \quad c_1 = \frac{e^{-A+C}}{\sin\alpha}\,, \quad dc_1 = 0\,, \tag{3.55}$$

where $\tilde{K}^i$ are necessarily $SU(2)$ invariant forms which furnish a frame for a round S^3. We have now without loss of generality determined the vielbein on M_4, which is a foliation of S^3 over an interval, and (3.52a)–(3.52c) are solved when

$$e^{-2A} = \frac{c_1 \sin\alpha\cos\alpha}{\rho}\,, \quad e^{2C} = e^{2A} c_1^2 \sin^2\alpha\,, \quad e^{2A-\Phi} = c_2\,, \quad dc_i = 0\,. \tag{3.56}$$

The only non-trivial 10d flux can be extracted from (3.52d) and is given by

$$F_3 = 2c_1^2 c_2 \nu\big(\sin^2\alpha - \rho\tan\alpha\,\partial_\rho\alpha\big)\mathrm{Vol}(S^3) + 2c_1^2 c_2\nu\big(\cos^2\alpha + \rho\cot\alpha\,\partial_\rho\alpha\big)\mathrm{Vol}(\tilde{S}^3)\,. \tag{3.57}$$

The pairing equation (3.52e) is equivalent to the Bianchi identity at this point; either one implies

$$d\alpha = 0\,. \tag{3.58}$$

The metric is of the form

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,2}) + e^{-2A}\left[d\rho^2 + \frac{\rho^2}{\cos^2\alpha} ds^2(S^3) + \frac{\rho^2}{\sin^2\alpha} ds^2(S^3)\right] \tag{3.59}$$

This solution has both an $SO(4)$ R-symmetry and $SO(4)$ flavour symmetry, and is S-dual to the one that we find in section 6, as will be explained in that section.

3.2.2. Subcase $\beta = \frac{\pi}{2}$

Here one can show that supersymmetry implies

$$d(e^{2A+2C-\Phi}\cos\alpha) - 2\nu e^{2A+C-\Phi} v_2 = d(e^C \cot\alpha w_2) = 0\,, \tag{3.60a}$$

$$e^C d\theta - 2\nu\tan\alpha w_1 = e^C \sin\theta d\phi + 2\nu\tan\alpha v_1 = 0\,, \tag{3.60b}$$

$$d(e^{-A-C}\sin\alpha) = d(e^{\Phi}\sin\alpha\tan\alpha)\wedge v_2 = d(e^{-A-C+\Phi}\tan\alpha)\wedge w_2 \wedge v_2 = 0\,, \tag{3.60c}$$

$$d(e^{A+C-\frac{\Phi}{2}}\tan\alpha\sqrt{\cos\alpha})\wedge w_2 + \frac{1}{2}\sqrt{\cos\alpha}\,e^{A+C-\frac{\Phi}{2}} H_0 v_1 \wedge w_1 = 0\,, \tag{3.60d}$$

$$d(e^{2A+2C-\Phi}\cos\alpha\cot^2\alpha)\wedge\mathrm{Vol}(S^2) = 0\,, \tag{3.60e}$$

$$H_3 + \frac{\nu}{2} e^C \cot\alpha w_2 \wedge \mathrm{Vol}(S^2) = d(e^{-2C}\tan\alpha)\wedge v_2 \wedge \mathrm{Vol}(S^2) = 0\,, \tag{3.60f}$$

$$e^{3A} \star_4 \lambda(G_+) - d(e^{3A-\Phi} w_2) + d(e^{3A-\Phi}\sin\alpha v_1 \wedge v_2 \wedge w_1) = 0\,, \tag{3.60g}$$

$$e^{3A+3C} \star_4 \lambda(G_-) + e^{3A+3C-\Phi} H_0 w_2 + d(e^{3A+3C-\Phi}\cos\alpha v_2 \wedge w_2) = 0\,, \tag{3.60h}$$

$$\Big((\sin\alpha w_2 - v_1\wedge v_2 \wedge w_1)\wedge\lambda(G_-) - \cos\alpha v_1 \wedge v_2 \wedge \lambda(G_+)\Big)\Big|_4 = 0\,. \tag{3.60i}$$

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There is no solution to this set of constraints, as we will now show. First, we note that $H_0=0$ is imposed by (3.60d). Let $\rho=e^{2A+2C-\Phi}\cos\alpha$. Due to (3.60a), we have $w_2\sim dx$, $v_2\sim d\rho$. It is then possible to rewrite (3.60d), (3.60e) as

$$d\left(\sqrt{\rho}\tan\alpha\right)\wedge dx=0\,,\quad d\left(\rho\cot^2\alpha\right)\wedge d\theta\wedge d\phi=0\,. \tag{3.61}$$

The first equation implies $\tan\alpha=\rho^{-1/2}f(x)$, which is incompatible with the second equation – thus this putative class contains no solutions.

3.2.3. Subcase $\delta=\alpha+\frac{\pi}{2}$, $H_0=0$

As explained below (3.46f), here we necessarily have $\beta>0$ and $0<\alpha<\frac{\pi}{2}$ – for this reason it will turn out the case contains no solutions. As the proof is similar to that of the previous section we shall be brief, this time only quoting sufficient supersymmetry conditions to prove this. In addition to the rotation of (B.3) we find it useful to send $v_1+iw_1\to e^{-i\beta}(v_1+iw_1)$, then a set necessary (but insufficient) conditions for supersymmetry are

$$d(e^{2A-\Phi}\cos\alpha\cos\beta)=0, \tag{3.62a}$$

$$d(e^{C}\cot\alpha w_2)=0, \tag{3.62b}$$

$$(v_1+iw_1)\tan\alpha+\frac{e^{C}\sin\beta}{2\nu}(d\theta+i\sin\theta d\phi),\quad d(e^{2A+3C-\Phi}(\sin\alpha\sin\beta w_2-\cos\beta v_2))=0, \tag{3.62c}$$

$$d(e^{2A+2C-\Phi}\cos\alpha\sin\beta)-2\nu e^{2A+C-\Phi}(\cos\beta\sin\alpha w_2+\sin\beta v_2)=0, \tag{3.62d}$$

$$e^{2A-\Phi}\cos\alpha\cos\beta H_3+d(e^{2A-\Phi}\cos\alpha\sin\beta v_1\wedge w_1)=0, \tag{3.62e}$$

$$\begin{aligned}&e^{2A+3C-\Phi}(\sin\alpha\sin\beta w_2-\cos\beta v_2)\wedge H_3\\&+d(e^{2A+3C-\Phi}v_1\wedge w_1\wedge(\sin\beta v_2-\cos\beta\sin\alpha w_2))=0.\end{aligned} \tag{3.62f}$$

As elsewhere we can take (3.62c)–(3.62d) as a local definition of the vielbein without loss of generality – v_1, w_1 are clearly the local vielbeine of a round S^2 in terms of the local coordinates $(\theta,\ \phi)$. For $v_2,\ w_2$ we introduce local coordinates x and $\rho=e^{A+C-\frac{1}{2}\Phi}\sqrt{\cos\alpha\sin\beta}$ such that

$$e^{2A+3C-\Phi}(\sin\alpha\sin\beta w_2-\cos\beta v_2)=dx. \tag{3.63}$$

We can then use (3.62e) to define H_3 without loss of generality, which leaves (3.62a), (3.62b) and (3.62f) to solve – this turns out to be impossible. To see this, one needs to consider the combination 4(3.62f)$+\big(f_1$(3.62a)$\wedge d\rho+f_2$(3.62a)$\big)\wedge\text{Vol}(S^2)$. When one tunes

$$f_1=e^{-A+5C+\frac{\Phi}{2}}\nu\frac{\cos^{\frac{3}{2}}\alpha\sin^{\frac{5}{2}}\beta}{\sin^2\alpha\cos\beta},\quad f_2=e^{2A+4C-\Phi}\cos\alpha\sin\beta\tan\beta, \tag{3.64}$$

this leads to

$$\cot\alpha\csc\alpha\sin\beta\tan\beta dx\wedge d\rho\wedge\text{Vol}(S^2)=0 \tag{3.65}$$

which cannot be solved without violating the initial assumptions that lead to this case. We conclude that there exist no solutions.

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3.2.4. Subcase: special value of β, $H_0 = 0$

The final case in IIB requires us to tune $\tan\beta$ to a specific value. After redefining $\delta \to \delta + \alpha$, this value is

$$\tan\beta = \sqrt{\frac{\cos(\alpha+\delta)\sin\delta}{\sin\alpha}}\,. \tag{3.66}$$

In addition, we rotate the vielbein (with respect to (3.44)) as

$$\begin{aligned}
v_1 + i w_1 &\to -i\frac{\cos\beta\cos(\alpha+\delta) + i\sin\alpha\sin\beta}{\sqrt{\cos^2\beta\cos^2(\alpha+\delta) + \sin^2\alpha\sin^2\beta}}(v_1 + i w_1),\\
v_2 + i w_2 &\to -i\frac{\sin\alpha\sin\beta + i\cos\beta\sin\delta}{\sqrt{\sin^2\alpha\sin^2\beta + \cos^2\beta\sin^2\delta}}(v_2 + i w_2)
\end{aligned} \tag{3.67}$$

In what follows we assume that the undefined functions of the spinor ansatz are bounded as $0 < 2\alpha < \pi$ and $0 < \delta + \alpha < \frac{\pi}{2}$, as the upper and lower limits have been dealt with in the proceeding sections. It is then possible to show that the necessary and sufficient conditions for supersymmetry for this case are

$$d(e^{A-C}\sin\alpha) = d\left(e^{2A-\Phi}\sqrt{\frac{\sin\alpha\sin(\alpha+\delta)}{\cos\delta}}\right) = d\left(e^{-\Phi}\sqrt{\frac{\sin 2(\alpha+\delta)}{\sin 2\delta}}\right) = 0, \tag{3.68a}$$

$$2ve^{-C}\sqrt{\tan\alpha\cot\delta}(w_1 - i v_1) - (d\theta + i\sin\theta d\phi) = 0, \tag{3.68b}$$

$$2vd\left(e^{-C}\tan\alpha\sqrt{\frac{\cos\alpha\sin(\alpha+\delta)}{\sin\delta}}w_2\right) - \text{Vol}(S^2) = d\left(e^{A+C}\sqrt{\frac{\cos\alpha\sin\delta}{\sin(\alpha+\delta)}}\right) - ve^A v_2 = 0, \tag{3.68c}$$

$$H_3 + \frac{1}{4}d\left(e^{2C}\frac{\cos^2\alpha\sin\delta}{\sin^2\alpha\sin(\alpha+\delta)\cos\delta}\sqrt{\cos(\alpha+\delta)\sin\alpha\sin\delta}\right)\wedge \text{Vol}(S^2) = d\alpha\wedge v_2 = 0 \tag{3.68d}$$

$$\begin{aligned}
&e^{3A}\star_4\lambda(G_+) - d_{H_3}\left(e^{3A-\Phi}\sqrt{\frac{\cos\alpha\cos(\alpha+\delta)}{\cos\delta}}\right)\\
&- d\left(e^{3A-\Phi}\sqrt{\frac{\sin\alpha\cos\alpha\sin\delta}{\cos\delta}}v_1\wedge w_1\wedge w_2\right) = 0,
\end{aligned} \tag{3.68e}$$

$$\begin{aligned}
&e^{3A+3C}\star_4\lambda(G_-) + d_{H_3}\left(e^{3A+3C-\Phi}\sqrt{\frac{\cos\delta\sin\alpha}{\sin(\alpha+\delta)}}\right)\\
&- d\left(e^{3A+3C-\Phi}\cos\alpha\sqrt{\frac{\sin\delta\cos(\alpha+\delta)}{\cos\delta\sin(\alpha+\delta)}}v_2\wedge w_2\right) = 0,\\
&\left(\left(\sqrt{\frac{\sin\alpha\cos\alpha\sin\delta}{\cos\delta}}v_2 - \sqrt{\frac{\cos\alpha\cos(\alpha+\delta)}{\cos\delta}}v_1\wedge w_1\wedge v_2\right)\wedge\lambda(G_-)\right.
\end{aligned} \tag{3.68f}$$

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$$
\begin{aligned}
&-\Big(\sqrt{\frac{\sin\alpha\sin(\alpha+\delta)}{\cos\delta}}-\cos\alpha\sqrt{\frac{\sin\delta\cos(\alpha+\delta)}{\cos\delta\sin(\alpha+\delta)}}v_1\wedge w_1\\
&-\sqrt{\frac{\sin\alpha\cos\delta}{\sin(\alpha+\delta)}}v_1\wedge v_2\wedge w_1\wedge w_2\Big)\wedge\lambda(G_+)\Big)\Big|_4=0,
\end{aligned}
$$

where $\text{Vol}(S^2)$ is the volume form on the S^2 spanned by $(\theta,\ \phi)$. We solve these conditions by first using (3.68b)–(3.68c) to define the vielbein locally without loss of generality as

$$
w_1=\frac{\nu}{2}e^C\sqrt{\frac{\cos\alpha\sin\delta}{\sin\alpha\cos\delta}}d\theta\ ,\quad v_1=-\frac{\nu}{2}e^C\sqrt{\frac{\cos\alpha\sin\delta}{\sin\alpha\cos\delta}}\sin\theta d\phi\ , \tag{3.69}
$$

$$
w_2=-\frac{\nu}{2}e^C\cot\alpha\sqrt{\frac{\sin\delta}{\sin(\alpha+\delta)\cos\alpha}}(d\psi+\cos\theta d\phi)\ ,\quad v_2=\nu e^{-A}d\rho\ , \tag{3.70}
$$

where we have taken θ,ϕ as local coordinates and introduced the additional coordinates ψ and

$$
\rho=e^{A+C}\sqrt{\frac{\cos\alpha\sin\delta}{\sin(\alpha+\delta)}}\ . \tag{3.71}
$$

We can invert this conditions then use (3.68a) to define A,C,Φ,δ in terms of α,ρ and some integration constants c_i as

$$
\begin{aligned}
&e^{-4A}=\frac{c_1^2\cos^2\alpha\sin^2\alpha}{\rho^2}-\frac{c_3\sin^2\alpha}{c_2^2},\quad e^{2C}=c_1^2\sin^2\alpha e^{2A},\\
&e^{-2\Phi}=e^{-4A}\left(c_2^2+\frac{c_3\rho^2}{c_1^2\sin\alpha^2}\right),\quad \cot(\alpha+\delta)=\frac{c_3\rho^2}{c_1^2c_2^2\sin\alpha\cos\alpha}\ .
\end{aligned} \tag{3.72}
$$

The second equality in (3.68d) implies that α is itself a function of ρ only, so we realise that ∂_ψ is an isometry of the solution and that M_4 is foliation of a ($SU(2)\times U(1)$ preserving) squashed 3-sphere over an interval.

We now turn our attention to the fluxes. We have that (3.68d) simply defines the NSNS flux in such a way that it is automatically closed, while the RR fluxes are defined through the 4d fluxes that follow from (3.68e). We could use the definitions of the functions in (3.72) and vielbein in (3.69) to calculate the 10d fluxes immediately, however we already have enough information to first fix α. The 3-form component of $\star_4\lambda(G_-)$ is necessarily parallel to $v_1\wedge w_1\wedge v_2$ from which it follows that the 10d flux F_1 is parallel to w_2. As this vielbein is fibred over the S^2 we have that $dF_1=0$ iff $(\star_4\lambda(G_-))_3=0$. For $\cos(\alpha+\delta)\neq 0$ this imposes $d(c_1^2c_2^2\sin^2\alpha+c_3\rho^2)=0$, which implies that

$$
\sin^2\alpha=c_4-\frac{c_3\rho^2}{c_1^2c_2^2},\quad dc_i=0, \tag{3.73}
$$

and as a result, every function has been solved in terms of ρ and the four integration constants c_i. We are now ready to calculate the fluxes. The non-trivial ones take the form

$$
B=\sqrt{c_3}\frac{1-c_4}{4c_4c_2}\rho^2\text{Vol}(S^2)\ ,\quad F_3=\frac{1}{4}\nu c_1^2c_2(1-c_4)d\psi\wedge\text{Vol}(S^2)+2\nu c_1^2c_2c_4\text{Vol}(S^3)\ , \tag{3.74}
$$

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$$F_5 = B \wedge F_3 + c_1 c_2^2 \nu \sqrt{c_3} d\left(\frac{\rho^2}{2(c_3\rho^2 - c_1^2 c_2^2 c_4)}(d\psi + \cos\theta d\phi)\right)$$

$$+ \frac{\nu c_1^2 \sqrt{c_3} c_4}{2} d\big(\rho^2(d\psi + \cos\theta d\phi)\big) \wedge \mathrm{Vol}(S^3) ,$$

where $dB = H$. Clearly the Bianchi identities of the fluxes are implied automatically and one can show that this is true of (3.68f) also. So this case contains a single example, expressed in terms of 4 integration constants. The 10d metric, warp factor and dilation then take the form

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,2}) + e^{-2A}\bigg[d\rho^2 + \frac{1}{1-c_4}\rho^2 ds^2(S^3) + \frac{\rho^2}{4(c_4 - \frac{c_3}{c_1^2 c_2^2}\rho^2)}(d\psi + \cos\theta d\phi)^2$$

$$+ \frac{\rho^2}{4c_4} ds^2(S^2)\bigg] , \quad e^{-4A} = (1-c_4)\left(\frac{c_1 c_4}{\rho^2} - \frac{c_3}{c_2^2}\right) , \quad e^{-4A+2\Phi} = \frac{1}{c_2^2} - \frac{c_3}{c_1^2 c_2^4 c_4}\rho^2 . \tag{3.75}$$

This solution preserves an $SO(4)$ R-symmetry realised by one $SU(2)$ factor of the round S^3 and the $SU(2)$ of the squashed sphere – the residual symmetries of the spheres make up an $SU(2) \times U(1)$ flavour symmetry. Despite our assumption that $\alpha + \delta \neq \frac{\pi}{2}$ (which is when $c_3 = 0$) when deriving (3.68a)–(3.68f) there is in fact no issue with taking this limit, which merely collapses this solution to that of section 3.2.1. There is good reason for this, as one can actually generate this solution from section 3.2.1 by first T-dualising on ∂_ψ then performing a formal U-duality[6] on the Mink$_3$ followed by another T-duality on ∂_ψ. Additionally, this solution is also contained in section 4.2.2: it can be obtained by imposing that the coordinate x there (which should be identified with ψ in this section) is an isometry and then T-dualising it.

This concludes our IIB classification, we shall now turn our attention towards IIA.

4. Mink$_3$ with an S^3 factor in IIA

The type IIA supersymmetry conditions obtained from plugging (2.14) into the seven-dimensional supersymmetry constraints (2.4) lead to the following constraints on the four-dimensional bispinors

$$d_{H_3}(e^{2A-\Phi}\mathrm{Im}\psi^1_-) = 0 , \tag{4.1a}$$

$$d_{H_3}(e^{2A+2C-\Phi}\psi^2_-) + 2i\nu e^{2A+C-\Phi}\psi^2_{\hat{\gamma}+} = 0 , \tag{4.1b}$$

$$d_{H_3}(e^{3A+2C-\Phi}\psi^2_+) + 2i\nu e^{3A+C-\Phi}\psi^2_{\hat{\gamma}-} = 0 , \tag{4.1c}$$

$$d_{H_3}(e^{2A+2C-\Phi}\mathrm{Re}\psi^1_-) - 2\nu e^{2A+C-\Phi}\mathrm{Im}\psi^1_{\hat{\gamma}+} = 0 , \tag{4.1d}$$

$$d_{H_3}(e^{2A+3C-\Phi}\mathrm{Re}\psi^1_{\hat{\gamma}_+}) - e^{2A+3C-\Phi}H_0\mathrm{Im}\psi^1_- = 0 , \tag{4.1e}$$

[6] T-dualities along the spatial components of Mink$_3$, then a lift to M-theory followed by a boost along the M-theory $U(1)$ before reducing to IIA, and finally undoing the spatial T-dualities. This process needs to be supplemented by rescaling the coordinates along the way, which is why we refer to this as a formal U-duality.

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$$d_{H_3}(e^{3A+2C-\Phi}\mathrm{Im}\psi^1_+) + 2\nu e^{3A+C-\Phi}\mathrm{Re}\psi^1_{\hat\gamma -} = 0\,, \tag{4.1f}$$

while the fluxes are defined through

$$d_{H_3}(e^{3A-\Phi}\mathrm{Re}\psi^1_+) + e^{3A}\star_4\lambda(G_-) = 0\,, \tag{4.2a}$$

$$d_{H_3}(e^{3A+3C-\Phi}\mathrm{Im}\psi^1_{\hat\gamma -}) + e^{3A+3C-\Phi}H_0\mathrm{Re}\psi^1_+ + \nu e^{3A+3C}\star_4\lambda(G_+) = 0\,, \tag{4.2b}$$

and must additionally satisfy

$$\left(\mathrm{Re}\psi^1_{\hat\gamma +}\wedge\lambda(G_+) + \mathrm{Im}\psi^1_-\wedge\lambda(G_-)\right)\Big|_4 = 0 \tag{4.3}$$

As before, we will first examine the 0-form constraints. These are given by two of the three constraints that were found for type IIB:

$$(\psi^2_{\hat\gamma})_0 = (\mathrm{Im}\psi^1_{\hat\gamma})_0 = 0\,. \tag{4.4}$$

Again, the solutions branch off similar to type IIB, with an $\alpha = 0$ and an $\alpha \neq 0$ branch. We parameterise Branch I as in (3.9) and Branch II as in (3.11).

4.1. Branch I: solutions with $\alpha = 0$

As was the case in IIB we first study the lower form conditions that follow from (3.1). After once more rotating the canonical frame of (B.3) by (3.13) we extract the necessary, but not sufficient, supersymmetry conditions

$$d(e^{2A+3C-\Phi}\cos\beta\sin\delta) + e^{2A+3C-\Phi}H_0\cos\beta w_1 = 0\,, \tag{4.5a}$$

$$e^{2A+3C-\Phi}H_0\sin\beta(\cos\delta w_1 - \sin\delta v_2)\wedge v_1\wedge w_2 + \cos\beta(\ldots) = 0\,, \tag{4.5b}$$

$$d(e^{2A-\Phi}\cos\beta w_1) = d(e^{3A+2C-\Phi}\cos\beta\cos\delta) - 2\nu e^{3A+C-\Phi}\cos\beta v_2 = 0\,, \tag{4.5c}$$

$$\cos\beta(\sin\delta v_1 + \frac{\nu}{2}e^C\cos\delta d\theta) = \cos\beta(\sin\delta w_2 - \frac{\nu}{2}e^C\cos\delta\sin\theta d\phi) = 0\,, \tag{4.5d}$$

$$\begin{aligned}&d(e^{2A+2C-\Phi}\sin\beta(\cos\delta w_1 - \sin\delta v_2)) + 2\nu e^{2A+C-\Phi}(\sin\beta w_1\wedge v_2 - \cos\beta\sin\delta v_1\wedge w_2)\\ &\quad - \cos\beta\cos\delta e^{2A+2C-\Phi}(d\theta\wedge w_2 + \sin\theta d\phi\wedge v_1) = 0,\end{aligned} \tag{4.5e}$$

where $\cos\beta(\ldots)$ represents further terms which vanish when $\cos\beta = 0$. These are sufficient to truncate the ansatz considerably. We note from (4.5a) that if $\sin\delta = 0$ then either $H_0 = 0$ or $\cos\beta = 0$, however the latter also leads to $H_0 = 0$ because of (4.5b) – so $\sin\delta = 0$ implies $H_0 = 0$. We also observe that if $\sin\delta = 0$ then (4.5c) requires $d\theta = d\phi = 0$, or naively $\cos\theta = 0$ but this is a subcase of the former (see (3.9)), so $\sin\delta = 0$ also implies $d\theta = d\phi = 0$. Our task now is to show that, as in IIB, $\sin\delta = 0$ is a necessary condition: first we note that if we set $\cos\delta = 0$ then there is no solution as (4.5c) sets the vielbein to zero, thus we can restrict our considerations to $0 < \sin\delta < \frac{\pi}{2}$ where $(w_1,\ v_2)$ must span an S^2. However, as was the case in IIB, (4.5e) can be rewritten as

$$d(e^{2A+2C-\Phi}\sin\beta(\cos\delta w_1 - \sin\delta v_2)) + 2\nu e^{2A+C-\Phi}(\sin\beta w_1\wedge v_2 + \cos\beta\sin\delta v_1\wedge w_2) = 0, \tag{4.6}$$

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using (4.5d) which excludes this because $w_1 \wedge v_2$ gives rise to a $\text{Vol}(S^2)$ term that can not be cancelled by the parts involving (v_1, w_2). Thus we can once more conclude that

$$H_0 = \delta = \theta = \phi = 0\,. \tag{4.7}$$

Given this, we can write the necessary and sufficient solutions for supersymmetry in the $\alpha = 0$ limit in a relatively simple way. After rotating the canonical frame, this time as in (3.16), we find

$$\begin{aligned}
&H_0 = d(e^{2A-\Phi}\cos\beta v_1) = d(e^{\frac{3}{2}A+C-\frac{\Phi}{2}}\sqrt{\cos\beta}) - \nu e^{\frac{3}{2}A-\frac{\Phi}{2}}\sqrt{\cos\beta}\,v_2 = 0\,,\\
&d(e^{2A+2C-\Phi}w) + 2\nu e^{2A+C-\Phi} w\wedge v_2\\
&= d(e^{2A+2C-\Phi}\sin\beta v_1) + 2\nu e^{2A+C-\Phi}\sin\beta v_1\wedge v_2 = 0\,,\\
&d(e^{3A+2C-\Phi}v_1\wedge w) - 2\nu e^{3A+2C-\Phi}v_1\wedge w\wedge v_2 = d(e^{3A-\Phi}\cos\beta)\wedge v_1\wedge v_2 = 0\,,\\
&d(e^{3A+2C-\Phi}\sin\beta w_1\wedge w_2) - 2\nu e^{3A+2C-\Phi}\sin\beta w_1\wedge w_2\wedge v_2 + e^{3A+2C-\Phi}\cos\beta H_3 = 0\,,\\
&e^{2A}H_3\wedge v_1 + \cos^2\beta d(e^{2A}\tan\beta)\wedge v_1\wedge w_1\wedge w_2 = d(e^{-A+\Phi}\sin\beta)\wedge v_1\wedge w_1\wedge w_2\\
&= w\wedge H_3 = 0\,,\\
&d(e^{3A-\Phi}\sin\beta) + d(e^{3A-\Phi}\cos\beta w_1\wedge w_2) - e^{3A-\Phi}\sin\beta H_3 - e^{3A}\star_4\lambda(G_-) = 0\,,\\
&d(e^{3A+3C-\Phi}\sin\beta v_2) + d(e^{3A+3C-\Phi}\cos\beta w_1\wedge w_2\wedge v_2)\\
&- e^{3A+3C-\Phi}\sin\beta H_3\wedge v_2 - e^{3A+3C}\star_4\lambda(G_+) = 0\,,\\
&\Big(\cos\beta v_1\wedge v_2\wedge\lambda(G_+) + (\cos\beta - \sin\beta w_1\wedge w_2)\wedge v_1\wedge\lambda(G_-)\Big)\Big|_4 = 0\,.
\end{aligned}\tag{4.8}$$

This is as far as we can go without making assumptions about β, which we now proceed to do.

4.1.1. *Subcase: $\beta = 0$*

Setting $\beta = 0$ in (4.8) immediately leads to $H_3 = 0$, the rest of the conditions are implied by

$$d(e^{\frac{3}{2}A+C-\frac{\Phi}{2}}) - \nu e^{\frac{3}{2}A-\frac{\Phi}{2}}v_2 = d(e^{2A-\Phi}v_1) = d(e^{-A}w) = 0\,, \tag{4.9a}$$

$$d(e^{2A})\wedge v_1\wedge v_2 = d(e^{-A+\Phi})\wedge w\wedge v_1 = 0\,, \tag{4.9b}$$

$$e^{3A}\star_4\lambda(G_-) - d(e^{3A-\Phi}w_1\wedge w_2) = 0\,, \tag{4.9c}$$

$$e^{3A+3C}\star_4\lambda(G_+) - d(e^{3A+3C-\Phi}w_1\wedge w_2\wedge v_2) = 0\,, \tag{4.9d}$$

$$\Big(v_1\wedge v_2\wedge\lambda(G_+) + v_1\wedge\lambda(G_-)\Big)\Big|_4 = 0\,. \tag{4.9e}$$

The first thing we note is that given (4.9c)–(4.9d), G_+ must be a 0-form and G_- a 1-form which means (4.9e) is solved automatically. Next, we solve (4.9a) by using it to define the vielbein

$$v_1 = e^{-2A+\Phi}g(x)dx,\quad w = e^A(d\psi_1 + i d\psi_2)\,,\quad v_2 = \nu e^{-3A/2+\Phi/2}d\rho,\quad \rho = e^{\frac{3}{2}A+C-\frac{\Phi}{2}}\,, \tag{4.10}$$

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where g is a function parametrising a potential coordinate transformation in x. From eq. (4.9c), we see that the combination $e^{A-\Phi}$ only depends on x and that e^A, e^Φ and e^C are functions of x, ρ only, so that ∂_{ψ_1} and ∂_{ψ_2} are isometries. We thus choose to parametrise

$$e^{A-\Phi} = f(x)\,, \quad g = -f\,, \tag{4.11}$$

the latter of which is a convenient choice we make without loss of generality. For the fluxes, we use the v_1, v_2, w_1, w_2 vielbein on M_4 to compute the Hodge duals from eq. (4.9c) and (4.9d), arriving at the ten-dimensional fluxes

$$F_0 = \partial_x f\,, \quad F_4 = \nu\rho^3\left(f\partial_\rho(f^{-1}e^{-4A})dx - \partial_x(f^{-1}e^{-4A})d\rho\right)\wedge \mathrm{Vol}(S^3)\,. \tag{4.12}$$

The Bianchi identities reduce to $dF_0 = dF_4 = 0$ which leads to

$$\partial_x^2 f = 0\,, \quad \partial_x^2(f^{-1}e^{-4A}) + f\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho(f^{-1}e^{-4A})) = 0\,, \tag{4.13}$$

the former of which can be immediately integrated as $f = (c + F_0 x)$, $dc = 0$. The metric takes the form

$$ds^2 = \frac{1}{\sqrt{fH}}ds^2(\mathbb{R}^{1,4}) + \sqrt{\frac{H}{f}}\left(d\rho^2 + \rho^2 ds^2(S^3)\right) + \sqrt{fH}dx^2, \quad H = f^{-1}e^{-4A}. \tag{4.14}$$

This solution corresponds to an intersecting D4–D8 brane system, where the localised D4-branes are embedded in the D8-branes [40].

4.1.2. *Subcase: $\beta = \frac{\pi}{2}$*

The $\beta = \frac{\pi}{2}$ limit of (4.8) leads to $H_3 \propto v_1 \wedge w_1 \wedge w_2$ with the remaining conditions implied by

$$d(e^{2A+2C-\Phi}u_i) + 2\nu e^{2A+C-\Phi}u_i \wedge v_2 = 0\,, \tag{4.15a}$$

$$d(e^{3A+2C-\Phi}\epsilon_{ijk}u_j \wedge u_k) - 2\nu e^{3A+C-\Phi}\epsilon_{ijk}u_j \wedge u_k \wedge v_2 = 0\,, \tag{4.15b}$$

$$d(e^{3A-\Phi}) - e^{3A-\Phi}H_3 + e^{3A}\star_4\lambda(G_-) = 0\,, \tag{4.15c}$$

$$d(e^{3A+3C-\Phi}v_2) - e^{3A+3C-\Phi}H_3\wedge v_2 + e^{3A+3C}\star_4\lambda(G_+) = 0\,, \tag{4.15d}$$

$$d(e^{-A-\Phi})\wedge v_1\wedge w_1\wedge w_2 = v_1\wedge w_1\wedge w_2\wedge\lambda(G_-)\Big|_4 = 0\,, \tag{4.15e}$$

where we introduce

$$u = (v_1, w_1, w_2), \tag{4.16}$$

to ease notation, and to make clear the cyclic property of these vielbein. The first thing we note is that, given (4.15c), the second of (4.15e) reads $\star_4 H_3 \wedge v_1 \wedge w_1 \wedge w_2 = 0$, but since $H_3 \propto v_1 \wedge w_1 \wedge w_2$ we must set

$$H_3 = 0\,. \tag{4.17}$$

Next one can show that both (4.15a) and (4.15b) together imply the useful identities

$$d(e^A u_i)\wedge u_j = \left[d(e^{A/2+C-\Phi/2}) - \nu e^{A/2-\Phi/2}v_2\right]\wedge u_i \wedge u_j = 0\,, \quad i \neq j, \tag{4.18}$$

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so we must have

$$d(e^A u_i) + \frac{c_i}{2}\epsilon_{ijk} u_j \wedge u_k = d(e^{A/2+C-\Phi/2}) - \nu e^{A/2-\Phi/2} v_2 = dc_i = 0\,. \tag{4.19}$$

Consistency of the first of these with (4.15a) implies that $c_i = 0$, and with this fixed (4.15b) is also followed from (4.19). We can use the standard trick of taking (4.19) to define a vielbein in terms of local coordinates, namely

$$u_i = e^{-A} dx_i\,, \quad v_2 = \nu e^{-A/2+\Phi/2} d\rho\,, \quad \rho = e^{A/2+C-\Phi/2}. \tag{4.20}$$

Having defined the vielbein, it is then a simple matter to solve the first of (4.15e) by introducing a free function

$$e^{-A-\Phi} = f(x_1, x_2, x_3). \tag{4.21}$$

All that remains is to calculate the fluxes, and impose their Bianchi identities. Using (4.20) to take the Hodge duals of 4.15c and 4.15d we find the 10d fluxes

$$F_2 = \frac{1}{2}\epsilon_{ijk}\partial_{x_i} f dx^j \wedge dx^k, \tag{4.22}$$

$$F_6 = -\nu\rho^3\left(\frac{1}{2}\epsilon_{ijk}\partial_{x_i}(f^{-1}e^{-4A})dx^j \wedge dx^k \wedge d\rho - f\partial_\rho(f^{-1}e^{-4A})dx_1 \wedge dx_2 \wedge dx_3\right)$$
$$\wedge \mathrm{Vol}(S^3),$$

which clearly means the Bianchi identities, away from localised sources, follow from

$$\partial^2_{x_i} f = 0\,, \quad \partial^2_{x_i}(f^{-1}e^{-4A}) + f\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho(f^{-1}e^{-4A})) = 0\,. \tag{4.23}$$

The metric takes the form

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{fH}} ds^2(\mathbb{R}^{1,2}) + \sqrt{\frac{H}{f}}\left(d\rho^2 + \rho^2 ds^2(S^3)\right) + \sqrt{fH}(dx_1^2 + dx_2^2 + dx_3^3)\,, \\ H &= f^{-1}e^{-4A}\,. \end{aligned} \tag{4.24}$$

The solution corresponds to an intersecting D2–D6 brane system [44].

4.1.3. Subcase: generic β

For $0 < \beta < \frac{\pi}{2}$ one is able to divide by $\sin\beta, \cos\beta$ freely when simplifying (4.8). Assuming that $\cos\beta \neq 0$ the result is

$$d(e^{2A-\Phi}\cos\beta v_1) = d(e^{-A}\sec\beta w) = 0\,, \tag{4.25a}$$

$$d(e^{\frac{3}{2}A+C-\frac{1}{2}\Phi}\sqrt{\cos\beta}) - \nu e^{\frac{3}{2}A-\frac{1}{2}\Phi}\sqrt{\cos\beta} v_2 = 0\,, \tag{4.25b}$$

$$d(e^{-A+\Phi}\sec\beta) \wedge v_1 \wedge w = d(e^{2A}) \wedge v_1 \wedge v_2 = d(e^{-2A}\tan\beta) \wedge v_1 = 0\,, \tag{4.25c}$$

$$e^{2A}\cos^2\beta H_3 + d(e^{2A}\sin\beta\cos\beta) \wedge w_1 \wedge w_2 = 0\,, \tag{4.25d}$$

$$d(e^{3A+3C-\Phi}\sin\beta v_2) + d(e^{3A+3C-\Phi}\cos\beta w_1 \wedge w_2 \wedge v_2)$$
$$- e^{3A+3C-\Phi}\sin\beta H_3 \wedge v_2 - e^{3A+3C}\star_4\lambda(G_+) = 0,$$

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$$d(e^{3A-\Phi}\sin\beta)+d(e^{3A-\Phi}\cos\beta w_1\wedge w_2)-e^{3A-\Phi}\sin\beta H_3-e^{3A}\star_4\lambda(G_-)=0\,, \tag{4.25e}$$

$$\Big(\cos\beta v_1\wedge v_2\wedge\lambda(G_+)+(\cos\beta-\sin\beta w_1\wedge w_2)\wedge v_1\wedge\lambda(G_-)\Big)\Big|_4=0\,, \tag{4.25f}$$

where H_3 is closed given (4.25a) and (4.25c). As usual we solve (4.25a) and (4.25b) by using them to define a vielbein in terms of local coordinates

$$\begin{aligned}&v_1=g(x)e^{-2A+\Phi}\sec\beta dx,\quad w=e^{A}\cos\beta(d\psi_1+id\psi_2),\\ &v_2=\nu e^{-\frac{3}{2}A+\frac{1}{2}\Phi}\sqrt{\sec\beta}d\rho,\quad \rho=e^{\frac{3}{2}A+C-\frac{1}{2}\Phi}\sqrt{\cos\beta},\end{aligned} \tag{4.26}$$

where $g(x_1)$ is a function parametrising a potential diffeomorphism in x_1. With local coordinates introduced we can solve (4.25c) in terms of them as

$$e^{A-\Phi}\cos\beta=f(x)\,,\quad A=A(\rho,x)\,,\quad \tan\beta=c(x_1)e^{2A}\,,\quad g=-f\,, \tag{4.27}$$

so that ∂_{ψ_i} are necessarily isometry directions. We can then calculate the ten-dimensional fluxes as before – first we note that

$$F_0=\partial_x f+\frac{\partial_x c f\tan^2\beta}{c}\,, \tag{4.28}$$

should be constant. We shall restrict ourselves to the case $\partial_x c=0$. For generic β then the fluxes may be expressed as

$$\begin{aligned}&F_0=\partial_x f,\quad F_2=F_0B_2,\quad B_2=-\frac{\sin^2\beta}{c}d\psi_1\wedge d\psi_2,\quad dc=0\\ &F_4=B_2\wedge F_2+\nu\rho^3\Big(f\partial_\rho(f^{-1}e^{-4A})dx-\partial_x(f^{-1}e^{-4A})d\rho\Big)\wedge \mathrm{Vol}(S^3).\end{aligned} \tag{4.29}$$

We note that the Bianchi identities follow when anything not coupled to B_2 is closed, and since these terms reproduce (4.12) the Bianchi identities imply the PDEs of (4.13) once more. This is because the class of solutions in this section can be generated via U-duality, from the intersecting D4–D8 system in section 4.1.1. For completeness the metric takes the form

$$\begin{aligned}&ds^2=\frac{1}{\sqrt{fH}}ds^2(\mathbb{R}^{1,2})+\frac{\cos^2\beta}{\sqrt{fH}}ds^2(T^2)+\sqrt{\frac{H}{f}}\Big(d\rho^2+\rho^2ds^2(S^3)\Big)+\sqrt{fH}dx^2,\\ &H=f^{-1}e^{-4A},\end{aligned} \tag{4.30}$$

where T^2 is spanned by $(\psi_1,\ \psi_2)$.

4.2. Branch II: α non-zero

For the second branch with $0<\alpha<\pi$, we begin by studying the lower form conditions that follow from (4.1). As in IIB we find it useful to rotate the canonical frame of (B.3) as (3.44). Necessary but insufficient conditions for supersymmetry are

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$$d(e^{2A+3C-\Phi}\cos\beta\cos(\alpha-\delta))+e^{2A+3C-\Phi}H_0(\cos\beta\cos\delta v_2+\sin\beta w_2)=0, \tag{4.31a}$$

$$d(e^{2A-\Phi}(\cos\beta\cos\delta v_2+\sin\beta w_2))=0, \tag{4.31b}$$

$$d(e^{3A+2C-\Phi}\cos\alpha\sin\beta)+2\nu e^{3A+C-\Phi}(\cos\beta\cos\delta w_2-\sin\beta v_2)=0, \tag{4.31c}$$

$$e^C\cos\alpha\sin\beta\sin\theta d\phi-2\nu(\cos\beta\cos\delta w_1-\sin\alpha\sin\beta v_1)=0, \tag{4.31d}$$

$$e^C\cos\alpha\sin\beta d\theta-2\nu(\cos\beta\cos\delta v_1+\sin\alpha\sin\beta w_1)=0, \tag{4.31e}$$

$$\begin{aligned}&d(e^{2A+2C-\Phi}(\sin\alpha\sin\beta v_2-\sin(\alpha-\delta)\cos\beta w_2))\\&+2e^{2A+C-\Phi}\nu\cos\beta(\sin\delta w_2\wedge v_2+\cos(\alpha-\delta)v_1\wedge w_1)=0,\end{aligned} \tag{4.31f}$$

First from (4.31a) we observe that when $\cos(\alpha-\delta)=0$ ($\cos\beta=0$ is a subcase of this) we necessary have $H_0=0$. Next we observe that generically (4.31b)–(4.31e) can be used to locally define the vielbein on M_4, the only exception is when $\sin\beta=0$ ($\cos\alpha=0$ is a subcase of this).

Setting $\sin\beta=0$ means that in order to solve (4.31d)–(4.31e) we must take $\delta=\frac{\pi}{2}$, additionally (θ,ϕ) drop out of the definition of the spinors so we can fix

$$\beta=\theta=\phi=(\delta-\frac{\pi}{2})=0 \tag{4.32}$$

without loss of generality. Interestingly one doesn't need to set $H_0=0$, however as we shall see in section 4.2.1 this case actually contains no solutions.

If one assumes $\sin\beta\neq0$ we see that $v_1,\ w_1$ must span S^2 while v_2, w_2 can be expressed in terms of local coordinates in such a way that they have no legs in S^2. This is a problem for (4.31f) which generically has an Vol(S^2) factor, due to the $v_1\wedge w_1$ term which sits orthogonal to everything else. Thus the only resolution is to fix $\cos(\alpha-\delta)=0$ which leads to $H_0=0$ also. This actually leads to a novel class of solutions that we shall derive in section 4.2.2.

4.2.1. Subcase: $\beta=0$

Upon setting $\beta=0$ we are led to the following conditions for supersymmetry

$$d(e^{2A+3C-\Phi}\sin\alpha)=d(e^{-A-\Phi}\cos^4\alpha)\wedge v_1\wedge w_1\wedge w_2=0\,, \tag{4.33a}$$

$$d(e^{2A+2C-\Phi}\cos\alpha u_i)+\nu e^{2A+C-\Phi}(\sin\alpha\epsilon_{ijk}u_j\wedge u_k+2u_i\wedge v_2)=0\,, \tag{4.33b}$$

$$d(e^{3A+2C-\Phi}(\epsilon_{ijk}u_j\wedge u_k+2\sin\alpha u_i\wedge v_2))-2\nu e^{3A+C-\Phi}\cos\alpha\epsilon_{ijk}u_j\wedge u_k\wedge v_2=0\,, \tag{4.33c}$$

$$\sin\alpha H_3-\cos\alpha H_0 v_1\wedge w_1\wedge w_2=0\,, \tag{4.33d}$$

$$e^{3A}\star_4\lambda(G_-)+d(e^{3A-\Phi})-e^{3A-\Phi}H_0\cot\alpha v_1\wedge w_1\wedge w_2=0\,, \tag{4.33e}$$

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$$e^{3A+3C}\star_4\lambda(G_+)+e^{3A+3C-\Phi}H_0+d(e^{3A+3C-\Phi}\cos\alpha v_2)$$
$$-e^{3A+3C-\Phi}H_0\csc\alpha v_1\wedge v_2\wedge w_1\wedge w_2=0,$$
$$\Big(\cos\alpha v_1\wedge w_1\wedge w_2\wedge\lambda(G_-)-(\sin\alpha-v_1\wedge v_2\wedge w_1\wedge w_2)\wedge\lambda(G_+)\Big)\Big|_4=0\,, \tag{4.33f}$$

where here as elsewhere $u=(v_1,w_1,w_2)$. Using the same techniques as are spelled out in section 3.1.2, it is possible to establish that

$$d(e^{\frac{1}{2}A+C-\frac{1}{2}\Phi})-\nu e^{\frac{1}{2}A-\frac{1}{2}\Phi}\cos\alpha v_2=d(e^{A}u_i)\wedge u_j=0,\quad\text{for } i\neq j \tag{4.34}$$

which we can use as in section 3.2.1 to define the vielbein in terms of the local coordinate $\rho=e^{\frac{1}{2}A+C-\frac{1}{2}\Phi}$ and a set of left invariant 1-forms such that

$$v_2=\nu e^{-\frac{1}{2}A+\frac{1}{2}\Phi}d\rho,\quad u_i=e^{-A}\cos\alpha c_i\tilde{K}_i,\quad d\tilde{K}_i=\frac{1}{2}\tilde{K}_j\wedge\tilde{K}_k, \tag{4.35}$$

under the assumption that $\alpha\neq 0$. Plugging this back into (4.33b) fixes

$$e^{2A}=\frac{c^4e^{-2\Phi}\sin^4\alpha}{\rho^4},\quad \sin\alpha=\frac{c_1}{\rho},\quad c_i=c \tag{4.36}$$

however plugging this back into (4.33a) leads to

$$d\rho\wedge\tilde{K}_1\wedge\tilde{K}_2\wedge\tilde{K}_3=0 \tag{4.37}$$

which cannot be solved.

4.2.2. Subcase: $\delta=\alpha+\frac{\pi}{2}$

The final case we consider is when $\delta=\alpha+\frac{\pi}{2}$ and contains $\beta=\frac{\pi}{2}$ as a subcase. In addition to the rotating the canonical frame (B.3) by (3.44) we find it useful to send $v_1+iw_1\to e^{-i\beta}(v_1+iw_1)$, then the necessary and sufficient conditions for supersymmetry are

$$d(e^{A-C}\sin\alpha)=H_0=0\,, \tag{4.38a}$$
$$d(e^{3A+2C-\Phi}\cos\alpha\sin\beta)-2\nu e^{3A+C-\Phi}k_1=d(e^{2A-\Phi}k_2)=0\,., \tag{4.38b}$$
$$e^{C}\cos\alpha\sin\beta d\theta+2\nu\sin\alpha v_1=e^{C}\cos\alpha\sin\beta\sin\theta d\phi+2\nu\sin\alpha w_1=0\,, \tag{4.38c}$$
$$H_3=\frac{1}{4}\Big(d(e^{2C}\cot^2\alpha\cos\beta\sin\beta)-2e^{C}\nu\cos\alpha w_2\Big)\wedge\text{Vol}(S^2), \tag{4.38d}$$
$$d\left(\frac{e^{-A+C}\sin\alpha}{\Delta}\right)\wedge k_1-e^{2A+C-\Phi}\cos\alpha\sin\beta d\left(\frac{e^{-3A+\Phi}\cos\alpha\cos\beta}{\Delta}\right)\wedge k_2=0\,, \tag{4.38e}$$
$$d\left(\frac{e^{-\frac{3}{2}A+C+\frac{1}{2}\Phi}\cos^{\frac{3}{2}}\alpha\sqrt{\sin\beta}\cos\beta}{\Delta}\right)\wedge k_1$$
$$-e^{\frac{1}{2}A-\frac{1}{2}\Phi}\sqrt{\cos\alpha\sin\beta}d\left(\frac{e^{-2A+C+\Phi}\cot\alpha\sin\beta}{\Delta}\right)\wedge k_2=0\,,$$

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$$e^{3A}\star_4\lambda(G_-)+d(e^{3A-\Phi}\cos\alpha\cos\beta)-d(e^{3A-\Phi}\cos\alpha\sin\beta v_1\wedge w_1)$$
$$-e^{3A-\Phi}\cos\alpha\cos\beta H_3=0\,,$$
$$e^{3A+3C}\star_4\lambda(G_+)+d(e^{3A+3C-\Phi}k_3)-d(e^{3A+3C-\Phi}v_1\wedge w_1\wedge k_1)+e^{3A+3C-\Phi}k_2\wedge H_3, \tag{4.38f}$$

$$\Big(\cos\alpha\sin\beta(1+\cot\beta v_1\wedge w_1)\wedge v_2\wedge w_2\wedge\lambda(G_+)-(k_2-k_4\wedge v_1\wedge w_1)\wedge\lambda(G_-)\Big)\Big|_4=0, \tag{4.38g}$$

where we introduce

$$k_1=(\sin\alpha\cos\beta w_2+\sin\beta v_2),\quad k_2=(\sin\alpha\cos\beta v_2-\sin\beta w_2),$$
$$k_3=(\cos\alpha v_2-\sin\alpha\sin\beta w_2),\quad k_4=(\cos\alpha w_2+\sin\alpha\sin\beta w_1),$$
$$\Delta=\sin^2\beta+\cos^2\beta\sin^2\alpha,\quad \mathrm{Vol}(S^2)=\sin\theta d\theta\wedge d\phi, \tag{4.39}$$

to ease presentation. We can use (4.38b)–(4.38d) to locally define the vielbein through

$$k_1=\nu\sqrt{\cos\alpha\sin\beta}e^{-\frac{3}{2}A+\frac{1}{2}\Phi}d\rho,\quad k_2=e^{-2A+\Phi}dx, \tag{4.40}$$

$$v_1=-\frac{1}{2\sin\alpha}\nu e^{-\frac{3}{2}A+\frac{1}{2}\Phi}\rho\sqrt{\cos\alpha\sin\beta}d\theta,$$

$$w_1=-\frac{1}{2\sin\alpha}\nu e^{-\frac{3}{2}A+\frac{1}{2}\Phi}\rho\sqrt{\cos\alpha\sin\beta}\sin\theta d\phi,$$

where (θ,ϕ,x) and

$$\rho=e^{\frac{3}{2}A+C-\frac{1}{2}\Phi}\sqrt{\cos\alpha\sin\beta} \tag{4.41}$$

are local coordinates on M_4. The ten-dimensional metric then takes the form

$$ds^2= \tag{4.42}$$
$$e^{2A}ds^2(\mathbb{R}_{1,2})+\frac{e^{-3A+\Phi}}{\cos\alpha\sin\beta}\rho^2ds^2(S^3)+\frac{e^{-4A+2\Phi}dx^2}{\Delta}$$
$$+\frac{e^{-3A+\Phi}\cos\alpha\sin\beta}{4\sin^2\alpha}\left(\frac{4\sin^2\alpha d\rho^2}{\Delta}+\rho^2ds^2(S^2)\right).$$

We can now turn our attention to (4.38a) and (4.38e) which lead to the PDEs

$$\partial_\rho(e^{A-C}\sin\alpha)=\partial_x(e^{A-C}\sin\alpha)=0\,, \tag{4.43a}$$

$$\nu\rho\partial_\rho\left(\frac{e^{-3A+\Phi}\cos\alpha\cos\beta}{\Delta}\right)+\partial_x\left(\frac{e^{-A+C}\sin\alpha}{\Delta}\right)=0\,, \tag{4.43b}$$

$$\nu\partial_\rho\left(\frac{e^{-2A+C+\Phi}\cot\alpha\sin\beta}{\Delta}\right)+\partial_x\left(\frac{e^{-\frac{3}{2}A+C+\frac{1}{2}\Phi}\cos^{\frac{3}{2}}\alpha\sqrt{\sin\beta}\cos\beta}{\Delta}\right)=0\,, \tag{4.43c}$$

and tell us that e^{2A}, e^{2C}, e^{Φ}, α, β are functions of $(\rho,\ x)$ only, which means these solutions support an additional $SU(2)$ isometry due to round S^2 spanned by $(\theta,\ \phi)$. Actually this $SU(2)$

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is an additional part of an enhanced R-symmetry which together with the $SU(2)_R$ of S^3 gives $SO(4)_R$ – there is also an $SU(2)$ flavour symmetry. It is now a simple matter to calculate the Hodge dual of the fluxes from (4.38f) and then the fluxes themselves given (4.38d) and our local vielbein (4.40). At first $G_\pm$ take a complicated form that we will not quote here, however, we are yet to impose also (4.38g): doing so and making extensive use of (4.43a)–(4.43c) one can express the ten-dimensional fluxes as:

$$\begin{aligned}
H &= \frac{1}{4}\bigg(d(e^{2C}\cot^2\alpha\cos\beta\sin\beta)\\
&\quad -\frac{2}{\Delta}(\rho e^{-3A+\Phi}\cos\alpha\cos\beta d\rho - \nu e^{-2A+C+\Phi}\cot\alpha\sin\beta dx)\bigg)\wedge \mathrm{Vol}(S^2)\ ,\\
F_0 &= 2\frac{\Delta}{\sin\alpha}e^{-2\Phi}\partial_x(e^{2A}\sin\alpha)\ ,\\
F_2 &= \frac{e^{-3A-\frac{1}{2}\Phi}\rho\sqrt{\cos\alpha}}{4\sin^2\alpha}\Big(F_0 e^{\frac{3}{2}\Phi}\rho\sqrt{\cos\alpha}\cos\beta - 2\nu e^{\frac{3}{2}A}\sin\alpha\sqrt{\sin\beta}\Big)\mathrm{Vol}(S^2)\ ,\\
F_4 &= -\mathrm{Vol}_3\wedge d(e^{3A-\Phi}\cos\alpha\sin\beta) + \frac{e^{-\frac{5}{2}A+\frac{1}{2}\Phi}\rho^2}{\sqrt{\cos\alpha\sin\beta\sin\alpha}}\bigg(d(e^{-2A}\cot\beta)\\
&\quad -\frac{2e^{-\frac{5}{2}A}\sin\alpha}{\sin\beta\Delta}\Big(\nu e^{\frac{1}{2}\Phi}\sqrt{\frac{\sin\beta}{\cos\alpha}}d\rho - e^{\frac{1}{2}A}\sin\alpha\cos\beta dx\Big)\bigg)\wedge\mathrm{Vol}(S^3)\ ,\\
F_6 &= -\mathrm{Vol}_3\wedge\big(d(e^{3A-\Phi}\cos\alpha\sin\beta v_1\wedge w_1) + e^{3A-\Phi}\cos\alpha\cos\beta H_3\big)\\
&\quad +\frac{\nu e^{-10A+2\Phi}\rho^5}{4\sin^2\alpha}\bigg(d(e^{3A-\Phi}\cos\alpha\cos\beta)\wedge d\rho\\
&\quad -\frac{e^{-A+\Phi}}{\cos\alpha\sin\beta}d(e^{3A-\Phi}\cos\alpha\cos\beta)\wedge dx\bigg)\wedge\mathrm{Vol}(S^2)\wedge\mathrm{Vol}(S^3)
\end{aligned} \tag{4.44}$$

where (4.38g) can be expressed in terms of F_0 as

$$\rho\sqrt{\sin\beta}\,\partial_\rho(\rho e^{2A}\sin\alpha) + \frac{\nu}{2}F_0\frac{e^{\frac{1}{2}A+\frac{3}{2}\Phi}\sin^2\alpha\sqrt{\cos\alpha}\cos\beta}{\Delta} = 0. \tag{4.45}$$

Imposing that F_0 is constant together with (4.43a)–(4.43c) and (4.45) then implies $dH = 0$ and the Bianchi identities of the remaining fluxes. This system is quite complicated, however taking inspiration from section 4.1 of [36] (which re-derives [49]) we anticipate that the system can be further simplified if we treat the cases $F_0 = 0$ and $F_0 \neq 0$ separately.

4.2.3. Subcase $F_0 = 0$

If we set $F_0 = 0$ then (4.45) and (4.44) impose that

$$\rho\, e^{2A}\sin\alpha = L^2, \quad dL = 0 \tag{4.46}$$

This leaves (4.43a)–(4.43c) to solve. We first integrate (4.43a) as

$$e^{A-C}\sin\alpha = c, \quad dc = 0, \tag{4.47}$$

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then use it to express (4.43b) as

$$c\partial_x\left(\frac{e^{-5A+\Phi}}{\cos\alpha\sin\beta\Delta}\right)+\nu\frac{1}{\rho}\partial_\rho\left(\frac{e^{-3A+\Phi}\cos\alpha\cos\beta}{\Delta}\right)=0. \tag{4.48}$$

We note that this defines an integrability condition that we can solve by introducing an auxiliary function $h(\rho,x)$ such that

$$c\frac{e^{-5A+\Phi}}{\cos\alpha\sin\beta\Delta}=\frac{\nu}{\rho}\partial_\rho h,\quad \frac{e^{-3A+\Phi}\cos\alpha\cos\beta}{\Delta}=-\partial_x h. \tag{4.49}$$

Plugging these definitions into (4.43c) and making use of (4.46)–(4.47) we arrive at

$$c^2\partial_x^2 h+\frac{1}{\rho}\partial_\rho(\rho\partial_\rho h)=0. \tag{4.50}$$

This is a 3d Laplacian expressed in axially symmetric cylindrical polar coordinates (up to rescaling x). Solution in this class are in one to one correspondence with solution to this Laplace equation. The physical data can be expressed in terms of h and the 2 constants $(c,\ L)$, as

$$L^4e^{-4A}=\rho^2\sin^2\alpha,\quad e^{-\Phi+5A}=\frac{\nu c\rho}{\cos\alpha\sin\beta\partial_\rho h\Delta},\quad \Delta=1-\frac{c^3\rho(\partial_\rho h)^2}{L^4\partial_\rho h(1-c\rho\partial_\rho h)},$$

$$\tan\alpha=\sqrt{\frac{L^4(1-c\rho\partial_\rho h)}{c^4\rho^2(\partial_x h)^2+L^4(1-c\rho\partial_\rho h)^2}-1},$$

$$\tan\beta=\frac{\nu\sqrt{1-c\rho\partial_\rho h}\sqrt{L^4\partial_\rho h(1-c\rho\partial_\rho h)-c^3\rho(\partial_x h)^2)}}{c^{\frac{3}{2}}\sqrt{\rho}\partial_x h}. \tag{4.51}$$

It is interesting that this class depends on axially symmetric Laplacian, indeed the same is true of the class of $AdS_5\times S^2$ solutions in IIA [50] one obtains by dimensionally reducing the M-theory class of Lin–Lunin–Maldecena [51]. The M-theory class actually depend on a 3d Toda equation, which is equivalent to the Laplacian only when one imposes an additional $U(1)$ isometry, which one then uses to reduce to IIA. As the class of this section is in massless IIA it can be lifted to M-theory, so an obvious question poses itself: Is there a class in M-theory governed by a 3d Toda from which the backgrounds in this section descend? It would be interesting to look into this and what connection, if any, this class has to $AdS_5\times S^2$ or indeed any AdS class.

4.2.4. Subcase: $F_0\neq 0$

We expect to be able to perform a similar simplification of the system of PDE's for $F_0\neq 0$ case, however up to this point we have failed to do so in general. However there is a special case which is far more simple, namely $\beta=\frac{\pi}{2}$. Here (4.43a)–(4.43c) and (4.45) force

$$e^{2A}=\frac{\rho}{g(x)^{\frac{1}{4}}\sin\alpha},\quad e^{\Phi-5A}=\frac{\cos\alpha\sin^2\alpha}{c^2\rho^2},\quad d\alpha=dc=0, \tag{4.52}$$

all that remains is to ensure that $dF_0=0$ which is ensured as long as

$$g=\tilde{c}-\frac{2F_0x\cos\alpha}{c_1^4}. \tag{4.53}$$

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This solution has all the generic fluxes except the internal part of F_6 turned on, it bears some resemblance to D8-branes on some sort of cone, but the precise picture depends on what values the free constant α takes.

We shall come back to study the solutions that follow from these massless and massive systems in [71].

5. The unique type II AdS$_4 \times S^3$ background

We have classified all Mink$_3 \times S^3$ with internal Killing spinors of equal norm, up to certain PDE determining various warp factors. As equal norms is a requirement[7] for the existence of AdS$_4$ it is reasonable to ask whether such solutions are contained within our classification. Any AdS$_4$ solution can be expressed as a Mink$_3$ solution, one needs only parametrise AdS as the Poincaré patch. This comes about quite naturally in terms of the Mink$_3 \times M_7$ set up by imposing that

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,2}) + ds^2(M_7) = e^{2\tilde{A}} \left(r^2 ds^2(\mathbb{R}^{1,2}) + \frac{dr^2}{r^2} \right) + ds^2(M_6) \,, \tag{5.2}$$

with $e^{2\tilde{A}}$ and M_6 independent of r. As we have local expressions on $M_7 = S^3 \times M_4$, this makes our task relatively easy. A quick scan through the various cases in section 3 and 4 indicates that the only class that are potentially compatible with AdS$_4$ are in sections 4.1.2 and 4.2.2 – the others manifestly cannot be put in the form (5.2). These are both in IIA, but closer inspection leads one to realise that the class of section 4.2.2 cannot work as (4.38a) would break the putative AdS isometry. This leaves only sections 4.1.2.

We will now show that there is a unique compact[8] AdS$_4$ solution, at least locally, for the class of solutions of section 4.1.2. This background corresponds to a foliation of AdS$_4 \times S^3 \times S^2$ over an interval and is the near-horizon of a D2–D6 brane intersection, and can also be obtained by dimensionally reducing a certain $\mathbb{Z}_k$ orbifold of AdS$_4 \times S^7$. Starting from M-theory one first parameterises S^7 as a foliation of $S^3 \times S^3$ over a closed interval, then performs both the orbifolding and reduction to IIA on the Hopf fibre of one of the S^3's (see e.g. [52]) – there by preserving 16 supercharges. For this purpose, we take the metric (4.24), expressed in the form

$$ds^2 = e^{2A} ds^2(\mathbb{R}^{1,2}) + e^{-A+\Phi} \left(d\rho^2 + \rho^2 ds^2(S^3) \right) + e^{-2A} (dx_1^2 + dx_2^2 + dx_3^3) \,,$$

and assume an AdS$_4$ factor, which requires $e^{2A} = r^2 e^{2\tilde{A}}$, with the rescaled warp factor $\tilde{A}$ as well as the dilaton Φ undetermined functions independent of the AdS$_4$ radial coordinate r. Further-

[7] It is established in Appendix D that the 7d spinors χ_1, χ_2 must obey the relation $|\chi_1|^2 \pm |\chi_2|^2 = c_\pm e^{\pm A}$ where $c_\pm$ are constants. We can, without loss of generality solve these conditions in terms of unit norm spinors χ_i^0 and an angle ζ as

$$\chi_1 = \frac{1}{\sqrt{2}} e^{\frac{A}{2}} \sqrt{1+\sin\zeta}\, \chi_1^0, \quad \chi_2 = \frac{1}{\sqrt{2}} e^{\frac{A}{2}} \sqrt{1-\sin\zeta}\, \chi_2^0, \quad c_+ = 1, \quad c_- = e^{2A} \sin\zeta \tag{5.1}$$

To make a Mink$_3$ solution AdS$_4$ requires us to fix the dependence of e^{2A} on the AdS radius, but since $e^{2A} \sin\zeta$ is constant, we must either set $\zeta = c_- = 0$ or fix ζ such that it also depends on the AdS radius. The latter contradicts the assumption of an $SO(2,3)$ isometry, so we conclude that AdS_4 requires $c_- = 0$; i.e. equal 7d spinor norms.

[8] Strictly speaking section 3.1.1 contains AdS$_5 \times S^5$ which can be expressed as a non-compact AdS$_4$ solution. We will disregard such higher dimensional AdS solutions.

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more, the background is only $SO(2,3)$ invariant if the internal metric is independent of r, which fixes ρ and the x_i to scale as $\rho \sim r^{1/2}$ and $x_i \sim r$. Keeping this in mind, we parametrise

$$
\begin{aligned}
x_1 &= r\, q(\mu)\, \sin\theta\, \cos\phi\,, \\
x_2 &= r\, q(\mu)\, \sin\theta\, \sin\phi\,, \\
x_3 &= r\, q(\mu)\, \cos\theta\,, \\
\rho &= r^{1/2}\, h(\mu)\,,
\end{aligned}
$$

where $q(\mu)$ and $h(\mu)$ are undetermined functions of some coordinate μ, and the (θ,ϕ) directions parametrise a 2-sphere such that the $\mathbb{R}^3$ spanned by x_i is written in polar coordinates with radius $r\,q(\mu)$. Now we have to ensure that the metric is diagonal with respect to the r-direction, i.e. set $g_{r\,\mu} = 0$, and that it shows the $1/r^2$ behaviour for g_{rr}, which amounts to imposing $g_{rr} = e^{2\tilde{A}}/r^2$. These two conditions lead to the following expressions for $\tilde{A}$ and Φ in terms of $q(\mu)$, $h(\mu)$ and independent of (θ,ϕ):

$$
\begin{aligned}
e^{4\tilde{A}} &= q(\mu)\left(q(\mu) - h(\mu)\frac{q'(\mu)}{2h'(\mu)}\right)\,, \\
e^{\Phi} &= -2e^{-\tilde{A}}\,\frac{q(\mu)\,q'(\mu)}{h(\mu)\,h'(\mu)}\,.
\end{aligned}
$$

These expressions imply, once inserted in the first eq. of (4.15e), the following ODE for the q and h functions:

$$
q'(\mu)\left[h'(\mu)^2 + h(\mu)\,h''(\mu)\right] = h(\mu)\,h'(\mu)\,q''(\mu)\,,
$$

which can be solved in closed form as $h = h\big(q(\mu)\big)$ and also implies the Bianchi identities of the fluxes. As h is a function of q, rather than μ, we can use diffeomorphism invariance to fix q such that h is simple, without loss of generality we choose

$$
q(\mu) = \frac{2L^3}{k}\cos^2\left(\frac{\mu}{2}\right)\,,
$$

where L and k are constants. This leads to

$$
h(\mu) = -2L^{3/2}\sin\left(\frac{\mu}{2}\right)\,.
$$

The resulting metric is of the form

$$
ds^2 = \frac{2L}{k}\cos\left(\frac{\mu}{2}\right)\left[ds^2(\mathrm{AdS}_4) + L^2\left(d\mu^2 + 4\sin^2\left(\frac{\mu}{2}\right)ds^2(S^3) + \cos^2\left(\frac{\mu}{2}\right)ds^2(S^2)\right)\right] \tag{5.3}
$$

with fluxes

$$
F_2 = -\frac{k}{2}\mathrm{Vol}(S^2)\,, \quad F_4 = \frac{3}{L}\mathrm{Vol}(\mathrm{AdS}_4)\,. \tag{5.4}
$$

This is the IIA reduction of $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ with length scale L and k D6-branes, as in eq. (2.8) of [52].

The fact that (θ,ϕ) are isometry directions of this solution means that there is an additional $SU(2)_{S^2}$ symmetry due to the round S^2 factor in the metric and fluxes. The spinors of this solution are then charged under $SU(2)_{S^2}$ and just one of the $SU(2)$'s of S^3 (see the ν dependence

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of (4.22)), $SU(2)_+$ say. Since S^2 and S^3 appear as a product the spinors are actually charged under $SU(2)_+ \times SU(2)_{S^2}$ which realises an enhanced $SO(4)$ R-symmetry as required by the $\mathcal{N}=4$ super-conformal algebra in 3d – $SU(2)_-$, under which the spinors are not charged, is a flavour symmetry.

6. Type II with a single Killing spinor

In the previous two sections, we have worked out the supersymmetry conditions making use of the pure spinor equations (2.4), which are valid only in case $|\chi_1|^2 = |\chi_2|^2$. Note that this is a necessary condition for the existence of D-branes which do not break background supersymmetry. The supersymmetry condition for a D$_p$-brane is given by $\Gamma^{(p)}\epsilon_1 = \epsilon_2$. Since $\Gamma^{(p)}$ is unitary, squaring this equation leads to the conclusion that left- and right-handside must have equal norm.[9] We will examine the simplest non-equal norm case, namely the one where

$$\epsilon_2 = 0\,. \tag{6.1}$$

We could either make use of the generalised geometrical reformulation of supersymmetry which incorporates $|\chi_1|^2 - |\chi_2|^2 \neq 0$ as deduced in appendix D, or use the actual Killing spinor equations. Considering the simplicity of this case, we will use the latter.

Much of the work has however already been done: the conditions for seven-dimensional pure NSNS solutions have been deduced in [29,56,57] up to some ansätze. We will merely show that the ansätze made in [29,56,57] (no warp factor, no external NSNS flux) are in fact enforced by supersymmetry, and then proceed to plug in the decomposition resulting from $M_7 = S^3 \times M_4$. This leads to a pair of explicit pure NS backgrounds: the NS5-brane and the U-dual to the IIB conical backgrounds of section 3.2.1. We will also analyse the seven-dimensional RR-sector, which is new, but the conclusion is that all RR-fluxes vanish.

6.1. Seven-dimensional decomposition

Our starting point are the democratic supersymmetry equations, which read as follows for $\epsilon_2 = 0$:

$$\left(\partial\!\!\!/\phi - \frac{1}{2}H\!\!\!\!/\,\right)\epsilon_1 = \left(\nabla_M - \frac{1}{4}H\!\!\!\!/_M\right)\epsilon_1 = 0\,, \quad \lambda F\!\!\!/\,\Gamma_M\epsilon_1 = 0\,. \tag{6.2}$$

As can be seen, the NSNS and RR sectors decouple. We impose a similar $3+7$ decomposition as before: the metric and RR flux is given by (2.1), while the Killing spinor ϵ_1 is given by (2.2) and $\epsilon_2 = 0$. We generalize the NSNS 3-form flux by allowing a term $h\, e^{3A}\mathrm{Vol}_3$.

Using the convention $\gamma_{\mu\nu\rho} = \epsilon_{\mu\nu\rho}$, plugging the above decompositions into (6.2) leads to the following 7d equations:

$$\left(\partial_m\phi\gamma^m - \frac{1}{12}H_{mnp}\gamma^{mnp} + \frac{1}{2}ih\right)\chi = \left(\nabla^{(7)}_m - \frac{1}{8}H_{mnp}\gamma^{np}\right)\chi = 0 \tag{6.3a}$$

$$\left(\frac{1}{4}e^A h - ie^A\partial_m A\gamma^m\right)\chi = 0 \tag{6.3b}$$

[9] The argument is slightly more complicated due to the fact that $Spin(1,9)$ spinors do not admit a non-trivial norm, and hence one should decompose to $Spin(9)$ first. See [53] for details. Also note that strictly speaking, the norms need only be equivalent on the brane.

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$$\lambda f \chi = \lambda f \gamma_m \chi = 0 \, . \tag{6.3c}$$

From (6.3b) it follows that

$$\partial_m A = h = 0 \, . \tag{6.4}$$

As can be seen, the NSNS and RR sector split and can thus be analysed independently.

The existence of a globally defined nowhere-vanishing $Spin(7)$ Majorana spinor χ reduces the structure group of M_7 to G_2. More concretely, the following bilinears can be defined:

$$\varphi_{mnp} = -i \chi^\dagger \gamma_{mnp} \chi \, , \qquad (\star_7 \varphi)_{mnpq} = \chi^\dagger \gamma_{mnpq} \chi \, , \tag{6.5}$$

where we have normalized χ. The other bilinears, i.e., the 1-, 2-, 5- and 6-form vanish. As has been deduced in [29,56,57], (6.3a) can be rewritten in terms of the G_2-structure as

$$d\varphi \wedge \varphi = d(e^{-2\Phi} \star_7 \varphi) = d(e^{-2\Phi} \varphi) - e^{-2\Phi} \star_7 H = 0 \, . \tag{6.6}$$

We will analyse the solution to the NSNS sector by requiring a further splitting of $M_7 = S^3 \times M_4$. On the other hand, we will show that the RR-fluxes vanish for any M_7.

6.2. NSNS sector

Considering the case $M_7 = S^3 \times M_4$, we further decompose the spinor as

$$\chi = \xi \otimes \big(\sin(\alpha/2)\eta + \cos(\alpha/2)\hat{\gamma}\eta\big) + \text{m.c.}, \qquad \eta^\dagger \eta = 1, \qquad \eta^\dagger \hat{\gamma} \eta = 0 \, , \tag{6.7}$$

with m.c. the Majorana conjugate. This leads to a further reduction of the structure group. Since S^3 is parallelisable, it has trivial structure group, leading to a $Spin(4)$ structure group on M_4. Generically, the structure group need not reduce on M_4.[10] In the case where either η_+ or η_- is nowhere vanishing, the structure group reduces to $SU(2)$, in case both are nowhere-vanishing, the structure group is trivial. As everywhere else, our analysis is purely local and we will work with a local trivial structure, parametrising possible vanishing of either chiral spinor by the angle α.

First, as in [58], we make use of an auxiliary $SU(3)$-structure (J, Ω) to express the G_2-structure as

$$\varphi = -v_2 \wedge J - \text{Im}\Omega, \qquad \star_7 \varphi = \frac{1}{2} J \wedge J + \text{Re}\Omega \wedge v_2 \, . \tag{6.8}$$

Next, we decompose the $SU(3)$-structure in terms of the vielbeine as

$$J = -\frac{1}{2}(K_1 \wedge w_1 + K_2 \wedge w_2 + K_3 \wedge v_1), \quad \Omega = e^{i\alpha}\Big(\frac{K_1}{2} + i w_1\Big) \wedge \Big(\frac{K_2}{2} + i w_2\Big) \wedge \Big(\frac{K_3}{2} + i v_1\Big) \tag{6.9}$$

Inserting this into (6.6), one finds

[10] Note that on any M_7 with a spin structure, an $SU(2)$-structure can be found [54] [55]. However, this is not necessarily the structure group defined by the spinors we are making use of, and so even when splitting $M_7 = M_3 \times M_4$ with M_3 parallelisable, M_4 need not admit a globally well-defined $SU(2)$-structure; consider for example $M_7 = S^3 \times S^4$.

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$$d(e^{3C-2\Phi}\cos\alpha v_2)=d(e^{2C-2\Phi}u_i)+\nu e^{C-2\Phi}(2u_i\wedge v_2+\sin\alpha\epsilon_{ijk}u_j\wedge u_k)=0\,, \tag{6.10a}$$

$$d(e^{2C-2\Phi}(\sin\alpha u_i\wedge v_2+\frac{1}{2}\epsilon_{ijk}u_j\wedge u_k))-\nu\epsilon_{ijk}e^{C-2\Phi}\cos\alpha u_j\wedge u_k\wedge v_2=0\,, \tag{6.10b}$$

$$d\alpha\wedge v_1\wedge w_1\wedge w_2=0\,,\quad u=(v_1,w_1,w_2), \tag{6.10c}$$

$$e^{-2\Phi}\star_7 H=-d(e^{-2\Phi}\cos\alpha v_1\wedge w_1\wedge w_2)-\nu\mathrm{Vol}(S^3)\wedge d(e^{3C-2\Phi}\sin\alpha). \tag{6.10d}$$

When $\alpha\neq\frac{\pi}{2}$, by taking linear combinations, exterior derivatives and wedge products with the vielbein of the equations in (6.6), one can derive

$$\begin{aligned}&\epsilon_{ijk}\big[d(e^{C-\Phi}\cos\alpha)-\nu e^{-\Phi}v_2\big]\wedge u_j\wedge u_k\\&+e^{5C+12\Delta}\cos^4\alpha\big[d(e^{-3C+2\Phi}\sec\alpha\tan\alpha)\wedge v_2\big]\wedge u_i=0,\end{aligned}$$

where, since u_i form a basis of independent 1-forms, the terms in square parentheses must vanish. This is sufficient to conclude that

$$d\alpha=0\,,\quad C=C(\rho)\,,\quad \Phi=\Phi(\rho)\,,\quad \rho=e^{C-\Phi}\,. \tag{6.11}$$

It is then not hard to establish that

$$du_i\wedge u_j=0\,,\quad i\neq j, \tag{6.12}$$

in a similar fashion. This means we can locally parametrise

$$v_2=\nu\sec\alpha e^{\Phi}d\rho\,,\quad du_i=c_i\epsilon_{ijk}u_j\wedge u_k\,,\quad dc_i=0\,. \tag{6.13}$$

Plugging this back into (6.6) we find that

$$c_i=e^{-C}\nu\tan\alpha, \tag{6.14}$$

so either $dC=0$ or $c=\alpha=0$.

Case 1: When $\alpha=0$, u_i are the vielbeine of T^3 so we can simply take

$$u_i=dx_i\,. \tag{6.15}$$

All that is left to do is calculate H and impose its Bianchi identity. We find

$$H=\nu\partial_\rho(e^{2\Phi})\rho^3\mathrm{Vol}(S^3)\,. \tag{6.16}$$

Closure of the flux then implies that $e^{2\Phi}$ is harmonic, leading to

$$e^{2\Phi}=g_s^2\left(1+\frac{c}{\rho^2}\right)\,,\qquad g_s,c\in\mathbb{R}\,, \tag{6.17}$$

which is consistent with the definition of ρ. Finally, we note that the metric is given by

$$ds^2=ds^2(\mathbb{R}^{1,5})+e^{2\Phi}\left(d\rho^2+\rho^2ds^2(S^3)\right)\,. \tag{6.18}$$

This is an NS5-brane, dual to the D5-brane solution in section 3.1.2 [41].

Case 2: When $0<\alpha<\frac{\pi}{2}$, u_i span the vielbeine of another S^3 so we take

$$u_i=\frac{e^{\tilde{C}}}{2}\tilde{K}_i,\quad d\tilde{K}_i+\frac{\tilde{\nu}}{2}\epsilon_{ijk}d\tilde{K}_j\wedge d\tilde{K}_k,\quad d\tilde{C}=0\,, \tag{6.19}$$

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where consistency requires that

$$\tilde{\nu} e^{C} \cos\alpha + e^{\tilde{C}} \nu \sin\alpha = 0 \,, \tag{6.20}$$

and find

$$H = -2e^{3C}\partial_\rho(e^{-\Phi})\cos^2\alpha \mathrm{Vol}(S^3) - 2e^{3\tilde{C}}\partial_\rho(e^{-\Phi})\cos\alpha\sin\alpha\mathrm{Vol}(\tilde{S}^3) \,. \tag{6.21}$$

By definition of ρ, the Bianchi identity is satisfied. After redefining $r = \exp\left(e^{-C}\cos\alpha\rho\right)$, it follows that the metric is given by

$$ds^2 = ds^2(\mathbb{R}^{1,2}) + dr^2 + e^{2C}ds^2(S^3) + \cot^2\alpha e^{2C} ds^2(\tilde{S}^3) \,. \tag{6.22}$$

Note that in IIB, this solution can be obtained from the solution of section 3.2.1 by means of the following S-duality transformation (up to redefining some constants):

$$\Phi \to -\Phi \,, \quad ds^2 \to e^{-\Phi} ds^2 \,, \quad F_3 \to -H \,. \tag{6.23}$$

6.3. RR-sector

The NSNS sector has been analysed by imposing a further decomposition $Spin(7) \to Spin(3) \times Spin(4)$ on the spinor. On the other hand, we will analyse the RR-sector in full generality.

Let us consider the RR-flux constraints equations (6.3c), repeated here for convenience:

$$\lambda f \chi = \lambda f \gamma_m \chi = 0 \,. \tag{6.24}$$

For type IIA, we have that

$$\lambda f = f_0 + -f_2 - i f_3 - i f_1 \,, \tag{6.25}$$

where we have defined $f_3 = \star_7 f_4$, $f_1 = \star_7 f_6$. For type IIB, one finds

$$\lambda f = f_1 - f_3 - i f_2 - i f_0 \,, \tag{6.26}$$

with $f_2 = \star_7 f_5$, $f_0 = \star_7 f_7$, hence up to some field redefinitions, the supersymmetry constraints are identical. The fluxes, a priori irreducible representations of $SO(7)$, decompose into representations of G_2 as follows: $\mathbf{7} \to \mathbf{7}$, $\mathbf{21} \to \mathbf{7} + \mathbf{14}$, $\mathbf{35} \to \mathbf{1} + \mathbf{7} + \mathbf{27}$. Concretely, we parametrise

$$\begin{aligned} f_{2|mn} &= \varphi_{mnp} f_2^p + f_{2|mn} \\ f_{3|mnp} &= f_3 \varphi_{mnp} + \psi_{mnpq} f_3^q + f_{3|q[m}\varphi_{np]}{}^q \,, \end{aligned} \tag{6.27}$$

where the $\mathbf{14}$ satisfies $f_{2|mn} = \frac{1}{2}\psi_{mnpq} f_2^{pq}$ and the $\mathbf{27}$ is corresponded to a symmetric traceless 2-tensor. Furthermore, we have introduced the notation $\psi = \star_7\varphi$ for convenience. Making use of the G_2-structure identities (A.5), (A.6), we find

$$\begin{aligned} f_1\chi &= f_{1|m}\gamma^m\chi & f_1\gamma_m\chi &= f_{1|m}\chi - i\varphi_{mnp} f_1^n \gamma^p \chi \\ f_2\chi &= 3i f_{2|m}\gamma^m\chi & f_2\gamma_m\chi &= 3i f_{2|m}\chi - \varphi_{mnp} f_2^n\gamma^p\chi - 2 f_{2|mn}\gamma^n\chi \\ f_3\chi &= 7i f_3\chi - 4 f_{3|m}\gamma^m\chi & f_3\gamma_m\chi &= -i f_3\gamma_m\chi + 4 f_{3|m}\chi + 6i f_{3|mn}\gamma^n\chi \,. \end{aligned} \tag{6.28}$$

Inserting the above into (6.24) and comparing representation by representation, it follows that all RR fluxes vanish.

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Appendix A. Conventions & identities

We decompose ten-dimensional gamma matrices as

$$\Gamma_\mu = \sigma_3 \otimes e^A \gamma_\mu \otimes \mathbb{I}\,, \quad \Gamma_m = \sigma_1 \otimes \mathbb{I} \otimes \gamma_m\,, \tag{A.1}$$

and seven-dimensional gamma matrices as

$$\gamma^{(7)}_\alpha = e^C \sigma_\alpha \otimes \hat{\gamma}\,, \quad \gamma^{(7)}_a = \mathbb{I} \otimes \gamma_a, \quad B_7 = \sigma_2 \otimes B_4, \quad B_4 B_4^* = -\mathbb{I}\,. \tag{A.2}$$

A.1. M_7

We consider gamma matrices satisfying

$$\gamma_{mnpqrst} = i\epsilon_{mnpqrst}\,, \quad \gamma_{(7-n)} = (-1)^{\frac{1}{2}n(n-1)} i \star_7 \gamma_{(n)}\,. \tag{A.3}$$

A nowhere-vanishing $Spin(7)$ spinor defines a G_2-structure on M_7 by means of the bilinear

$$\varphi_{mnp} = -i\chi^\dagger \gamma_{mnp}\chi\,. \tag{A.4}$$

Defining $\psi = \star_7\phi = \chi^\dagger\gamma_{mnpq}\chi$, the G_2-structure satisfies the following identities [59]:

$$\begin{aligned} \psi_{mnrs}\psi_{pq}{}^{rs} &= -2\psi_{mnpq} + 4\delta_{mp}\delta_{nq} - 4\delta_{mq}\delta_{np}\,,\\ \psi_{mnrs}\varphi_p{}^{rs} &= -4\varphi_{mnp}\,,\\ \varphi_{mrs}\varphi^{nrs} &= 6\delta^n_m\,, \end{aligned} \tag{A.5}$$

as well as

$$\begin{aligned} \gamma_{mn}\chi &= i\varphi_{mnp}\gamma^p\chi\,,\\ \gamma_{mnp}\chi &= i\varphi_{mnp}\chi - \psi_{mnpq}\gamma^q\chi\,,\\ \gamma_{mnpq}\chi &= -4i\varphi_{[mnp}\gamma_{q]}\chi + \psi_{mnpq}\chi\,. \end{aligned} \tag{A.6}$$

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A.2. S^3

We consider Pauli-matrices playing the role of gamma-matrices. They satisfy

$$\sigma_{\alpha\beta\gamma} = i\epsilon_{\alpha\beta\gamma} \ , \quad \sigma_{(3-n)} = (-1)^{\frac{1}{2}n(n-1)} \star_3 \sigma_{(n)} \ , \tag{A.7}$$

with $\alpha, \beta, \gamma = 1, 2, 3$ indices on S^3. The non-vanishing spinor bilinears of S^3 are given by

$$\xi^{c\dagger}\sigma^\alpha\xi = \frac{1}{2}\left(K_1^\alpha + iK_2^\alpha\right) \ , \quad \xi^\dagger\sigma^\alpha\xi = \frac{1}{2}K_3^\alpha \ . \tag{A.8}$$

The real 1-forms K_i, $i = 1, 2, 3$, define a trivial structure on S^3 (i.e., a vielbein, up to normalisation). Note that S^3 is parallelisable and hence the trivial structure is globally well-defined. We will always normalise the volume form as

$$K_1 \wedge K_2 \wedge K_3 = -8\text{Vol}(S^3) \ , \tag{A.9}$$

regardless of which specific vielbein is used.

Appendix B. The bispinors of M_4

We consider gamma matrices satisfying

$$\gamma_{abcd} = \epsilon_{abcd} \ , \quad \gamma_{(4-n)} = (-1)^{\frac{1}{2}n(n+1)} \star_4 \gamma_{(n)}\hat{\gamma} \ , \tag{B.1}$$

with $\hat{\gamma} = \gamma_{1234}$ the chirality matrix. Given a globally well-defined nowhere vanishing chiral spinor η_+, one can construct the bilinears

$$J_{ab} = i\eta_+^\dagger\gamma_{ab}\eta_+ \ , \quad \omega_{ab} = i\eta_+^{c\dagger}\gamma_{ab}\eta_+ \tag{B.2}$$

which furnish an $SU(2)$-structure. Given two globally well-defined nowhere vanishing chiral spinors of opposite chirality, the structure group reduces to a trivial structure [60]. Generically, supersymmetry requires a nowhere vanishing spinor η, which can admit a chiral locus. This ensures that, although the structure group of M_4 cannot be globally reduced, it is possible to reduce the structure group of the generalised cotangent bundle $TM_4 \oplus T^*M_4$ to $SU(2) \times SU(2)$, completely analogously to the well-known situation of $SU(3)$-structures [30]. Since the supersymmetry constraints are local, we will always work with the vielbeine determining the local trivial structure. Using the conventions of [61] with $\eta = (\eta_+, \eta_-)$, we set

$$v = v_1 + iv_2 = \eta_-^\dagger\gamma_a\eta_+ dx^a, \quad w = w_1 + iw_2 = \eta_-^{c\dagger}\gamma_a\eta_+ dx^a \ . \tag{B.3}$$

Although some care must be taken on the chiral locus, where the above 1-forms all vanish, it turns out that no solutions exist on the chiral locus, as discussed in sections 3 and 4. We can expand the locally defined 4d components of the Killing spinors $\eta_{1,2}$ in terms of η as

$$\eta^\dagger\eta = 1 \ , \quad \eta^\dagger\hat{\gamma}\eta = 0 \tag{B.4}$$

as

$$\eta_1 = \cos\left(\frac{\alpha}{2}\right)\eta + \sin\left(\frac{\alpha}{2}\right)\hat{\gamma}\eta \ , \quad \eta_2 = a\eta + b\hat{\gamma}\eta + c\eta^c + d\hat{\gamma}\eta^c \tag{B.5}$$

where a, b, c and d are subject to

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1 \ . \tag{B.6}$$

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We can then calculate the 4d bispinors appearing in (3.1a)–(3.1f) and (4.1a)–(4.1f), where to do so we find it useful to parametrise

$$a = a_1 + ia_2\,, \quad b = b_1 + ib_2\,, \quad c = c_1 + ic_2\,, \quad d = d_1 + id_2\,, \tag{B.7}$$

for a_i, b_i, c_i and d_i real. However we first note that in both IIA and IIB we must solve the 0-form constraints

$$(\psi^2_{\hat{\gamma}})_0 = (\mathrm{Im}\psi^1_{\hat{\gamma}})_0 = 0\,, \tag{B.8}$$

which reduce to

$$b_2 \cos\frac{\alpha}{2} + a_2 \sin\frac{\alpha}{2} = d_2 \cos\frac{\alpha}{2} + c_2 \sin\frac{\alpha}{2} = d_1 \cos\frac{\alpha}{2} + c_1 \sin\frac{\alpha}{2} = 0\,. \tag{B.9}$$

We can solve these in general by fixing

$$\begin{aligned}
&a_2 = \lambda_1 \cos(\frac{\alpha}{2}) \qquad b_2 = -\lambda_1 \sin(\frac{\alpha}{2}) \qquad c_1 = \lambda_2 \cos(\frac{\alpha}{2})\,,\\
&c_2 = -\lambda_3 \cos(\frac{\alpha}{2}) \qquad d_1 = -\lambda_3 \sin(\frac{\alpha}{2}) \qquad d_2 = -\lambda_1 \sin(\frac{\alpha}{2})\,,
\end{aligned} \tag{B.10}$$

which turns (B.6) into

$$a_1^2 + b_1^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1. \tag{B.11}$$

In terms of this parametrisation the 4d bispinors are given by

$$\begin{aligned}
\psi^1_+ &= a_1 - i\,\lambda_1 - i\,b_1 v_1 \wedge v_2 - (\lambda_2 - i\lambda_3) v_1 \wedge (w_1 - i\,w_2) + (i\,a_1 + \lambda_1) w_1 \wedge w_2\\
&\quad + b_1 v_1 \wedge v_2 \wedge w_1 \wedge w_2\,,\\
\psi^1_- &= (a_1 - i\,\lambda_1) v_1 - i\,b_1 v_2 - (\lambda_2 - i\,\lambda_3)(w_1 - i\,w_2) + (i\,a_1 + \lambda_1) v_1 \wedge w_1 \wedge w_2\\
&\quad + b_1 v_2 \wedge w_1 \wedge w_2\,,\\
\psi^2_+ &= -(\lambda_2 + i\lambda_3) - (a_1 + i\,\lambda_1) v_1 \wedge (w_1 - i\,w_2) - i\,b_1 v_2 \wedge (w_1 - i\,w_2)\\
&\quad - i(\lambda_2 + i\,\lambda_3) w_1 \wedge w_2\,,\\
\psi^2_- &= -(\lambda_2 + i\,\lambda_3) v_1 - (a_1 + i\,\lambda_1)(w_1 - i\,w_2) - i\,b_1 v_1 \wedge v_2 \wedge (w_1 - i\,w_2)\\
&\quad + (\lambda_3 - i\,\lambda_2) v_1 \wedge w_1 \wedge w_2\,,\\
\psi^1_{\hat{\gamma}+} &= b_1 - (i\,a_1 + \lambda_1) v_1 \wedge v_2 - i\,(\lambda_2 - i\,\lambda_3) v_2 \wedge (w_1 - i\,w_2) + i\,b_1 w_1 \wedge w_2\\
&\quad + (a_1 - i\,\lambda_1) v_1 \wedge v_2 \wedge w_1 \wedge w_2\,,\\
\psi^1_{\hat{\gamma}-} &= -b_1 v_1 + (i\,a_1 + \lambda_1) v_2 + (\lambda_3 + i\,\lambda_2) v_1 \wedge v_2 \wedge (w_1 - i\,w_2) - i\,b_1 v_1 \wedge w_1 \wedge w_2\\
&\quad - (a_1 - i\,\lambda_1) v_2 \wedge w_1 \wedge w_2\,,\\
\psi^2_{\hat{\gamma}+} &= (i\,\lambda_2 - \lambda_3) v_1 \wedge v_2 - b_1 v_1 \wedge (w_1 - i\,w_2) - i(a_1 + i\,\lambda_1) v_2 \wedge (w_1 - i\,w_2)\\
&\quad - (\lambda_2 + i\,\lambda_3) v_1 \wedge v_2 \wedge w_1 \wedge w_2\,,\\
\psi^2_{\hat{\gamma}-} &= (\lambda_3 - i\,\lambda_2) v_2 + b_1 (w_1 - i\,w_2) + i(a_1 + i\lambda_1) v_1 \wedge v_2 \wedge (w_1 - i\,w_2)\\
&\quad + (\lambda_2 + i\,\lambda_3) v_2 \wedge w_1 \wedge w_2\,.
\end{aligned} \tag{B.12}$$

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Appendix C. The $SU(2)$ doublets of S^3

There exist two independent spinors on S^3 that obey the Killing spinor relations

$$\nabla_a \xi_\pm = \pm \frac{i}{2} \gamma_a \xi_\pm, \tag{C.1}$$

each of which preserves two supercharges. Additionally the global isometry group of S^3 can be decomposed as $SO(4) = SU(2)_+ \times SU(2)_-$, so S^3 supports two sets of $SU(2)$ Killing vectors $K^i_\pm$, $i = 1, 2, 3$, that are dual to one forms that obey

$$dK^\pm_i \pm \frac{1}{2} \epsilon_{ijk} K^\pm_j \wedge K^\pm_k, \tag{C.2}$$

i.e. the right-/left-invariant forms of $SU(2)$. It is possible to use the spinors on S^3 to construct $SU(2)_\pm$ doublets. Consider the following vector with spinor entries

$$\xi^\alpha_\pm = \begin{pmatrix} \xi_\pm \\ \xi^c_\pm \end{pmatrix}^\alpha . \tag{C.3}$$

These transform under the action of the spinorial Lie derivative as[11]

$$\mathcal{L}_{K^\pm_i} \xi^a_\pm = \pm \frac{i}{2} (\sigma_i)^a{}_b \xi^\beta_\pm, \quad \mathcal{L}_{K^\pm_i} \xi^a_\mp = 0, \tag{C.5}$$

for σ_i the Pauli matrices, which means that $\xi^a_\pm$ transforms as a doublet under local $SU(2)_\pm$ transformations and a singlet under $SU(2)_\mp$.

Appendix D. Supersymmetry conditions for three-dimensional external spacetimes

In [31], supersymmetry conditions for 3+7 dimensional compactifications are given in terms of bispinors. The repackaging of the supersymmetry conditions was done under the following conditions:

- The external space is Minkowski.
- The spinors have equivalent length.
- The NSNS flux H does not have an external component.

In this section, we will look at relaxing the latter two conditions to obtain more general solutions. Our starting point will be the ten-dimensional bispinor description of the supersymmetry constraints, as described in [37]:

[11] The spinorial Lie derivative along a Killing vector K is defined as

$$\mathcal{L}_K \epsilon = K^\mu \nabla_\mu \epsilon + \frac{1}{8} (dK)_{\mu\nu} \gamma^{\mu\nu} \epsilon. \tag{C.4}$$

The easiest way to see that this leads to the claimed transformation property, is to parametrise the vielbein on S^3 as $e^1 = \frac{1}{2} d\theta$, $e^2 = \frac{1}{2} \sin\theta d\phi$, $e^3 = \frac{1}{2}(d\psi + \cos\theta d\phi)$ and take the flat space gamma-matrices to be the Pauli matrices σ_i. Then (C.1) is solved by $\xi_+ = e^{\frac{i}{2}\theta\sigma_1} e^{\frac{i}{2}\phi\sigma_3} \xi^0_+$, $\xi_- = e^{-\frac{i}{2}\psi\sigma_3} \xi^0_-$ for $\xi^0_\pm$ constant 2d spinors. The $SU(2)_\pm$ forms are then precisely $K^+_i = -i\mathrm{Tr}(\sigma_i dg g^{-1})$, $K^-_i = -i\mathrm{Tr}(\sigma_i g^{-1} dg)$ for $g = e^{\frac{i}{2}\phi\sigma_3} e^{\frac{i}{2}\theta\sigma_2} e^{\frac{i}{2}\psi\sigma_3}$. The result is then not hard to show.

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$$\mathrm{d}_H\left(e^{-\Phi}\Psi\right)+\tilde{K}\wedge F+\iota_K F=0 \tag{D.1a}$$
$$\mathrm{d}\tilde{K}=\iota_K H \tag{D.1b}$$
$$\mathcal{L}_K g_{10}=0 \tag{D.1c}$$
$$\left(e_{+1}\cdot\Psi\cdot e_{+2},\Gamma^{MN}\left(\pm d_H\left(e^{-\Phi}\Psi\cdot e_{+2}\right)+\frac{1}{2}e^{\Phi}d^\dagger\left(e^{-2\Phi}e_{+2}\right)\Psi-F\right)\right)=0\,, \tag{D.1d}$$
$$\left(e_{+1}\cdot\Psi\cdot e_{+2},\left(d_H\left(e^{-\Phi}\Psi\cdot e_{+1}\right)-\frac{1}{2}e^{\Phi}d^\dagger\left(e^{-2\Phi}e_{+1}\right)\Psi-F\right)\Gamma^{MN}\right)=0\,, \tag{D.1e}$$

with

$$\Psi=\epsilon_1\otimes\bar{\epsilon}_2\,,\quad K_M=\frac{1}{32}\left(\bar{\epsilon}_1\Gamma_M\epsilon_1+\bar{\epsilon}_2\Gamma_M\epsilon_2\right)\,,\quad \tilde{K}_M=\frac{1}{32}\left(\bar{\epsilon}_1\Gamma_M\epsilon_1-\bar{\epsilon}_2\Gamma_M\epsilon_2\right)\,. \tag{D.2}$$

The final two equations are known as the pairing equation; we refer to [37] for more details, and will follow along the lines of section 4.2 in the following.

We consider the case where the Killing spinors are given by (2.2). Due to the properties of $Spin(1,2)$, we can define

$$\frac{1}{2}\bar{\zeta}\gamma_\mu\zeta=v_\mu\,,\quad \frac{1}{2}\bar{\zeta}\gamma_{\mu\nu}\zeta=(\star_3 v)_{\mu\nu}\,, \tag{D.3}$$

with the other bilinears vanishing. Since we are considering flat space, ζ are covariantly constant, hence $dv=0$. Making use of the spinor decomposition, it follows that

$$8e^{-A}\Psi=v\wedge\Phi_\mp-\star_3 v\wedge\Phi_\pm\,,\quad K=\frac{1}{8}e^A(|\chi_1|^2+|\chi_2|^2)v\,,\quad \tilde{K}=\frac{1}{8}e^A(|\chi_1|^2-|\chi_2|^2)v, \tag{D.4}$$

where we have defined $\Phi_+ + i\Phi_- = 8e^{-A}\chi_1\otimes\chi_2^\dagger$ and $K,\tilde{K}$ should be read as 1-forms in ten dimensions. Using the flux decomposition

$$F=f+e^{3A}\mathrm{Vol}_3\wedge\star_7\lambda(f)\,,\quad H=H_3+e^{3A}h\mathrm{Vol}_3\,. \tag{D.5}$$

We will first solve (D.1c): since by construction, v, K are Killing vectors, we must have

$$|\chi_1|^2+|\chi_2|^2=c_+e^A. \tag{D.6}$$

Next let's consider (D.1b), which leads to

$$c_+e^{3A}h=0\,,\quad |\chi_1|^2-|\chi_2|^2=c_-e^{-A}\,. \tag{D.7}$$

Next, let us consider (D.1a). We find that

$$\begin{aligned}\mathrm{d}_{H_3}\left(e^{3A-\Phi}\Phi_\pm\right)&=c_+e^{3A}\star_7\lambda(f)\\ \mathrm{d}_{H_3}\left(e^{2A-\Phi}\Phi_\mp\right)&=c_-f\,.\end{aligned} \tag{D.8}$$

In addition, the fact that $c_-\neq 0$ does not change the argument of [37], so the pairing equations remain unchanged, leading to

$$(f,\Phi_\mp)=0\,. \tag{D.9}$$

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Appendix E. M-theory

The focus of this paper are backgrounds in type IIA and type IIB. In this appendix, we will discuss M-theory backgrounds on $\mathbb{R}^{1,2} \times S^3 \times M_5$. Given equivalent internal spinor norms, our (massive) IIA classification is complete (up to finding solutions to PDE). Therefore, a significant number of backgrounds one would obtain from a similar analysis of M-theory are those which one can obtain from uplifting our massless IIA backgrounds. Novel solutions from a complete M-theory analysis would be backgrounds satisfying one of the two conditions: either M_5 does not admit an S^1 factor to be integrated out to perform the dimensional reduction to IIA, or the internal component of the Killing spinor on $M_8 = S^3 \times M_5$ is such that after the reduction, the resulting seven-dimensional internal components of the IIA spinors are not of equal norm.

Such a full M-theory classification is beyond the scope of this paper. Instead, we aim to make contact with the literature of M-theory on $\mathbb{R}^{1,2} \times M_8$, which is much studied (see for example [62–67]). We will derive the decomposed supersymmetry conditions, and give several simple classes of solutions.

For $\mathcal{N} = 1$ solutions to the supersymmetry constraints on $\mathbb{R}^{1,2} \times M_8$, the Killing spinor ϵ decomposes as

$$\epsilon = \xi \otimes (\chi_+ + \chi_-) \ , \tag{E.1}$$

where ξ is a Majorana spinor of $Spin(1,2)$ and $\chi_\pm$ are chiral Majorana spinors of $Spin(8)$. Generically, $\chi_\pm$ can have zeroes, and the structure group of M_8 is $SO(8)$, although a $Spin(7)$-structure can be defined on the auxiliary space $M_8 \times S^1$ [66]. In the case where one of the two does not vanish, the structure group reduces to $Spin(7)$ [65]. If both chiral spinors have no zeroes, both of them define a $Spin(7)$-structure: the intersection of the two leads to a reduction of the structure group to G_2.[12] The reduction of the structure group leads to the existence of globally defined invariant tensors. Instead, we will work locally, and consider patches where either one or both are non-zero.

E.1. *Spin*(7) *holonomy*

Let us first examine the case with

$$\epsilon = \zeta \otimes \chi_+ \ . \tag{E.2}$$

Following the conventions of [65], the general solution to the M-theory supersymmetry constraints with these ansätze is that

$$ds^2 = e^{2\Delta} ds^2(\mathbb{R}^{1,2}) + e^{-\Delta} ds^2(M_8) \ , \quad G = \text{Vol}_3 \wedge d(e^{3\Delta}) + F \ , \tag{E.3}$$

where $ds^2(M_8)$ a metric of $Spin(7)$ holonomy. The four-form F lies in the **27** of $Spin(7)$, i.e., it satisfies

$$F^{pqr}{}_m \Psi_{npqr} = 0 \ , \tag{E.4}$$

with $\Psi_{mnpq} = \bar{\chi}\gamma_{mnpq}\chi$ the invariant four-form defining the $Spin(7)$-structure. In addition, the Bianchi identity and equation of motion for F require that F is harmonic and satisfies

[12] Note that this is only the case for two $Spin(7)$-structures defined in terms of opposite chirality spinors. Given two same chirality globally well-defined nowhere vanishing spinors, the structure group instead reduces to $Spin(6) \simeq SU(4)$.

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$$d(\star_8 d(e^{-3\Delta})) + \frac{1}{2} F \wedge F = 0 \tag{E.5}$$

away from M2-brane sources.

Let us now impose $M_8 = S^3 \times M_5$. The internal metric and Killing spinor decompose as

$$ds^2(M_8) = e^{2C} ds^2(S^3) + ds^2(M_5)\,, \quad \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes (\xi \otimes \eta + \xi^c \otimes \eta^c)\,, \tag{E.6}$$

and the gamma matrices decompose as

$$\gamma_\alpha^{(8)} = \sigma_1 \otimes \sigma_\alpha \otimes \mathbb{I}\,, \quad \gamma_a^{(8)} = \sigma_2 \otimes \mathbb{I} \otimes \gamma_a\,, \quad B^{(8)} = \sigma_3 \otimes \sigma_2 \otimes B_5\,, \tag{E.7}$$

with the charge conjugation matrix satisfying $B_5 * = -B_5$, α an index on S^3 and a an index on M_5. The pseudoreal $Spin(5)$ spinor η of unit norm gives rise to an $SU(2)$-structure on M_5, via [68]

$$\eta \otimes \eta^\dagger = \frac{1}{4}(1+V) \wedge e^{-iJ}, \quad \eta \otimes \eta^{c\dagger} = \frac{1}{4}(1+V) \wedge \omega\,. \tag{E.8}$$

The (local) $SU(2)$-structure consists of a real one-form V, real two-form J and a complex two-form ω such that $J \wedge J = \frac{1}{2}\omega \wedge \omega^*$ and $\iota_V J = \iota_V \omega = J \wedge \omega = \omega \wedge \omega = 0$.

By making use of this decomposition, the $Spin(7)$ four-form decomposes in terms of the $\mathbb{I} \times SU(2)$-structure as

$$\Psi = -\frac{1}{2} J \wedge J + \frac{e^{\tilde{C}}}{2} V \wedge (K_1 \wedge \mathrm{Re}\omega + K_2 \wedge \mathrm{Im}\omega + K_3 \wedge J) \tag{E.9}$$

$$- \nu \frac{e^{2\tilde{C}}}{4}(dK_1 \wedge \mathrm{Re}\omega + dK_2 \wedge \mathrm{Im}\omega + dK_3 \wedge J) - \nu e^{3\tilde{C}} V \wedge \mathrm{Vol}(S^3)\,. \tag{E.10}$$

Using the decomposed Ψ, we examine the supersymmetry conditions. First, we consider the flux component F, which we decompose as

$$F = e^{3C}\mathrm{Vol}(S^3) \wedge F_1 + \star_5 \tilde{F}_1\,, \tag{E.11}$$

with F_1, $\tilde{F}_1$ one-forms on M_5. Inserting this and (E.9) into (E.4), it follows from $(m,n) = (a,b)$ that $F_1 \sim \tilde{F}_1$, $F_1^a V_a = 0$ and from $(m,n) = (\alpha, a)$ that $F_1^a \omega_{ab} = F_1^a \omega^*_{ab} = F_1^a J_{ab} = 0$. Hence $F_1 = \tilde{F}_1 = 0$, hence $F = 0$. The equation of motion for the flux (E.5) thus reduces to the following constraint on the warp factors:

$$d(e^{3C} \star_5 d(e^{-3\Delta})) = 0\,. \tag{E.12}$$

Next, the requirement that M_8 is of $Spin(7)$ holonomy is equivalent to the closure of Ψ, which is equivalent to

$$d(e^{3\tilde{C}} V) = d(e^{2\tilde{C}} J) + 2\nu e^{\tilde{C}} V \wedge J = d(e^{2\tilde{C}}\omega) + 2\nu e^{\tilde{C}} V \wedge \omega = d(J \wedge J) = 0\,. \tag{E.13}$$

In general, this means that locally $V = e^{-3C} d\tau$, and we will write

$$ds^2(M_8) = e^{2C} ds^2(S^3) + e^{-6C} d\tau^2 + ds_4^2 \tag{E.14}$$

Let us give some simple classes of examples for which the above conditions are solved.

- In the case that $C = C(\tau)$, the metric ds_4^2 is conformally Calabi–Yau. Let $J = e^{2(W-C)}\tilde{J}$, $\omega = e^{2(W-C)}\tilde{\omega}$. Then provided that we define $W(\tau)$ to satisfy

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$$W' + \nu e^{-4C} = 0\,, \tag{E.15}$$

we find that $d\tilde{J} = d\tilde{\omega} = 0$.

• By taking $C = -\frac{1}{4}\log(4\tau)$, we find that $dJ = d\omega = 0$, hence ds_4^2 is a Calabi–Yau metric.[13] Introducing $\rho = (4\tau)^{1/4}$, the metric reduces to

$$ds^2(M_8) = d\rho^2 + \rho^2 e^{2C} ds^2(S^3) + ds_4^2\,, \tag{E.16}$$

and thus $M_8 = \mathbb{R}^4 \times Y_2$, where Y_2 is a Calabi–Yau two-fold.

• Next, let us examine Sasaki–Einstein structures, as well as a class of generalizations. It can be shown that any five-dimensional Sasaki–Einstein can be defined by means of a set of real forms $(\tilde{V}, \omega_j)$, $j = 1, 2, 3$ with $\tilde{V}$ a one-form and ω_j two-forms. These satisfy [15]

$$d\omega_1 = d(\omega_2 + i\omega_3) + 3i\tilde{V} \wedge (\omega_2 + i\omega_3) = 0\,. \tag{E.17}$$

A more general class of spaces are the so-called hypo manifolds [69], satisfying $d\omega_1 = d(\tilde{V} \wedge \omega_2) = d(\tilde{V} \wedge \omega_3) = 0$, which themselves are a subclass of balanced manifolds [70], satisfying

$$d(\omega_1 \wedge \omega_1) = d(\tilde{V} \wedge \omega_2) = d(\tilde{V} \wedge \omega_3) = 0\,. \tag{E.18}$$

By setting $J = \omega_1$, $e^c V = \tilde{V}$, $\mathrm{Re}\omega = \omega_2$, $\mathrm{Im}\omega = \omega_3$, it follows that any solution to the supersymmetry constraints is a balanced metric. On the other hand, any solution to the supersymmetry constraints which is hypo automatically is such that ds_4^2 is Calabi–Yau. This leads to the conclusion that the spinors do not define a Sasaki–Einstein on M_5, as the base space of a Sasaki–Einstein manifold is not Ricci-flat. Another way to see this is to note that Sasaki–Einstein metrics can be written as a fibration over a Kähler–Einstein base, but it is clear that since the supersymmetry constraints are invariant under permutations of $(J, \mathrm{Re}\omega, \mathrm{Im}\omega)$, ds_4^2 cannot be non-Calabi–Yau Kähler.

E.2. G_2-structure

Next, let us examine the case where both internal chiral Killing spinors are (locally) non-vanishing. Again following [65], the Killing spinor is given by

$$\epsilon = e^{-\Delta}\theta \otimes (\chi_+ + \chi_-)\ , \tag{E.19}$$

leading to the solution

$$ds^2 = e^{2\Delta}\Big(ds^2(\mathbb{R}^{1,2}) + ds^2(M_8)\Big)\,, \quad G = e^{3\Delta}(\mathrm{Vol}_3 \wedge f + F). \tag{E.20}$$

This time, the metric is not of special holonomy. Instead, it allows a G_2-structure with non-trivial torsion. The norms of $\chi_\pm$ can be parametrised as

$$|\chi_\pm|^2 = 1 \pm \sin\zeta\ , \tag{E.21}$$

with $\sin\zeta$ a function of M_8 such that the norms of $\chi_\pm$ are non-vanishing. The bilinears of $\chi_\pm$ defining the G_2-structure are given by

[13] We have redefined τ to absorb the sign ν.

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$$K_m = \frac{1}{\cos\zeta}\chi_+^\dagger \gamma_m^{(8)}\chi_- , \quad \varphi_{mnp} = \frac{1}{\cos\zeta}\chi_+^\dagger \gamma_{mnp}^{(8)}\chi_- . \tag{E.22}$$

In terms of these, the constraints

$$d(e^{3\Delta}\cos\zeta\, K) = 0\,, \tag{E.23a}$$

$$K \wedge d(e^{6\Delta}\iota_K \star_8 \varphi) = 0\,, \tag{E.23b}$$

$$d(e^{12\Delta}\cos\zeta\, \varphi \wedge \iota_K \star_8 \varphi) = 0\,, \tag{E.23c}$$

$$\cos\zeta\, d(\varphi)\wedge\varphi + 4 \star_8 d\zeta - 2\cos\zeta \star_8 f = 0\,, \tag{E.23d}$$

$$d(e^{3\Delta}\sin\zeta) - e^{3\Delta} f = d(e^{6\Delta}\cos\zeta\varphi) + e^{6\Delta}\star_8 F - e^{6\Delta}\sin\zeta\, F = 0, \tag{E.23e}$$

are locally equivalent to the supersymmetry conditions.

Next, we split $M_8 = S^3 \times M_5$, leading to the following decomposition of the metric and flux:

$$ds^2(M_8) = e^{2C} ds^2(S^3) + ds^2(M_5)\,, \quad F = e^{3C} F_1 \wedge \text{Vol}(S^3) + F_4\,. \tag{E.24}$$

The spinors decompose as

$$\chi_+ = \sqrt{1+\sin\zeta}\begin{pmatrix}1\\0\end{pmatrix}\otimes(\xi\otimes\eta_1 + \xi^c\otimes\eta_1^c)\,, \quad \chi_- = \sqrt{1-\sin\zeta}\begin{pmatrix}0\\1\end{pmatrix}\otimes(\xi\otimes\eta_2 + \xi^c\otimes\eta_2^c) \tag{E.25}$$

and the gamma matrices again decompose as (E.7). The $Spin(5)$ spinors can be expanded in a common basis as

$$\eta^1 = \eta, \quad \eta^2 = a_0\eta + a\eta^c + \frac{b}{2}\overline{w}\eta, \quad |a_0|^2 + |a|^2 + |b|^2 = 1 \tag{E.26}$$

where b can be made real by rotating the 1-form w and η is unit norm. We assume $w = w_1 + i w_2$ is locally non-vanishing.
As a result, a second locally non-vanishing 1-form can be defined as

$$u = \frac{1}{2}\iota_{w^*}\omega\,, \tag{E.27}$$

with ω defined as in (E.8). We thus see that the local $SU(2)$-structure defined on M_5 by η reduces further to a trivial structure, with the local vielbein defined by (V, w_1, w_2, u_1, u_2).

We now express the G_2-structure (E.22) in terms of the trivial structure of $S^3 \times M_5$. The first bilinear we calculate is

$$K = \frac{e^C}{2}(\text{Im}a_0 K_1 + \text{Re}a_0 K_2 - \text{Im}a\, K_3) - bu_2 - \text{Re}a\, V, \tag{E.28}$$

and the only way to make this compatible with (E.23a) is to set

$$a_0 = \text{Im}a = 0 \tag{E.29}$$

so that the spinors $\eta_{1,2}$ are nowhere parallel. We are now free to parametrise

$$b = \cos\alpha, \quad \text{Re}a = \sin\alpha \tag{E.30}$$

and rotate to a frame where

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$$K = V. \tag{E.31}$$

Having done this the other bilinears take the form

$$\begin{aligned}
\varphi &= -\cos\alpha e^1 \wedge e^2 \wedge e^3 - e^{3C}\sin\alpha \mathrm{Vol}(S^3) - \nu\frac{e^{2C}}{4}\cos\alpha e^i \wedge dK_i,\\
&- \frac{e^C}{2}K_i \wedge (u_1 \wedge e^i + \frac{1}{2}\epsilon_{ijk}\sin\alpha\, e^j \wedge e^k),\\
\iota_K \star_8 \varphi &= u_1 \wedge (\sin\alpha e^1 \wedge e^2 \wedge e^3 - e^{3C}\cos\alpha \wedge \mathrm{Vol}(S^3))\\
&+ \nu\frac{e^{2C}}{4}(\sin\alpha\, u_1 \wedge e^i + \frac{1}{2}\epsilon_{ijk}e^j \wedge e^k) \wedge dK_i - \frac{e^C}{4}\cos\alpha\epsilon_{ijk}u_1 \wedge e^j \wedge e^k \wedge K_i,
\end{aligned} \tag{E.32}$$

where we have defined

$$e = (w_1,\ w_2,\ -u_2), \tag{E.33}$$

for ease of presentation. Remark that

$$\varphi \wedge \iota_K \star_8 \varphi = 7\nu\,\mathrm{Vol}_7, \tag{E.34}$$

where Vol_7 is the volume form of the manifold spanned by the warped left-invariant forms of S^3 and the vielbein, with orientation

$$\left\{\frac{e^C}{2}K_1, \frac{e^C}{2}K_2, \frac{e^C}{2}K_3, u_1, e^1, e^2, e^3\right\}.$$

Inserting these definitions for φ and $\iota_K \star_8 \varphi$ into (E.23a)–(E.23e) lead to the 5d conditions

$$d(e^{3\Delta}\cos\zeta V) = d(e^{6\Delta+3C}\cos\alpha u_1) \wedge V = d(\frac{e^{-6\Delta-3C}}{\cos^3\alpha\cos^2\zeta}u_1) \wedge e^1 \wedge e^2 \wedge e^3 = 0\,, \tag{E.35a}$$

$$d(e^{6\Delta+2C}\cos\zeta\cos\alpha e^i) + \nu\, e^{6\Delta+C}\cos\zeta(2u_1 \wedge e^i + \sin\alpha\epsilon_{ijk}e^j \wedge e^k) = 0\,, \tag{E.35b}$$

$$\left(d(e^{6\Delta+2C}(\sin\alpha u_1 \wedge e^i + \frac{1}{2}\epsilon_{ijk}e^j \wedge e^k)) + \nu e^{6\Delta+C}\cos\alpha\epsilon_{ijk}u_1 \wedge e^j \wedge e^k\right) \wedge V = 0\,, \tag{E.35c}$$

$$d(e^{-2C}\cos\alpha\epsilon_{ijk}e^j) \wedge e^k \wedge u_1 \wedge V = \epsilon_{ijk}u_1 \wedge du_1 \wedge e^j \wedge e^k = 0\,, \tag{E.35d}$$

$$-2\cos\zeta d\alpha \wedge e^1 \wedge e^2 \wedge e^3 + 2\star_5 d\zeta - \cos\zeta \star_5 f = d(e^{3\Delta}\sin\zeta) - e^{3\Delta}f = 0\,, \tag{E.35e}$$

$$d(e^{6\Delta+3C}\cos\zeta\sin\alpha) + e^{6\Delta+3C}(-\star_5 F_4 + \sin\zeta F_1) = 0\,, \tag{E.35f}$$

$$d(e^{6\Delta}\cos\zeta\cos\alpha e^1 \wedge e^2 \wedge e^3) + e^{6\Delta}(\star_5 F_1 - \sin\zeta F_4) = 0\,, \tag{E.35g}$$

where we have used that $\cos\zeta \neq 0$ and one can show that $\cos\alpha = 0$ is inconsistent with supersymmetry. In addition, the Bianchi identities and equations of motion for the flux reduce to

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$$\begin{aligned} d\left(e^{3\Delta} f\right) = d\left(e^{3\Delta+3C} F_1\right) = d\left(e^{3\Delta} F_4\right) &= 0 \\ d\left(e^{6\Delta+3C} \star_5 F_4\right) - e^{6\Delta+3C} f \wedge F_1 &= 0 \\ d\left(e^{6\Delta} \star_5 F_1\right) - e^{6\Delta} f \wedge F_4 &= 0 \\ d\left(e^{6\Delta+3C} \star_5 f\right) + e^{6\Delta+3C} F_1 \wedge F_4 &= 0\,. \end{aligned} \tag{E.36}$$

Note that the signs are such that supersymmetry together with the Bianchi identities imply the first two equations of motions. All Hodge duals in the above are with respect to the unwarped five-dimensional metric.

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4. Results

In this thesis we have studied a number of AdS solutions to type II supergravity, obtained applying NATD to some relevant backgrounds, in addition to classifying certain Mink_3 solutions. For the newly generated AdS geometries, we systematically extracted information from the supergravity fields, intending to provide some basic hints about the putative dual SCFT. This included the derivation of the quantised charges and holographic central charge[1] or free energy.

In the examples considered, a non-flat Kalb-Ramond B_2 field was generated, carrying NS5-brane charge along non-trivial 2-cycles in the new background. In some cases, the 3-sphere on which NATD was applied shrank to a point in the seed geometry, which was then mapped to a singularity in the dual background. This was interpreted as localized NS5-brane sources putting an end to the space, with the NS-NS sector behaving as that of a smeared NS5 near-horizon geometry close to the singularity, as happens also under ATD.

Advancing along the non-compact direction ρ arising from NATD required applying large gauge transformations repeatedly to the Kalb-Ramond field B_2, as we moved from a certain interval in this direction to the next[2]. Each LGT has the effect of inducing one unit of NS5-brane charge and also shifting the Page charges associated to the RR-fluxes, which are the ones being quantized in the presence of B_2. In particular, if the original background had colour D(p-1)-branes, in the new solution we typically found colour Dp- and D(p+2)-branes wrapping the same directions, up to an extra two-sphere for the higher-dimensional branes. Their charges were related as $N_p = nN_{p+2}$, where n is the number of LGTs applied. Indeed, these two different kind of branes are thought to be associated to the same degrees

[1] The prescriptions used are explained in section 4.2 of [42] and the references therein.

[2] Large gauge transformations are introduced in order to enforce the $0 \leq \left| \int_{\Sigma_2} B_2 \right| \leq 4\pi^2\alpha'$ condition, where Σ_2 is any non-trivial two cycle for which the holonomy of B_2 is non-vanishing. This is a requirement for the partition function of an Euclidean string winding around Σ_2 to be well-defined.

of freedom, in the sense that the charge of the Dp-branes are dissolved in D(p+2)-branes. For this reason, we picked up only the lower-dimensional realization of the colour branes for the holographic interpretation of the new backgrounds.

As a result, brane set-ups read-off from these charges were not only infinite[3] but also presented an increasing number of colour branes as we move along ρ, realizing an infinite linear quiver with nodes of increasing rank.

Precisely, it was for the IIA $\mathcal{N}=1$ AdS_5 and the IIB $\mathcal{N}=4$ AdS_4 solutions, that we also managed to regularize the brane set-ups by an adequate introduction of flavour branes. We could therefore propose consistent, finite dual quiver field theories.

In particular, for the background obtained in [13, 12] applying NATD on $S^3 \hookrightarrow T^{1,1} \to S^2$ of the $\mathrm{AdS}_5 \times T^{1,1}$ gravity dual of the Klebanov-Witten theory [36], we obtained in [24] (section 3.5) a D4-NS5-NS5' brane set-up, with NS5 and NS5'-branes mutually transversal and also perpendicular to the D4-branes, whose number augments every time an NS5 is crossed. This generalises [72], where the number of D4's is constant, but is still expected to realize a mass deformation of some $\mathcal{N}=2$ theory realized by a D4-NS5 system with all NS5-branes parallel among them[4].

The regularisation of this brane set-up and the corresponding linear quiver was achieved through the addition of flavour D6-branes, but only after a specific brane re-ordering, compatible with the fixed point gravity dual. This was necessary for the quiver description to be compatible with a mass deformation[5], taking as UV theory the $\mathcal{N}=2$ quiver theories of [25]. We relied on the exact field theoretical central charges computed via a-maximization at both UV and IR fixed points [73], which were related by the Tachikawa-Wecht relations [74], as expected for an RG flow triggered by a mass deformation. By construction, the famous 27/32 ratio was recovered in the holographic limit (this is implied in the Tachikawa-Wecht result), when also the field theoretical central charges coincided with their holographic counterparts[6].

[3]The non-compact $\rho \in \mathbf{R}^+$ direction realizes a new field-theory direction along which the colour branes are extended. Not bounding ρ therefore implies an infinite brane set-up.

[4]The mass deformation is realized adding mass terms to the superpotential of the $\mathcal{N}=2$ UV theory, triggering an RG-flow to a $\mathcal{N}=1$ IR fixed-point. In the dual brane picture, one starts with a D4-NS5 intersection with at least two mutually-parallel 5-branes orthogonal to the D4-branes, and rotates one of the NS5's w.r.t. the other (the angle of rotation being related to the mass-deformation parameter) until they end up being perpendicular between them and also to the D4-branes [37].

[5]In particular, a hard cut-off for the brane set-up (introducing flavours at fixed ρ position) was not compatible with a $\mathcal{N}=1$ theory with a quartic superpotential, as expected for the mass deformation of the cubic superpotential of the $\mathcal{N}=2$ theory.

[6]The holographic constrain $a \approx c$ for the 4D central charges holds in the *long quiver*

It is worth stressing that conjecturing the supergravity background found directly after the application of NATD to be related to a D4-NS5-NS5' brane set-up, allowed us to propose a consistent dual field theory, even in this $\mathcal{N}=1$ scenario with less (super)isometries. It was indeed the relation with $\mathcal{N}=2$ Gaiotto-Maldacena geometries, exploited through the $\mathcal{N}=2$ theories proposed in [25], that provided the information we needed for the proposal of the $\mathcal{N}=1$ field theory.

Regarding the previously mentioned IIB $\mathcal{N}=4$ AdS_4 solution, this was obtained in [23] (section 3.4) applying NATD on the 3-sphere of the $\mathrm{AdS}_4 \times S^3 \times S^2$ background arising as the near-horizon limit of a D2-D6 intersection. Supersymmetry was fully preserved by the transformation in this case[7]. The new $\mathrm{AdS}_4 \times S^2 \times S^2$ geometry fitted in the classification of [32], and the Page charges were compatible with a D3-D5-NS5 Hanany-Witten brane set-up. The holographic dictionary of [34] eased the interpretation of the worldvolume theory of the former brane configuration as a three-dimensional Gaitto-Witten $T^{\hat{\rho}}_{\rho}(SU(N))$ theory, the strong coupled fixed-point of which would yield the $\mathcal{N}=4$ SCFT dual. The regularisation of the infinite array of branes was achieved in this case as a hard cut-off by the addition of a certain number of semi-inifinite D5-branes, acting as flavours, in such a way that the linking numbers (read off from the brane set-up as described in [34]) were consistent with the conditions for the field theory to flow to a non-trivial fixed point in the IR.

Still considering AdS_4 spaces, a new $\mathcal{N}=2$ solution in 11D supergravity supported by magnetic G_4 flux was discovered in [43] (section 3.2) applying a chain of a NATD followed by an ATD to the $\mathcal{N}=6$ $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background, the IIA reduction of the M-theory gravity dual of ABJM [58], followed by uplitf to 11D. The new solution is found to be the only explicit example of this class and holographic candidate for the 3d-3d duality, besides the uplift of the Pernici-Sezgin solution. We remark that, unlike the aforementioned cases, supersymmetry is not fully preserved, but reduced from $\mathcal{N}=6$ to $\mathcal{N}=2$ because of the NATD transformation[8]. The new background possesses a $U(1) \approx SO(2)$ isometry interpreted as the geometrical realization of the R-symmetry of the dual SCFT. Indeed, it rotates the pure spinors inducing a $U(1)$-worth of the corresponding G-structure in $\mathcal{N}=1$ language.

limit, i.e. for a large number of nodes or, equivalently, NS5-branes [24] (section 3.5).

[7]The reduction to IIA of the $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ solution already reduced supersymmetry from $\mathcal{N}=8$ to $\mathcal{N}=4$. The resulting spinors were independent of the NATD-directions, so that no further supersymmetry was broken by the transformation.

[8]It was possible to apply the second transformation, in this case an ATD, on an isometric circle of the intermediate background found in [68], under which the Killing spinors were uncharged, thus preserving the remaining supersymmetry.

This novel $\mathcal{N} = 2$ AdS$_4$ solution belongs to the class of [45], which describes $\mathcal{N} = 2$ AdS$_4$ backgrounds with magnetic flux in 11D supergravity whose Killing spinors satisfy the same projection conditions as M5-branes wrapping calibrated 3-cycles (a more general class of AdS$_4$ spaces was engineered in [46], where electric flux was also allowed). The classification could therefore include also AdS$_4$ geometries which are not identifiable as near-horizon limits of wrapped M5-branes. This could be the case for our $\mathcal{N} = 2$ AdS$_4$ solution, as the holographic free energy does not exhibit the expected N^3 scaling, but the $N^{3/2}$ inherited from ABJM[9].

Regarding AdS$_3 \times S^2$ geometries preserving $\mathcal{N} = (0, 4)$ supersymmetry, an extension of the existing classification [55, 45] was motivated by the solution found in [54] (section 3.1) applying NATD on one of the two 3-spheres of the type IIB realization of the AdS$_3 \times S^3 \times S^3 \times S^1$ background, supported by D1 and D5-brane fluxes [75, 76]. Even if the new background belongs to massive IIA, after Abelian T-dualizing twice (on different $U(1)$ isometries), a massless IIA solution could be reached, which allowed for and uplift to 11D supergravity, providing a solution lying outside the Kim class [55].

Taking as an Ansatz 1/4-BPS $SO(2, 2) \times SO(3)$ invariant solutions in 11D supergravity, with the internal spinors realizing an $SU(2)$-structure, three classes were identified in [56] (section 3.3): the six-dimensional internal manifold either has an $SU(3)$-holonomy, i.e. Calabi-Yau, or corresponds to an $SU(2)$-structure manifold with an emergent $U(1)$ or $SU(2)$ isometry. The first case belongs to the Kim class, included in [45], with magnetic G_4 flux only along the S^2 and where the extra $U(1)$ isometry does not correspond to an R-symmetry but to the M-theory circle. The latter case represents the extension of the classification, allowing for a general, but still magnetic G_4 flux supporting an AdS$_3 \times S^2 \times S^2 \times CY_2$ geometry. In this case, the extra $SU(2)$ isometry realizes an emergent R-symmetry, corresponding to a large superconformal algebra [53]. This is in agreement with the holographic central charge, whose scaling with the quantized charges (or levels of the affine $SU(2)$ algebras) does not change under (non-)Abelian T-dualities, nor upon uplift to 11D. A solution within this novel class was found in [54] applying a chain of Abelian T-dualities to the type IIB AdS$_3 \times S^3 \times S^3 \times S^1$. It is worth highlighting that the central charge scales like $c \sim N^2$ instead of N^3 as expected for M5-brane geometries. Furthermore, this solution was not considered in the wrapped M5-brane geometries of [45]. These facts point to an unclear M5-brane origin, similarly to the case of the $\mathcal{N} = 2$ AdS$_4$ solution.

Remark however that our extension of the Kim class still lacks the gener-

[9]NATD does not seem to alter the scaling with the number of colour branes N of holographic quantities computed from the volume of the internal space.

ality required to accommodate the uplift to 11D of the non-Abelian T-dual of $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$, also found in [54], which does include both electric and magnetic components for the G_4 flux.

Concerning the aforementioned issues arising from the discrepancy between the expected and observed scaling for the central charges/free energies with the quantized charges, it was observed that these holographic observables depend on the volume of the internal space. Therefore, the way the regularization is imposed to the NATD-generated background may influence drastically their scaling behaviour. Most typically, a hard cutoff was applied upon computation of such an observable c, with the following effect: if the scaling in the original background was like $c \sim {N_p}^q$ with the N_p the number of colour Dp-branes, then for the new geometry it exhibited the behaviour $c \sim n^3 N_{p+2}^q \sim n^{3-q} {N_p}^q$. Recall that $N_p = n\, N_{p+2}$ with n the number of NS5-branes, due to the Myers dielectric effect relating the Dp and D(p+2) colour branes, as explained at the beginning of this section.

However, a regularization for the geometry other than a hard cut-off may be necessary in order to get a consistent dual field theory. An explicit example of this was given for the new $\mathcal{N} = 4$ AdS_4 background in [23] (section 3.4). Getting the holographic free-energy into the form of the field-theoretical computation of [77], required quite a non-trivial (though physically motivated) continuation of the local geometry obtained directly from NATD.

Let us remark that this regularization-scheme dependence might be causing also the discrepancy of the holographic observables of the new $\mathcal{N} = (0, 4)$ $\mathrm{AdS}_3 \times S^2$ and $\mathcal{N} = 2$ AdS_4 solutions w.r.t. the N^3 behaviour expected for geometries arising from wrapped M5-branes [45]. On the other hand, for $c \sim n^3 N^q$ with n roughly giving the number of NS5-branes, one might still wonder about the possibility of the corresponding geometry arising as a near-horizon limit of some configuration of wrapped M5-branes.

The most general class of solutions considered in this thesis were type II $\mathcal{N} > 1$ backgrounds on $\mathrm{Mink}_3 \times S^3$, for which a classification was initiated in [67] (section 3.6). Several backgrounds arising as near-horizons of well-known brane intersections were recovered in both type IIA and IIB, assuming the two internal spinors to be of equal norm (what is a requirement for AdS spaces), apart from pure NS backgrounds found when setting one of the internal spinors to vanish.

Our main result is that the only type II $\mathrm{AdS}_4 \times S^3$ compact solution is the $\mathcal{N} = 4$ AdS_4 obtained from the near-horizon limit of intersecting D2–D6 branes (also as a IIA reduction of $\mathrm{AdS}_4 \times S^7/\mathbf{Z}_k$ preserving $\mathcal{N} = 4$ supersymmetry). Besides, we report on a novel IIA class of $\mathcal{N} = 4$ solutions on $\mathrm{Mink}_3 \times S^2 \times S^3$ preserving an $SO(4)$ R-symmetry (with no emergent AdS_4 factor), with *a priori* all fluxes turned on. Assuming a further $U(1)$

isometry in this new class, ATD can be applied on this circle to get a new IIB background with $\mathrm{Mink}_3 \times S^3 \times S^3_{sq}$, where the squashed 3-sphere admits in general an $SU(2) \times U(1)$ isometry. For the unsquashed case, this solution is connected by S-duality to a new pure NS background on $\mathrm{Mink}_3 \times S^3 \times S^3$. These new solutions were found separately in our classification.

5. Conclusions

Throughout the papers in which this thesis is based, we have investigated the power of non-Abelian T-duality as a solution generating technique in type II supergravity, exploring the AdS/CFT correspondence for the new solutions. These were also used to either probe known classifications, providing explicit geometries, or challenge their generality, motivating extensions thereof.

We focused on NATD applied on $SU(2)$ isometries acting without isotropy in the target space. One of the main strengths of this incarnation of the procedure is that new solutions generated this way may still preserve some supersymmetry, if not all. On the other hand, the NATD transformation produces by construction non-compact spaces even if the isometry group is compact: it exchanges the coordinates associated to this isometry group, which realize a compact manifold[1] in target space, for Lagrange multipliers living in the Lie algebra of the group.

Given that a well-defined SCFT cannot be dual to a geometry with an infinite internal space, trying to assign such a theory to an AdS solution generated with NATD reduces to the question of whether a consistent, physically-motivated regularization can be found for this new, uncompact geometry. We partially answered this question with the particular proposals of [23, 24], see sections 3.4 and 3.5, based on the seminal work of [25].

One of the most important open questions concerning the solution generating technique aspect of NATD in the context of holography is the effect of the transformation on the field theory side. Only systematics known to date is the transformation rule for the RR-fluxes presented in [6]. Already the identification of the relevant d.o.f.s in the putative dual theory is far from straightforward, due to e.g. the scaling behaviour of the central charge/free energy depending on how the regularization of the geometry is performed. An extension of the results so far on quiver completions will probably be necessary before some general pattern might be derived.

In this direction, a straightforward generalization of our results concerning

[1] In this context, "compact manifold" is used to denote a manifold with a finite volume integral.

the compatibility of mass deformations and NATD might be achieved. We will study the effect of the latter on the $\mathbf{CP}^1$-worth of fixed points that may be reached from the $\mathcal{N}=2$ $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$ solution applying different mass deformations. This "moduli space" of fixed points include as particular cases a version of the Pilch-Warner solution [78], gravity dual to the Leigh-Strassler CFT [79], apart from the Klebanov-Witten background studied in this thesis.

Large gauge transformations played a central role in the holographic interpretation of the new gravity backgrounds. They were responsible for the rise in the number of colour and flavour branes at each stack of the brane set-up, introducing also almost all NS5-branes separating these stacks.

Abelian T-duality arises for large values of the radial ρ direction generated with NATD. In this large-ρ limit - equivalent to a large number n of LGTs - for a background generated with NATD we recover the background generated with ATD from the same seed solution, as explained in appendix B of [23], included in section 3.4. In this limit, the NATD central charge/free energy differs from its ATD counterpart by $\mathcal{O}(1/n)$ "corrections". In the spirit of [57], the linear quivers generated with NATD could be seen as some "open up" limit of the circular quivers arising with ATD. This connection has been made clearer for the NATD and ATD backgrounds obtained from the $\mathcal{N}=4$ near-horizon limit of the D2-D6 system. It was shown in [80] that both geometries could be seen as arising in different limits of the same circular quiver construction. It would be interesting to see if this result underlies a more general phenomenon, extensible to AdS/CFT pairs in other dimensions.

We remarked in the previous section that the classification of $\mathcal{N}=(0,4)$ $\mathrm{AdS}_3 \times S^2$ geometries achieved in [56] (section 3.3) still could not accommodate the uplift of the non-Abelian T-dual of the $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ solution. We require the internal spinors to realize an identity structure, which is beyond our $SU(2)$-structure Ansatz, in order to be able to reproduce both electric and magnetic components for the G_4 flux. A relevant question is whether geometries described by this would-be classification could realize some M2-M5 brane intersection unknown to date. Furthermore, the possible M5-brane origin of new geometries with purely magnetic flux remains unclear. A better understanding thereof could allow for the exploitation of the anomaly inflow [81] to calculate corrections to the entropy of the putative black holes, which could then be compared to results from other methods.

Finally, explicit solutions to the IIA class of $\mathcal{N}=4$ $\mathrm{Mink}_3 \times S^3 \times S^2$ backgrounds were left to be found in [67] (section 3.6), including, through U-dualities, type IIB and pure NS realizations. There are both massive and massless IIA candidate solutions compatible with supersymmetry, which look very promising both for finding Mink_3 solutions with compact internal space, but also possibly geometries that asymptote to AdS.

Conclusiones

A lo largo de los artículos en los que se basa esta tesis, hemos investigado el potencial de T-dualidad no abeliana (NATD) como técnica para generar soluciones en supergravedad de tipo II, explorando la correspondencia AdS/CFT para las nuevas soluciones. Éstas también fueron empleadas para sondar las clasificaciones conocidas, proporcionando geometrías explícitas, o poniendo a prueba su generalidad, motivando así la extensión de las mismas.

Nos centramos en NATD aplicada a isometrías $SU(2)$ actuando sin isotropía en el espacio ambiente. Una de las principales ventajas de esta encarnación del procedimiento es que las nuevas soluciones así generadas pueden preservar algo de supersimetría, si no toda. Por otro lado, la transformación NATD produce espacios no compactos por construcción, incluso si el grupo de isometría es compacto: intercambia las coordenadas asociadas al grupo de isometría, las cuales dan lugar a una variedad compacta[2] en el espacio ambiente, por los multiplicadores de Lagrange que son elementos del álgebra del grupo.

Dado que una SCFT bien definida no puede ser dual a una geometría con un espacio interno infinito, la cuestión de intentar asignar una teoría de este tipo a una solución AdS generada con NATD se reduce a si es posible encontrar una regularización consistente y físicamente motivada para esta nueva geometría no compacta. Respondimos, al menos parcialmente, a esta cuestión con las propuestas particulares hechas en [23, 24], véase las secciones 3.4 y 3.5, basándonos en el trabajo seminal de [25].

Una de los problemas abiertos de mayor relevancia concerniente a NATD, como técnica para generar soluciones en el contexto de holografía, es el efecto de la transformación en el lado de la teoría de campos. A día de hoy, sólo las reglas de transformación del sector RR, presentadas en [6], se conocen de forma sistemática. Incluso la identificación de los grados de libertad relevantes en la supuesta teoría dual no es en absoluto evidente, debido p.ej. a la dependencia del escaleo de la carga central o la energía libre en la forma

[2] En este contexto, por “variedad compacta” queremos decir que su integral de volumen es finita.

en que se regulariza la geometría. Probablemente sea necesario extender los resultados actuales en compleciones de *quivers* para poder llegar a derivar algún patrón general.

En esta dirección, una generalización directa de nuestros resultados en lo concerniente a la compatibilidad de deformaciones masivas y NATD se podría conseguir estudiando el efecto de esta última en el conjunto de puntos fijos, de los que hay tantos como elementos en $\mathbf{CP}^1$, que se puede alcanzar aplicando diferentes deformaciones masivas a la solución $\mathcal{N}=2$ $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$. Este "espacio de móduli" de puntos fijos incluye como casos particulares una versión de la solución de Pilch-Warner [78], dual gravitatorio de la CFT de Leigh-Strassler [79], aparte del espacio de Klebanov-Witten estudiado en esta tesis.

Las transformaciones de *large gauge* (LGT) han jugado un papel fundamental en la interpretación holográfica de las nuevas soluciones gravitatorias, ya que son responsables del aumento en el número de branas de color (y de sabor, si las hubiera) en cada pila de la configuración de branas, así como de la introducción de casi todas las branas NS5 que separaban las distintas pilas.

T-dualidad abeliana (ATD) surge para valores altos de la dirección radial ρ generada con NATD. En este límite de ρ alto - equivalente a un gran número n de LGT - para una espacio generado con NATD, se recupera el espacio generado con ATD a partir de la misma solución original, tal y como se explica en el apéndice B de [23], incluido en la sección 3.4. En este límite, la carga central o energía libre derivadas de la aplicación de NATD difiere de su opuesta abeliana por “correciones” de orden $\mathcal{O}(1/n)$. En la línea de razonamiento de [57], se puede ver los *quivers* lineales generados con NATD como un límite en el que los *quivers* circulares dados por ATD se "abren". Esta conexión se hizo más patente para las soluciones NATD y ATD obtenidas del límite *near-horizon* $\mathcal{N}=4$ del sistema D2-D6, mostrándose en [80] que ambas geometrías se podían contemplar como distintos límites de una misma construcción de *quiver* circular. Sería intersante comprobar si este resultado se debe a un fenómeno mas general, extrapolable a pares AdS/CFT en otras dimensiones.

En la sección anterior señalábamos que la clasificación de geometrías $\mathcal{N}=(0,4)$ $\mathrm{AdS}_3 \times S^2$, conseguida en [56] (sección 3.3), aún no podía acomodar la elevación a 11D del T-dual no abeliano de la solución $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$. Se require que los espinores internos den lugar a una estructura identidad, la cual va más allá de nuestro Ansatz con una estructura $SU(2)$, para poder reproducir tanto una componente eléctrica como una magnética para el flujo G_4. Una pregunta de interés es si las geometrías descritas por una tal clasificación podrían provenir de alguna intersección de M2-M5 aún desconocida.

De hecho, el posible origen en branas M5 de las nuevas geometrías con flujo puramente magnético sigue sin estar claro. Una mejor comprensión del mismo podría permitir que se aprovechase la afluencia de anomalías [81] para calcular correciones a la entropía de los presuntos agujeros negros, las cuales se podrían comparar entonces a los resultados obtenidos con otros métodos.

Por último, en [67] (sección 3.6) no se pudieron encontrar soluciones explícitas de la clase IIA de espacios $\mathcal{N} = 4$ $\mathrm{Mink}_3 \times S^3 \times S^2$, que incluye, mediante U-dualidades, versiones de tipo IIB y puramente NS de esas mismas geometrías. Las posibles soluciones, masivas y no masivas de tipo IIA, podrían ser compatibles con supersimetría, y resultan muy prometedoras de cara a encontar tanto geometrías Mink_3 con espacio interno compacto, como posiblemente soluciones que asintoten a AdS.

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